Selected Approaches to Probabilistic Assessment of Fatigue Crack Growth in Metallic Materials with Fairly Reduced Homogeneity

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ABSTRACT. *The aim of the work was to compare different methods of probabilistic assessment of fatigue crack growth (FCG) in specific case of an Al-alloy. Actual FCG data measured in four different specimens of an Al-alloy with fairly reduced homogeneity were evaluated using three differently sophisticated methods of probabilistic assessment, namely (i) assessment on the basis of deterministic integration of FCG dependencies considering statistically evaluated tolerance limits along regression line, (ii) probabilistic assessment using ALIAS HIDA software developed in the framework of a European research and development project, with Monte-Carlo simulations and (iii) highly sophisticated probabilistic assessment according to Lauschmann, published in the past. Estimations performed by the three methods were quite comparable, particularly for not too extreme probability values, between 10 % and 90 %. The examples demonstrate an importance of the use of probabilistic methods in damage tolerance estimations with fatigue cracks.*

INTRODUCTION

Estimation of safety and reliability of engineering structures and components containing cracks or crack-like defects are one of the most important application field of fracture mechanics particularly in components, where limited defects can be accepted due to the component size, their high costs and, first of all, loading character. Such the design philosophy, usually called “damage tolerance”, formerly “safe life”, enables to postpone partial or general repair or put out the structure of operation, which is connected with significant financial savings. In such cases, safety and reliability of further operation, residual life assessment, eventually specification of interval of damage development inspections are important issues.

Linear fracture mechanics is a powerful tool enabling, with a considerable extent, to transfer results measured in standard laboratory specimens to actual structures in operation. In case of cyclic loading, the damage process is described by the well known Paris-Erdogan equation of fatigue crack growth (FCG) rate on stress intensity factor range $da/dN = C\ \Delta K^m$, when dependencies of K-factor on crack length in standard specimens are known and for complicated components, it can be calculated mostly by finite element method or, even better, by boundary integral equations. If inaccuracies caused by different constraint factors are not considered, such transfer of results is basically quite correct.

There is, however, a problem consisting in different type and extent of material inhomogeneity and related scatter of local FCG rate values. In Fig. 1, taken from [1], three different characters of material variability are schematically shown, namely low, medium and high variabilities, whereas this classification is dependent on specimen size, where FCG rate is evaluated. For example, in small size specimens, the case 3 occurs much more frequent than in large scale components of the same material. Therefore, data basis of FCG rate has to be evaluated using more than one specimen and mean FCG rates are eventually statistically evaluated with regression line. An application of purely deterministic crack growth assessment and evaluation of residual life in a real component just on the basis of the regression line is dangerous and irresponsible, because due to the material inhomogeneity and scatter of local FCG rates, such the assessment can be several times more optimistic than the reality. Therefore, probabilistic approaches have been recently further intensively studied and applied particularly for service life of structures and components exploited to the maximum extent, which is typical for recent years [2-5].

In this paper, three probabilistic approaches are applied and compared using an actual set of FCG data measured in a specific Al-alloy: (i) estimation using method of deterministic integration of statistically evaluated tolerance limits along the regression line, (ii) probabilistic assessment using Monte-Carlo simulations as a part of ALIAS HIDA software developed in the project of the EU Framework Programme “HIDA Applicability” [6-9] and (iii) highly sophisticated and exact approach according to Lauschmann [1].

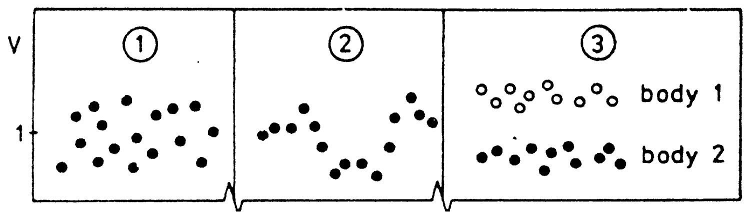

Figure 1. Scheme of three basic types of material variability

PROBABILISTIC ASSESSMENT USING TOLERANCE LIMITS

The work was carried out using the set of experimental points of crack lengths a_i measured as a dependence of corresponding number of cycles N_i in totally four single-edge-notch specimens of an Al-Cu4-Mg1 aircraft alloy [1]. The specimen width was W = 56 mm, thickness 5 mm. All the specimens were loaded with the constant stress range $\Delta\sigma$ = 60 MPa at stress asymmetry R = 0.05.

In the first step of this work, the points (N_i, a_i) were evaluated by standard procedures as the Paris dependence of $da/dN = C\ \Delta K^m$. Then, tolerance limits for different values of ratio of experimental points were evaluated. It could be reminded that, simplifiedly told, tolerance limit (upper or lower) for the specific ratio of points P evaluated statistically with the probability α along regression line divides the area of the graph to two parts, one of them containing the ratio P of experimental points and the other part contains ratio 1-P of the experimental points. The evaluated Paris dependence with the tolerance limits for P = 0.50, 0.90 and 0.99 are in Fig. 2.

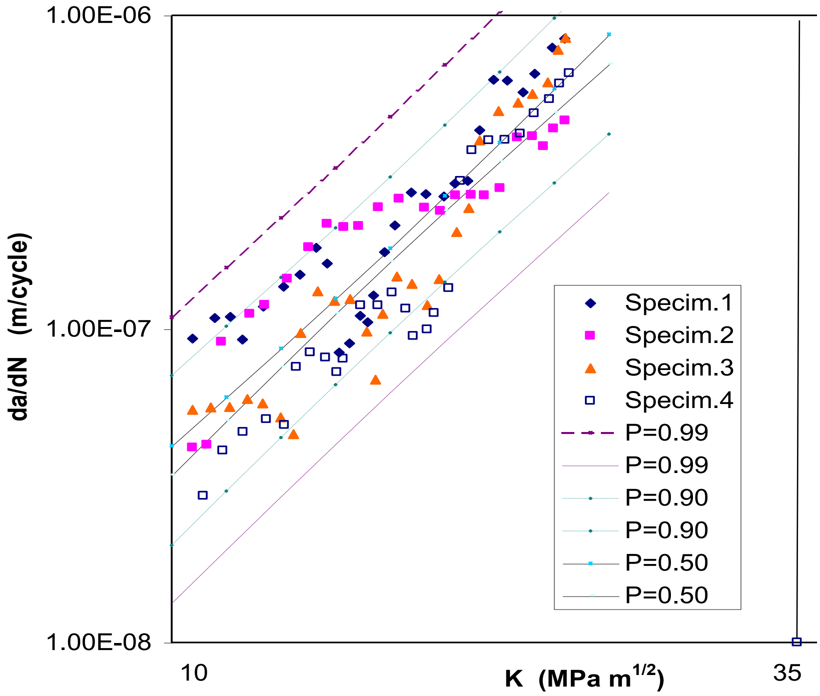

Figure 2. FCG data evaluated as Paris dependence with tolerance limits

The philosophy of FCG probabilistic assessment using tolerance limits appears from the integration of the tolerance limit curve evaluated for the specific P value, i.e. that during integration, points of the tolerance limit are used instead of the values of the mean regression line $da/dN = C\ \Delta K^m$.

Basic equations for statistical evaluation of tolerance limits are generally known [10, 11]. The most complicated part of the evaluation is computation of fractile of uncentral Student´s distribution, which has to be evaluated by two dimensional numerical integration according to two variables.

As a verification example of the probabilistic evaluation, FCG was computed by the integration of tolerance limits for two values of ratio P, namely P = 0.25 and 0.75. In the ideal case and FCG measurement in a large number of specimens, the integrated curves would represent limits, where 75% and 25% of actual crack growth curves, respectively, would be on the left side and right side of the integrated curves, respectively. The results of the verification example are in Figure 3. Though there are measurements only in four actual specimens, the trend and good agreement are quite well indicated.

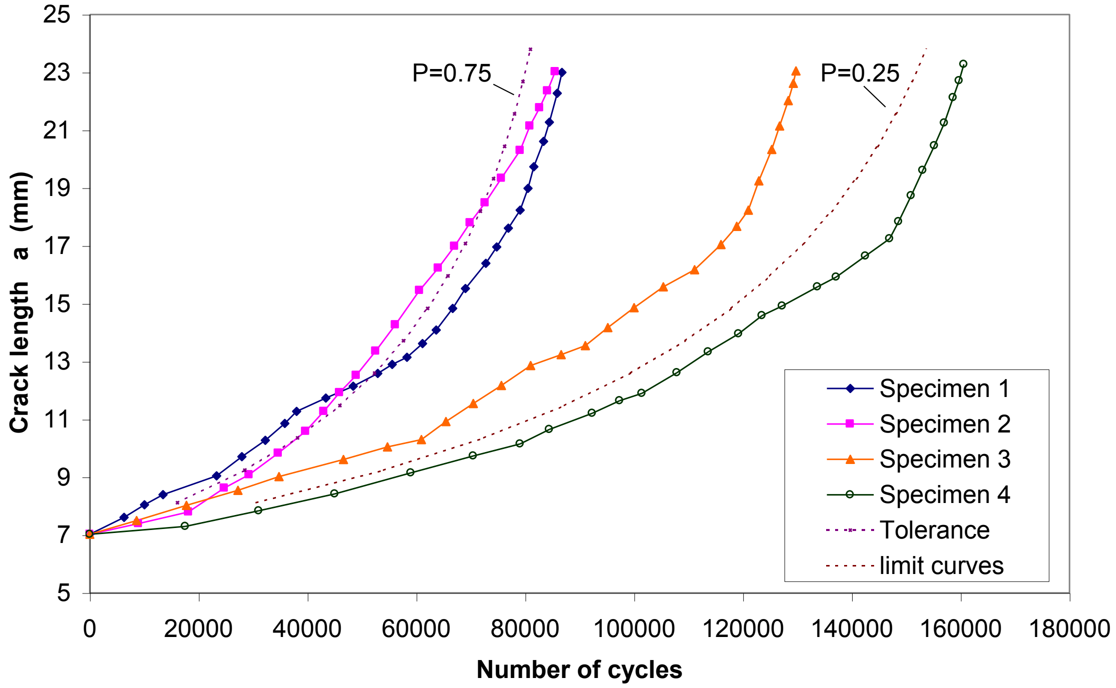

Figure 3. Comparison of actual FCG rates with probabilistic curves calculated by integration of tolerance limits for ratio of points P = 0.25 and 0.75, respectively

PROBABILISTIC ASSESSMENT USING SOFTWARE ALIAS HIDA

The same set of data as in the previous section was used as a basis for verification of possibilities of the use of ALIAS HIDA software for a real FCG probabilistic assessment. In this case, probabilistic computations were performed considering a single random parameter, namely the constant C in the Paris law. Computations using both the parameters, namely C and m as random values do not provide realistic results because of their existing mutual correlation [12]. The statistical set of data of the 106 experimental points in Fig. 2 was evaluated by the following procedure:

- The regression line in logarithmic coordinates, i.e. power regression, common for all the points was evaluated. The corresponding regression coefficients were C = 1.351E-8 (da/dN values expressed in mm/cycle) and m = 3.450, respectively.
- The value m was fixed for further calculations.
- Individual random values C_i of the parameter C were calculated for each individual experimental point [K_i, $(da/dN)_i$] so that the result corresponded to the instantaneous value of crack growth rate $(da/dN)_i$ of the specific point, Eq. 1:

$$C_i = (da/dN)_i / K_i^{3.450} \qquad (1)$$

Actually, the calculated values C_i represent individual values of the parameter C describing individual positions of each of the 106 experimental points in Figure 2, when the value m = 3.450.

During the probabilistic assessment using the ALIAS HIDA software, a possibility to calculate distribution function from the set of data was used. This file was represented by the 106 values of C_i evaluated according to the procedure described above. The best approximation was reached when Weibull distribution was applied – Fig. 4. However, almost the same level of fitting corresponded to logarithmic – normal distribution.

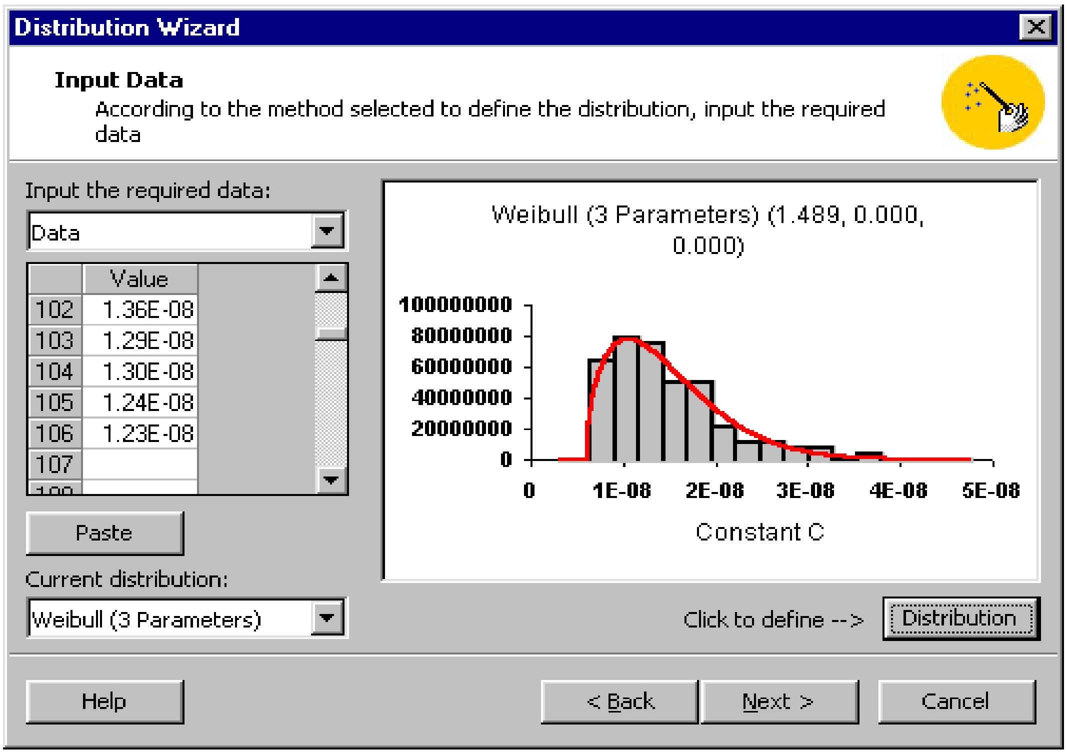

Figure 4. Weibull distribution applied to the set of random values C_i of the parameter C

In the next step, stochastic simulation of crack growth using the ALIAS HIDA software, i.e. using algorithms of Monte Carlo method. An example of the calculated probabilistic curve is in Figure 5.

In the final step, the results of the probabilistic assessment using the ALIAS HIDA software were compared with the two alternative methods, namely the method of integration of tolerance limits and the method according to Lauschmann [1]. The comparison is in Figure 6.

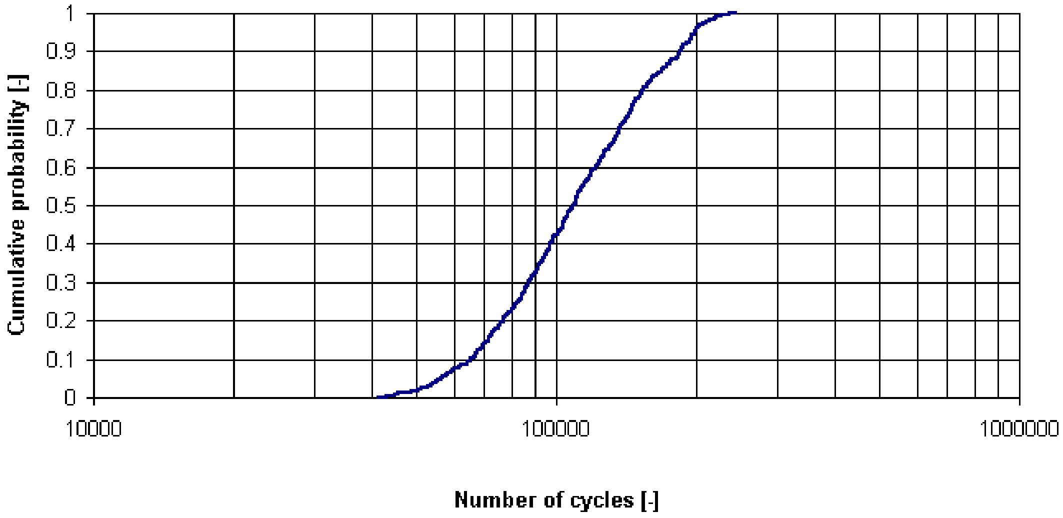

Figure 5. Example of cumulative probabilistic curve of failure of the Al-alloy specimens

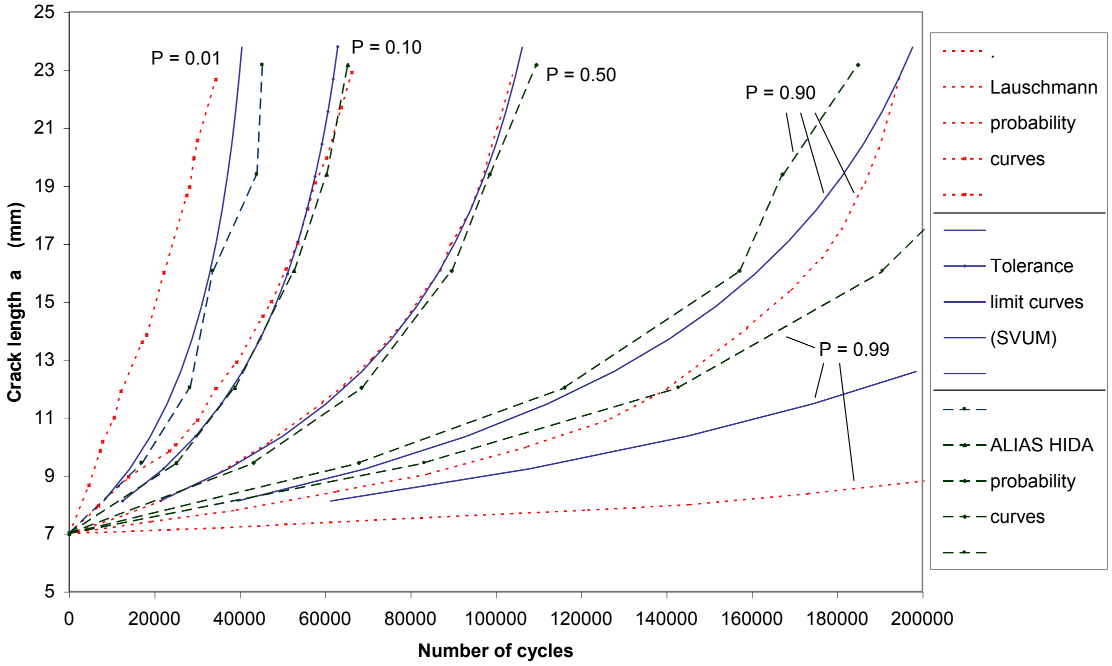

Figure 6. Comparison of probabilistic assessment of the crack growth using the three methods

Quite a good agreement of the results obtained using the three different methods follows from Figure 6, particularly for cases with not too extreme probability values P, in the range between 0.10 and 0.90. As regards low probability values, the agreement is better than in case of high probability values. One of the reasons of some differences between the three methods can be the fact that correct evaluation of tolerance limits is made only on the assumption that the distribution of experimental points along the regression line has a constant scatter in the whole evaluation interval. In the presented case, this assumption was not exactly valid. It can be seen in Figure 2 that in the region of lower crack growth, the scatter is somewhat higher.

The method using the ALIAS HIDA software with parameter C as the only random variable provides the most conservative results for high probabilities P and the less conservative results for low values of P, respectively. The most conservative results for low probabilities P were obtained by the method according to Lauschmann, which is likely the most precise, because it calculates with both the parameters C and m as random variables. On the other hand, this method is quite complicated. For an engineering assessment, the other two methods, not too complicated, provide quite satisfactory results.

CONCLUSIONS

Two different probabilistic approaches were applied to a set of data of fatigue crack growth rates in an Al-alloy, namely (i) assessment using deterministic integration of statistically evaluated tolerance limits along regression line and (ii) probabilistic assessment using ALIAS HIDA software elaborated within the project HIDA Applicability within the EU Framework Programme. Results were compared with an exact, sophisticated method according to Lauschmann [1]. The results can be summarised as follows:

- The agreement of the three methods was good, particularly for not extreme values of probability, between 0.10 and 0.90.
- The method of integration of tolerance limits, which is quite simple, provides satisfactory results with possibilities of practical use in engineering assessments, if extreme probability values are not considered.
- Probabilistic assessment using the ALIAS HIDA software, on the basis of Monte Carlo simulations can be carried out considering C parameter in the Paris law as the single random variable.
- Though the method according to Lauschmann, which is obviously the most precise, results in fairly conservative results for low probability values, the other two methods, which are simpler, look to be adequate for ordinary assessments of residual fatigue life.

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