Damage Induced Criticality and Scaling Laws of Fatigue Crack Path

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ABSTRACT. *Specific type of critical phenomena in mesodefect ensembles – structural-scaling transitions - allowed us to link the multiscale evolution of damage, the mechanisms of structural relaxation and damage-failure transition related to the dynamics of collective modes in mesodefects ensembles. Different types of collective modes are the consequence of qualitative changes of the group properties of evolution equations governing the mechanisms of structural (plastic) relaxation and damage-failure transition in the process zone at the crack tip. The dynamics of collective modes can be considered as physical mechanism providing the dynamic crack path and the universality of phenomenological laws for fatigue crack path in advanced materials. Experimental and structural study of crack advance supported theoretical results concerning the links of specific criticality of damage kinetics and the variety of scenario of crack path.*

INTRODUCTION

The interaction of the main crack with the ensemble of the defects in the so-called process zone at the crack tip is the subject of intensive experimental and theoretical studies in the problem of the crack path. The long-standing problem is the path of dynamic cracks in quasi-brittle materials after some critical crack velocity. The linear elastic theory predicted that a crack should continuously accelerate up to the Rayleigh wave speed, V_R, however, the experiments on a number of brittle materials showed that the crack will seldom reach even the half of this value and then the pronounced branching regime of crack path can appear [1,2].

The increasing interest is observed in the study of crack path in the condition of cyclic loading of the high-strength brittle materials (ceramics, intermetalics) and traditional materials (metals, alloys) after the structure reforming due to the compacting of metal (alloy) powder or granules. Such materials have the improved specific strength at high temperatures and simultaneously reveal the pronounced lack of damage tolerance due to the distinct features of fatigue crack propagation related to the nonlinear behavior of defects in the area of crack tip. It is the subject of intensive discussion concerning the physical nature of the Paris law since the experimentally measured Paris exponents typically vary between 2 and 4 for ductile materials, and can be considerably larger for

brittle materials, i.e., approaching 10 in low toughness metals [3] and even higher in intermetallics and ceramics [4]. The principal question is the scales related to the mechanisms responsible for the crack advance and the sensitivity of this process to the scales of damage evolution.
These phenomena demonstrate the qualitative new features of a crack behavior caused by the interaction of cracks with the ensemble of defects in the process zone. Statistically based phenomenology of collective behavior of mesodefects allowed the interpretation of damage kinetics as the structural-scaling transitions in defect ensembles and to establish the self-similar laws of damage evolution related to the collective modes of defects. Qualitative different non-linearity was established for characteristic stages of damage evolution that allows the original interpretation of the scenario of crack dynamics, the power universality in the Paris law related to the evolution of collective modes of mesodefects and interaction with main crack.

STRUCTURAL ASPECTS OF DYNAMIC AND FATIGUE CRACK PATH

The variety of scenario of crack path can be linked with the interaction of crack propagation and nonlinear mechanisms of structural relaxation, damage-failure transitions and failure in the process zone near the crack tip. The recent experimental study of dynamic crack propagation in quasi-brittle materials revealed the limiting steady state crack velocity, a dynamical instability to micro-branching [5,6], the formation of non-smooth fracture surface [7], and the sudden variation of fracture energy (dissipative losses) with a crack velocity [8]. This renewed interest was the motivation to study the interaction of mesodefects at the crack tip area (process zone) with a moving crack. The still open problem in the crack evolution is the condition of crack arrest that is related to the question whether a crack velocity smoothly approaches to zero as the loads is decreased from large values to the Griffith point [9].

Several types of damage have been identified as the governing factors of fatigue crack path [10]: persistent slip bands (PSB); roughness profile of extrusion; microcracks formed at the interfaces between PSB and matrix, in the valleys of surface roughness of PSB surface profile; fatigue damage at grain boundaries. Most of the damage causing defects range from 1 μm to $1\,mm$ which is below the in-service non-destructive evaluation (NDE limit $\sim 1\,mm$) inspection limit. Hence studies on nucleation and growth kinetics of these cracks become a necessary part of assessing the total life. Crack initiation, as well as the whole fatigue process, is controlled by the cyclic plastic deformation and fatigue crack path are indicated by the positions of the higher cyclic plastic deformation than average. In a wide range of deformation conditions the cyclic plastic deformation are localized within the stacks of highly active primary slip planes forming persistent slip bands (PSBs), while the surmounting material accommodates an appropriate two orders of magnitude smaller plastic strain amplitude. The PSBs are imbedded into a second phase commonly known as “matrix”, which consists of irregularly arranged dislocation reach regions –“veins”. The dislocation density within the veins is of the order of $10^{15}\ m^{-2}$ which corresponds to a mean dislocation spacing of $30\,nm$. The veins are separated by channels, which are relatively dislocation-free and are of a size comparable to that of the veins. At the early stage of fatigue the veins contribute to rapid hardening by partly impeding dislocation motion on the primary slip system. Increasing the number of cycles leads to an increase in both the dislocation density within the veins and the number of veins per unit volume.

STRUCTURAL-SCALING TRANSITIONS IN MESODEFECTS ENSEMBLE AND DAMAGE EVOLUTION SCENARIO

Statistical theory of collective behavior of mesodefect ensemble [11,12,13] allowed interpretation of plastic deformation and failure as non-equilibrium structural-scaling transition and to propose the phenomenology of solids with mesodefects based on the statistically predicted form of the free energy of solid with defects. Non-equilibrium free energy F represents the generalization of the Ginzburg-Landau expansion in terms of the order parameters - the defect density tensor (defect induced deformation $p = p_{zz}$ in uniaxial case) and structural scaling parameter δ

$$F = \tfrac{1}{2} A(\delta,\delta_*)p^2 - \tfrac{1}{4} Bp^4 - \tfrac{1}{6} C\,(\delta,\delta_c)p^6 - D\sigma\, p + \chi(\nabla_l\, p)^2\,, \qquad (1)$$

where $\sigma = \sigma_{zz}$ is the stress, χ is the non-locality parameter, A, B, C, D are the material parameters, δ_* and δ_c are characteristic values of structural-scaling parameter (bifurcation points). Structural-scaling parameter represents the ratio of two characteristic scales for the given structural level– characteristic size of defects and the distance between defects: $\delta \sim \left(R/r_0 \right)^3$. These areas correspond to characteristic types of collective modes generated in different ranges of δ, that are responsible for plastic relaxation and damage-failure transition. These collective modes are the self-similar solutions $p(x,t)_S$ of evolution equations for mentioned order parameters that have the form of spatial-periodical defect distribution for the fine-grain state S_1, the solitary waves for plastic strain localization area S_2 and “blow-up” dissipative structures S_3 for damage localization “hotspots” (Fig.1)

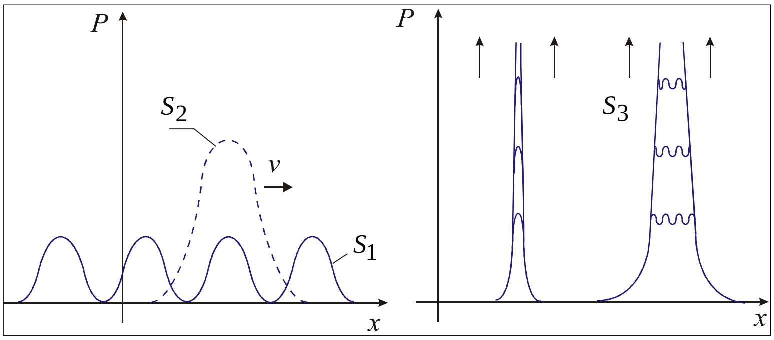

Figure 1. Characteristic types of collective modes of defects

Transitions over the bifurcation point δ_* leads to the replace of stable material response for fine grain materials to the metastable one for the ductile materials with the intermediate grain size that occurs for the value of $\delta = \delta_* \approx 1.3$, when the interaction between orientation modes of defects has more pronounced character. It means that the metastability has the nature of the orientation ordering in the defect ensemble that allowed the explanation of the violation of the Hall-Petch law under the grain refining [14]. The pass over the bifurcation point δ_c leads to the qualitative change of the potential non-linearity approaching to the form that is similar to the potential for the Griffith crack instability. The generation of collective modes in defect ensemble that are localised on some characteristic spatial scales allows the description of the appearance of multiscale dislocation substructures (PSBs, damage localisation areas) in some universal way. These modes play the role of current collective variables responsible for the scaling transitions when the initial structural scales are replaced by the scales of collective modes.

Structural-scaling Transitions and Dynamic Crack Path in Quasi-brittle Materials

The generation of collective modes under the loading provides the change of the system symmetry and initiates specific mechanisms of the momentum transfer and failure. For instance, the plastic deformation is realized as the development of strain localization areas, which consist of the arranged dislocations substructures propagating with some group velocity of the solitary wave fronts. The length of front represents the scale of the orientation ordering of dislocation substructures. The damage-failure scenario includes the multiscale "blow-up" damage localisation kinetics as the precursor of crack nucleation. The self-similar nature of mentioned collective modes has the importance for the explanation of crack propagation scenario both in quasi-brittle and ductile materials. The examples for this situation are the transition from the steady-state to the branching regimes of crack propagation in quasi-brittle materials.

The understanding of self-similar scenario of damage-failure transition related to the evolution of collective modes stimulated our experimental study of crack dynamics

[15, 16] with the aim of explanation of mechanisms of transition from the steady-state to the branching regime of crack path in the preloaded (by external stress σ) PMMA plate, Fig.2.

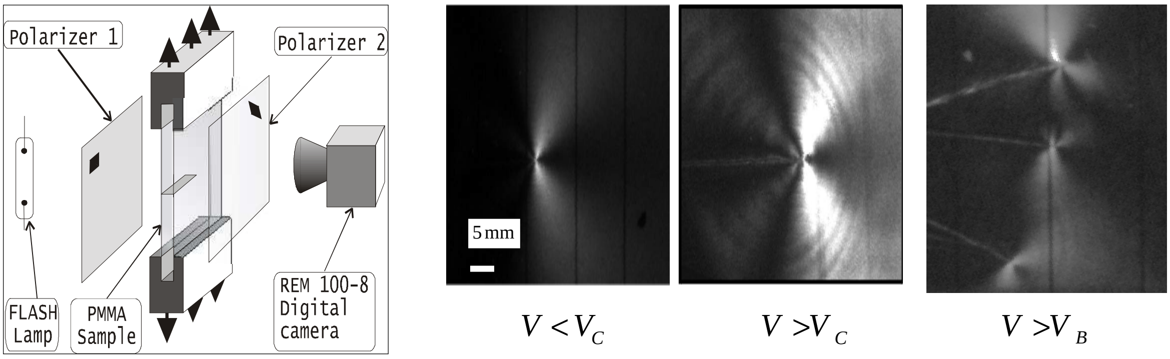

Figure 2. Scheme of experiment and characteristic pattern of dynamic crack propagation

High-speed framing was realized with the usage of digital camera Remix REM 100-8 and the photo-elasticity method. Three characteristic regimes of crack dynamics were established in different ranges of crack velocity: steady-state $V < V_C$, branching $V > V_C$ and fragmenting $V > V_B$, when the multiply branches of the crack have the autonomous behavior (Fig.2, 3).

Steady-state regime of crack dynamics is the consequence of the subjection of damage kinetics to the self-similar solution of the stress distribution at the crack tip (mechanically speaking to the stress intensity factor). Bifurcation point V_C corresponds to the transition to the regime, when the "second attractor" (with the symmetry properties related to the number of multiscale blow-up collective modes) disturbs the steady-state regime due to the excitation of numerous new failure hotspots (the daughter cracks having the image of mirror zones on the fracture surface). The change of the symmetry properties were studied under the recording of dynamic stress signal (polarization of laser beam) at the front of propagating crack in the point deviated on 4 mm from the main crack path (Fig.4).

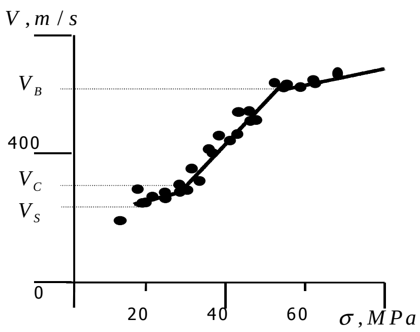

Figure 3.

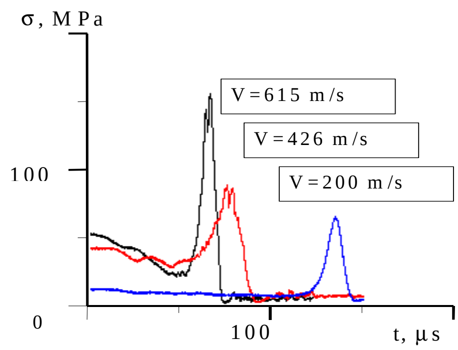

Figure 4

The stress signals, corresponding phase portraits $\dot{\sigma} \sim \sigma$ and correlation index ν (the characteristics of dynamic system symmetry) are presented in Fig. 5,6. The scaling properties as the above attractor were studied in the term of the correlation integral calculated from the stress phase pattern using the formula [17] $C(r) = \lim_{m\to\infty} \frac{1}{m^2} \sum_{i,j=1}^{m} H\left(r - \left|x_i - x_j\right|\right) \approx r^{\nu}$, where x_i, x_j are the coordinates of the points in the $\dot{\sigma} \sim \sigma$ space, $H(...)$ is the Heaviside function. The existence of the scales $r > r_0$ with the stable correlation index was established for the regimes $V < V_C$ and $V_B > V > V_C$. The values of the correlation indexes show the existence of two scaling regimes with the deterministic ($V = 200 m/s$, $\nu \approx 0.8$) and stochastic ($V = 426, 613 m/s, \nu \approx 0.4$) dynamics. The extension of the portions with a constant indexes determines the scale of the process zone L_{PZ}. The length of the process zone increases with the growth of the crack velocity in the range $V_B > V > V_C$. Numerical simulation of the damage kinetics in the process zone allowed us to conclude that this scaling is the consequence of the subjection of crack advance to the blow-up collective modes, which determines the collective behavior of defects ensemble in the process zone [18,19].

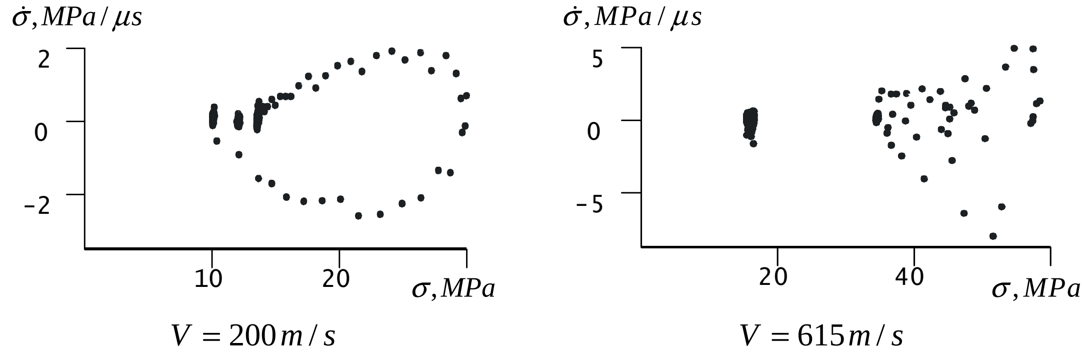

Figure 5. The phase portraits $\dot{\sigma} \sim \sigma$.

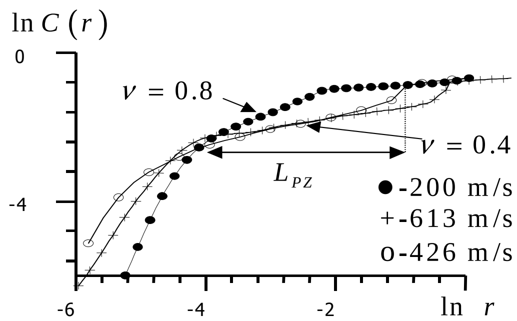

Figure 6.

The morphology of fracture surface corresponding to different correlation index is presented in Fig.7. The difference of the image of fracture surface reflects the influence of damage localization kinetics on the mechanisms of the crack path.

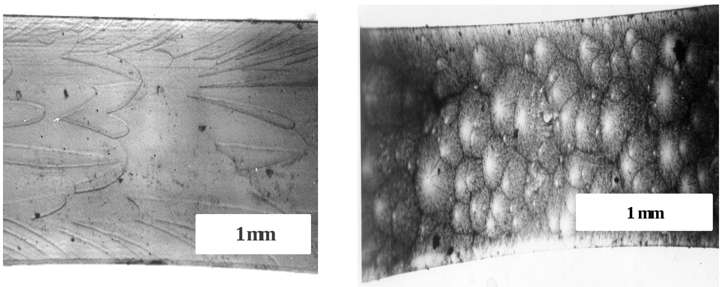

Figure 7.

Energy Absorbing and Scaling Transitions under Fatigue Crack Path

The early stage of High Cycle Fatigue (HCF) is followed by a “saturation plateau”, where structural changes take place within the matrix to accommodate high values of plastic strains because the dislocation veins in the matrix can not accommodate strain in excess of approximately 10^{-4}. PSBs structure is generated due to the initial blocking of glide dislocations and the formation of parallel wall (ladder) structures, which occupy about 10%, by volume, of the PSBs. The PSBs are composed of a large number of slip planes which form a flat lamellar structure. In strain-controlled experiments, the co-existence of matrix and PSB goes along with plateau in the cyclic stress-strain curve. If the applied strain amplitude is raised, this is accommodated by an increase in the volume fraction occupied by the PSBs. According to *TEM* observations of cyclically deformed *Cu* the labyrinth and cell dislocation structures are formed after saturation plateau [20] and can be considered as the dislocation arrangement precursor of fatigue crack nucleation and early crack growth occur in the PSBs. The final stage of fatigue damage corresponds to an increase in the peak resolved shear stress.

The description of damage kinetics reveals the specific system behavior in the ranges of scaling parameter $\delta_C < \delta < \delta_*$ and $\delta < \delta_C$ that can be qualified as the condition of the self-criticality [21]. It means that defect density parameter p influences on the correlation properties of the nonlinear system (in term of the δ kinetics) and provides the conditions of continuous reorganization of dislocation substructures according to non-linear (group) properties of damage evolution equations and types of collective modes generated in different ranges of structural-scaling parameter. The existence of two ranges of δ characterizes the qualitative difference of mechanisms of evolution of dislocation substructures: the orientation ordering in the form of generation and propagation of solitary waves and generation of multiscale blow-up dissipative structures as damage localization areas nucleation.

The scenario of defects evolution in the range of structural-scaling transition $\delta_C < \delta < \delta_*$ leads to the qualitative change of relaxation properties and, as the consequence, the energy absorbing. This anomaly develops in the course of generation of collective modes (solitary waves as the PSB image) that reveal the features of the "slow dynamics" [22]. According to the non-linear form of potential (1) the kinetics of δ provides the continuous ordering of dislocation substructures under scaling transitions in the metastabilty area, Fig.8.

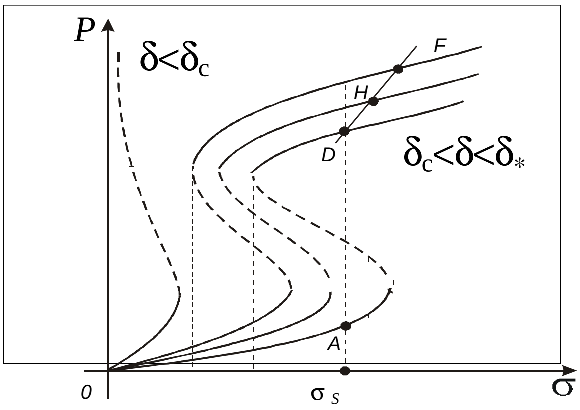

Figure 8. Scaling transitions in PSBs controlled fatigue stage.

The high value of the structural relaxation time in respect of the loading time $\tau_l \approx (\dot{\varepsilon})^{-1}$ coupled with the self-similar features of structure rearrangement in the scaling transition regime explain the anomaly of the energy absorbing under cycle load starting from some characteristic level of strain. This strain corresponds to the saturation stress σ_S providing the start of the scaling transition (the path *ADHF*, Fig. 8) and the anomaly of energy absorbing in the condition of structural-scaling transition in dislocation substructures.

The Paris Law of Crack Kinetics in HCF

The microscopic mode of fatigue crack growth is strongly affected by the slip characteristics of material, microstructure scales, applied stress level and the extent of near tip plasticity. In ductile solids, cyclic crack growth is observed as a process of intense localized deformation in slip bands near the crack tip which leads to the creating of new crack surfaces by shear decohesion. A number of mechanisms have been proposed to clarify the linkage of above stages with crack growth path. The important feature of cyclic loading conditions when the onset of crack growth from pre-existing defects can occur at stress intensity values that are well below the quasi-static fracture toughness. This observation was used as a physical basis for the Paris model [23] when small scale yielding assumption allowed the formulation of the crack kinetics in the form

$$\frac{da}{dN} = C\,\Delta K^{m}, \tag{2}$$

in the term of the stress intensity factor range defined as $\Delta K = K_{\max} - K_{\min}$, where $K_{\max}$ and $K_{\min}$ respectively are the maximum and minimum stress intensity factors, C and m are empirical constants which a functions of both material properties and microstructure and the loading parameters (R ratio for instance) as seen hereafter. This formula predicted the Paris exponent of $m \approx 4$ in agreement with experiments for most metals. Since the crack growth kinetics is linked with the temporal ability of material to the energy absorbing at the crack tip area the understanding of the saturation nature can be the key factor for the explanation of the 4th power universality. It was shown that the saturation nature can be considered as a consequence of the anomaly of energy absorbing in the course of structural-scaling transition in dislocation ensembles with the creation of PSBs and long-range interaction of dislocation substructures due to the internal stresses, which provide the "self-criticality" scenario of structure evolution at some constant σ_S value of external stress. The saturation plateau is very pronounced feature of the structure controlled regime with the low sensitivity to the applied stress starting from some critical value σ_S. Since the damage kinetics along the path *DHF* leads finally to the nucleation of the crack hotspots after the second bifurcation point at $\delta = \delta_C$, the stress controlled regime corresponds to the set of states *D,…H,…, F,...* with the kinetics of this path approaching to the 4th power of damage kinetics $\dot{p} \sim A\sigma_S^4$. This result supports the phenomenological law proposed by Paris for the HCF crack growth kinetics. It is interested to note that this channel (scaling transitions due to the generation of the multiscale dislocation substructures similar to PSBs) is very powerful in the sense of the energy absorbing. For instance, similar to the Paris law, the 4th power law $\dot{\varepsilon} \approx A\sigma_A^4$ for plastic strain rate $\dot{\varepsilon}_p$ on the stress amplitude σ_A, was established at the steady-state plastic wave front for the wide class of shocked materials [24].

DISCUSSION

The Paris law has found multiple conformations for different materials and numerous experimental data reveal the power $\mathrm{m} \sim 3-4$ in an intermediate range of $\Delta\mathrm{K}$. The weak dependence of this power on material microstructure, loading ratio and environmental conditions manifested some universality features of material responses concerning damage evolution scenario providing the fatigue crack advance. There are two deviations from the Paris law. The crack growth is decreased in the region, where the stress intensity factor is below the threshold value $\Delta\mathrm{K}_{\mathrm{th}}$ that depends significantly on the material microstructure and the loading ratio. The third area corresponds to the stress intensity factor $\mathrm{K}_1 \to \mathrm{K}_{1\mathrm{C}}$, where the Griffith-Irwin crack growth instability appears.

The Paris law in the form (2) provides indication of many factors that affect fatigue-crack propagation behavior. Clearly, the average growth rate depends upon a series of variables in addition to the stress-intensity range; these include (i) the nature of the loading, i.e., the load ratio, $R = K_{\min}/K_{\max}$, cyclic frequency v and time t , (ii) materials properties, notably elastic modulus E and K_{Ic}, and (iii) some characteristic scales.

The observation of power law of crack propagation is related according to the results [25- 27] to the intermediate self-similarity of the solution for the stress distribution at the crack tip area with the size, that is essentially larger than the microstructural scales, but smaller with another dimensions (crack size, specimen characteristic sizes). The problem of scaling of fatigue crack growth in the general context of scaling processes was studied in [28] to introduce the list of meaningful variables for the crack advance function f

$$\frac{da}{dN} = f\left(\Delta K, R, E, K_{IC}, h\right), \tag{3}$$

where R is the loading ratio, E is elastic modulus, h is characteristic spatial scale.

Scaling analysis based on the assumption of incomplete similarity allowed the conclusion that the parameters C and m in the Paris law are not generally the material characteristics and they can depend at least on the symmetry of cycle and some characteristic length. It was noticed in [29] that incomplete similarity is related to the asymptotic invariance of the mathematical model (renormalization group). Size-scale dependencies of m and C are quite difference for metals and quasi-brittle materials. The typical range for metals is $m = 2 - 5$ and $m = 10 - 50$ for quasi-brittle materials [30], that is the consequence of qualitative different mechanisms of crack advance related to the nonlinear scenario of damage evolution at the crack tip area. In the attempt to link the Paris law and mentioned non-linear scenario of multiscale defect evolution in the process zone two characteristic scales can be introduced: the size of the process zone at the crack tip area L_{PZ} and characteristic correlation scale l_{SC}. Length L_{PZ} is the size of process zone, where the multiscale damage accumulation provides the current crack advance; l_{SC} is the correlation scale, which provides the correlated behavior of defects on the scale of process zone. Following the assumption of the incomplete similarity the following kinetic equation for the crack advance can be proposed:

$$\frac{da}{dN} = \frac{L_{pz}}{N_a}\left(\frac{\Delta K}{E\sqrt{l_{sc}}}\right)^2 \Phi\left(\frac{\Delta K}{E\sqrt{l_{sc}}}, R, Z\right), \tag{4}$$

where N_a is characteristic number of cycles for the crack advance over the size of process zone L_{pz}, $Z = E\sqrt{L_{PZ}}/K_{IC}$. Assuming the natural limit for $Z \to 0$ the representation of function Φ

$$\Phi = \left(\frac{\Delta K}{E\sqrt{l_{sc}}}\right)^{\alpha} \Phi_1\left(R, Z\right), \tag{5}$$

leads to the Paris law in the form

$$\frac{da}{dN}=\frac{L_{pz}}{N_a}\left(\frac{\Delta K}{E\sqrt{l_{sc}}}\right)^{2+\alpha}\Phi_1(R,Z) \tag{6}$$

with parameters:

$$C=\frac{L_{pz}}{N_a}\frac{\Phi_1(R,Z,)}{\left(E\sqrt{l_{sc}}\right)^{2+\alpha}}, \qquad m=2+\alpha(R,Z). \tag{7}$$

The kinetics (7) is close to the law proposed by Herzberg [31]

$$\frac{da}{dN}=b\left(\frac{\Delta K_{eff}}{E\sqrt{b}}\right)^3,$$

where b is the Burgers vector, which is nearly constant for many materials. This law with the power exponent 3 is in very satisfactory agreement with the data, when crack closure effects are removed [32].

Experimental and theoretical study allowed us to establish new type of critical phenomena – structural-scaling transition related to the multiscale defects evolution that provides the mechanisms of structural relaxation and damage-failure transition according to the dynamics of specific collective modes in mesodefects ensembles. The properties of these modes are given by different classes of self-similar solutions of statistically based evolution equations for damage parameter (defect density tensor - defect induced strain) and structural-scaling parameter, that describes the scale transitions under multiscale defects evolution. Different types of collective modes are the consequence of qualitative changes of the group properties of evolution equations for the defect density parameter in the course of structural-scaling transitions. The mechanisms of structural (plastic) relaxation and damage-failure transition in the process zone of advanced crack depend on the dynamics of collective modes of defects that can be considered as physical mechanism providing the variety of the dynamic crack path and universality of phenomenological laws for fatigue crack path in advanced materials.

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