# Mixed mode crack path investigation in a limestone rock using two circular shaped samples –An experimental and theoretical study

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**ABSTRACT.** *The mixed mode I/II crack growth behavior of Guiting limestone was investigated using centre cracked circular disc (CCCD) and the edge cracked semi circular bend (SCB) specimens. It was observed that for similar mode mixities in the two different shaped specimens, the fracture paths grew in two different trajectories. The deviation of crack path from the initial crack line was more pronounced for the SCB specimen and also for mode II dominant loading conditions in both samples. It is shown that the observed crack path and the fracture initiation angle can be predicted theoretically by using the fracture parameters ($K_I$, $K_{II}$ and T-stress) of the rock samples via a generalized form of the maximum tangential stress criterion. The main difference in the fracture trajectory was found to be related to the magnitude and sign of T-stress in the CCCD and SCB samples. Accordingly, the SCB specimen which has a considerable positive T-stress in mixed mode loading would have a larger fracture initiation angle in comparison with the CCCD specimen which has a very high negative T-stress for all mode mixities.*

## INTRODUCTION

In many practical applications of rock engineering, like rock cutting, rock fragmentation, excavation and rock slope stability analysis, the investigation of the fracture path is an important task for optimizing the size of rock pieces or controlling the stability of cracked rock structures. Most of the fractures in rock structures occur under complex loading states and usually under a combination of opening and sliding deformation (mixed mode I/II). Under mixed mode loading conditions, fracture of cracked components and structures may grow along curvilinear paths and not necessarily along the direction of original crack. Furthermore, when an estimate of crack arrest is required, the direction of fracture initiation from existing cracks must be determined particularly under mixed mode loading. There are a number of theoretical

models and various experimental techniques to investigate mixed mode crack growth. The available fracture models often use the stress intensity factors ($K_I$ and $K_{II}$) for predicting the crack growth direction. The most commonly used test samples for conducting fracture tests and study of crack growth resistance in rock materials are the centre cracked circular disc (CCCD), subjected to diametral compression, (and is often called the Brazilian disc) and the edge cracked semi circular bend (SCB) samples, subjected to three-point bend loading. The major advantages in using these two specimens are: the specimens can be easily extracted from rock cores, simple geometry, simple loading configuration, easy test set-up procedure, few machining operations, application of compressive loads rather than the tensile loads and the ability of introducing different mode mixities from pure mode I to pure mode II. Hence these test samples have been used frequently to investigate mixed mode crack growth of rock materials [1-5].

In this paper the mixed mode I/II crack growth behavior of a sedimentary soft rock (Guiting Limestone) is investigated experimentally and theoretically using both CCCD and SCB specimens. It is shown that the geometry and the type of loading can influence strongly the fracture path under mixed mode loading. The effect of $T$-stress (which is a geometry dependent stress term) in addition to $K_I$ and $K_{II}$ is considered as an additional parameter for evaluating the observed fracture initiation angle and its path in these samples.


**MIXED MODE FRACTURE EXPERIMENTS**

Fig. 1 shows the geometry and loading condition of CCCD and SCB specimens used for mixed mode I/II fracture tests. In the CCCD specimen the orientation of the centre crack of length $2a$ relative to the applied load $P$ (i.e. angle $\alpha$), varies the state of crack deformation and different combinations of mode I and II are obtained. Similarly for the SCB specimen, by changing the inclination angle ($\alpha$) of edge crack of length $a$ with respect to the applied load $P$, various mode mixities can be achieved. For both specimen shapes, $\alpha = 0^o$ corresponds to pure mode I (opening mode) loading. Also by increasing the loading angle $\alpha$ from zero, shear deformation (mode II component) is introduced in the CCCD and SCB samples. Pure mode II is achieved in each sample at a specific angle $\alpha$ depending on the specimen geometry and loading conditions. The pure mode II angle for CCCD specimen varies typically between $20^o - 30^o$ and for the SCB specimen between $35^o$ and $60^o$ [6].

A sedimentary soft limestone (Guiting limestone) was used for the experiments. This rock is a homogenous material composed of calcite. It is a porous limestone that is beige in color which is widely found in the UK. For the sake of comparison, the basic dimensions of CCCD and SCB specimens were considered to be the same and were as follows: $2R = 100$ mm, $B$ (disc thickness) = 40 mm and $a = 15$ mm. Thus, the ratio of $a/R$ was equal to 0.3 in both specimens. To investigate different combinations of modes I and II, five mode mixities were considered for each configuration. These mode mixities can be expressed in terms of the parameter, $M^e$ which is a mixity parameter

defined as: $M^e = \frac{2}{\pi}\arctan(\frac{K_{\mathrm{I}}}{K_{\mathrm{II}}})$. The values of $M^e$ varied from 1 (pure mode I), 0.75, 0.5, 0.25 and finally 0 for pure mode II.

To create the test samples a series of disc and semi disc specimens were prepared from a rock core of Guiting limestone. A fret saw with a thin saw blade of 0.4 mm thickness was used to introduce cracks into the specimens. The samples were then tested using a 25 kN servo hydraulic test machine. The tests were carried out under displacement control conditions with a constant cross head speed of 0.08 mm/min. The CCCD samples were loaded through two flat plate fixtures and the SCB specimens were tested using a three point bend fixture with a span (2$S$) of 43 mm (i.e. $S/R = 0.43$).
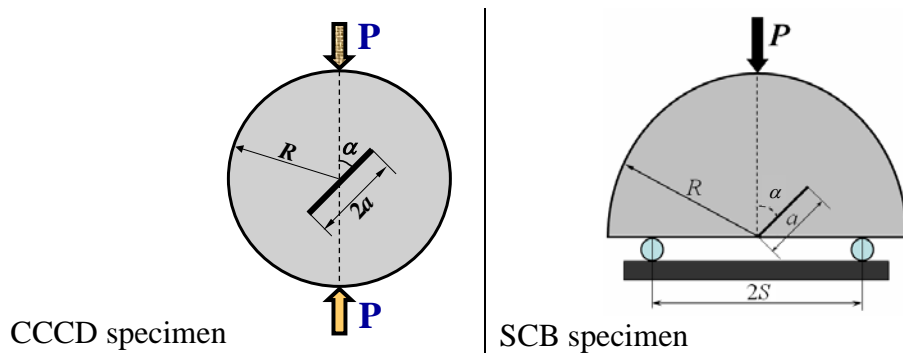


| CCCD specimen | SCB specimen |

Figure 1. Geometry and loading conditions of CCCD and SCB specimens subjected to mixed mode I/II loading; $P$ is the applied force, $R$ is radius of discs, $\alpha$ is crack inclination angle, $a$ is crack length and $S$ is the half span length for the SCB specimen.

The loading set up and the fracture patterns resulting from fracture of the test samples are shown in Fig. 2. Fracture initiated in all samples from the crack tip and then extended along a curvilinear path, finally terminating at the location of applied compressive load. However, while the crack growth trajectory for mode I loading was self similar (i.e. along the initial crack line) in both CCCD and SCB samples, their fracture paths were not the same for any other combinations of mixed mode I/II loading. In the next section the direction of fracture initiation and the path of fracture growth are investigated theoretically for both CCCD and SCB samples.
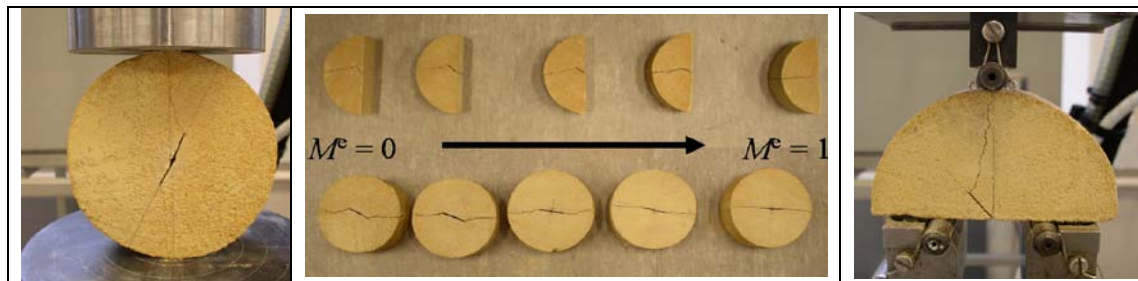


Figure 2. Loading setup for the CCCD and SCB specimens and the curved fracture trajectory observed for mixed mode loading conditions.

## THEORETICAL INVESTIGATION OF CRACK GROWTH PATH

To predict the crack growth path under mixed mode conditions it is necessary to determine the fracture initiation angle at the crack tip. Some mixed mode fracture theories has been proposed for determining the direction of brittle fracture. The maximum tangential stress (MTS) criterion [7] is one of the well known and commonly used fracture criteria for predicting the direction of fracture angle. Based on the MTS criterion the direction of fracture initiation for any mode mixity can be obtained from:

$$K_\mathrm{I} \sin\theta_0 + K_\mathrm{II}(3\cos\theta_0 - 1) = 0 \qquad (1)$$

According to Eq. 1, for each mode mixity a unique value can be determined for the direction of crack initiation ($\theta_0$) in any cracked geometry. But the observed fracture patterns for the CCCD and SCB specimens reveal that the crack initiation angles and fracture paths were not the same for mixed mode I/II loading. For example, the fracture path of broken samples for similar $M^e$ values in both sample shapes, have been compared in Fig. 3. It can seen in this figure, except for pure mode I ($M^e = 1$) conditions which the fracture is along the initial crack line, fracture paths were not the same for any other combinations of mixed mode I/II loading. In other words, for any two similar mode mixities (or $K_\mathrm{I}/K_\mathrm{II}$ ratio) in both sample shapes, the fracture paths differed. The deviation of crack path from the initial crack line was more pronounced for the SCB geometry and also for mode II dominant loading conditions in both sample shapes.
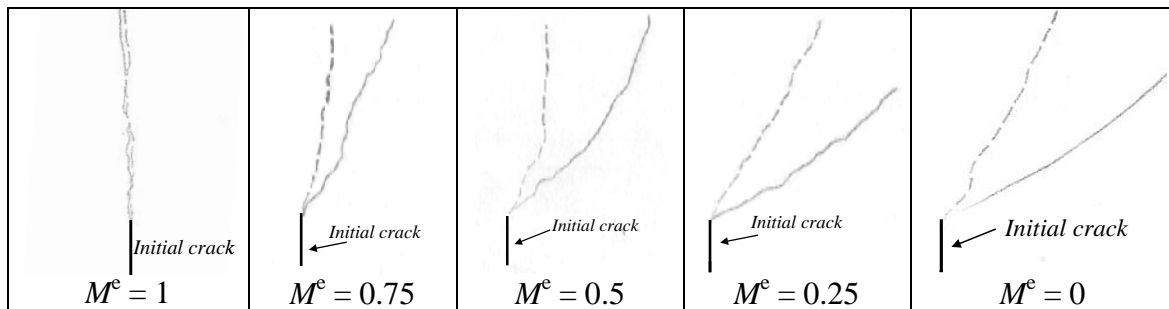


Figure 3. Mixed mode fracture path of the tested limestone material; dashed line for the CCCD specimen and the solid line for the SCB specimen.

However, since the conventional MTS criterion fails to provide good predictions for the direction of crack initiation angle for the tested CCCD and SCB samples a generalized form of the MTS criterion is employed here to predict the observations. The generalized MTS criterion states that mixed mode fracture initiates from the crack tip along the direction of maximum tangential stress. The tangential stress ($\sigma_{\theta\theta}$) component can be written as [8]:

$$\sigma_{\theta\theta} = \frac{1}{\sqrt{2\pi r}}\cos\frac{\theta}{2}\left[K_\mathrm{I}\cos^2\frac{\theta}{2} - \frac{3}{2}K_\mathrm{II}\sin\theta\right] + T\sin^2\theta + O\left(r^{1/2}\right) \qquad (2)$$

where $r$ and $\theta$ are the crack tip co-ordinates. $T$ is a non-singular and constant stress term which is independent of the distance from the crack tip and usually called the $T$-stress. $O\left(r^{1/2}\right)$ represent the remaining terms of the series expansion and are negligible near the crack tip. Accordingly, the direction of fracture initiation ($\theta_0$) can be determined based on the generalized MTS criterion as [9]:

$$\left.\frac{\partial \sigma_{\theta\theta}}{\partial \theta}\right|_{\theta=\theta_0} = 0 \;,\; \frac{\partial^2 \sigma_{\theta\theta}}{\partial \theta^2} < 0 \;\;\Rightarrow\;\; \left[K_{\mathrm{I}}\sin\theta_0 + K_{\mathrm{II}}\left(3\cos\theta_0 - 1\right)\right] - \frac{16T}{3}\sqrt{2\pi\, r_c}\,\cos\theta_0 \,\sin\frac{\theta_0}{2} = 0 \qquad (3)$$

By ignoring the effect of $T$-stress in Eq. (3), the generalised MTS will be identical with the conventional MTS criterion. Generally, the mixed mode crack initiation angle decreases for those specimens having a negative $T$-stress and conversely $\theta_0$ increases for the specimens with a positive $T$-stress. Therefore, unlike the conventional MTS criterion the generalised MTS criterion can predict different values of $\theta_0$ for any mode mixity and for any cracked geometry. In the next section the the generalised MTS criterion is used to predict the observed results.

## DISCCUSSION

In order to use the generalized MTS criterion (Eq. 3), three parameters, $K_{\mathrm{I}}$, $K_{\mathrm{II}}$ and $T$, should be known for any test specimen and for various mode mixities. These parameters for the CCCD and SCB samples are[6]:

$$\left[\; K_i = \begin{cases} Y_i\,\dfrac{P}{RB}\sqrt{\dfrac{a}{\pi}} \\[2mm] Y_i\,\dfrac{P\sqrt{\pi a}}{2RB} \end{cases} \;,\; T = \begin{cases} \dfrac{T^*P}{\pi RB\left(R-a\right)} & \text{for CCCD specimen} \\[2mm] \dfrac{T^*P}{2RB} & \text{for SCB specimen} \end{cases} \;\right] \quad i = \mathrm{I},\mathrm{II} \qquad (4)$$

where $Y_{\mathrm{I}}, Y_{\mathrm{II}}$ and $T^*$ are the normalized forms of $K_{\mathrm{I}}$, $K_{\mathrm{II}}$ and $T$, respectively. These parameters are functions of crack length ratio ($a/R$), crack initiation angle (or mode mixity) and also the location of loading supports ($S/R$ in the SCB specimen). The variations of $Y_{\mathrm{I}}, Y_{\mathrm{II}}$ and $T^*$ for the tested CCCD and SCB specimens made of limestone rock are given in Fig. 4 for various combinations of mode I and mode II.

By introducing the corresponding values of $K_{\mathrm{I}}$, $K_{\mathrm{II}}$ and $T$ for each specimen and any required mode mixity, the direction of the initial fracture angle was determined from Eq. 3. To predict $\theta_0$ in both CCCD and SCB specimens a fixed value of $r_c = 1.2$ mm was selected and assumed to be the size of fracture process zone in the limestone. This value was assumed to be constant for all mode mixities. The measured fracture initiation angles and the predictions using the generalized MTS criterion for both CCCD and SCB samples are shown in Fig. 5.
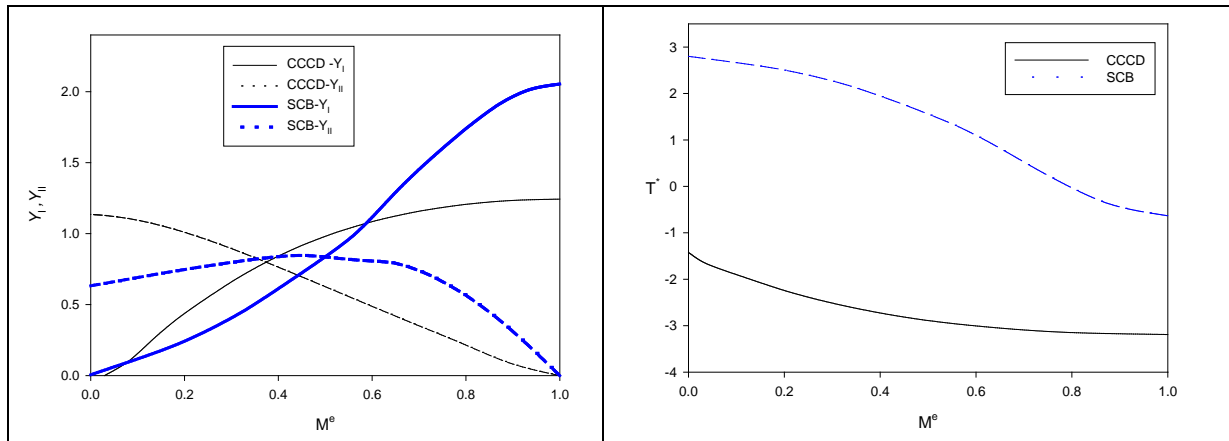
Figure 4. Variations of $Y_I$, $Y_{II}$ and $T^*$ in the tested BD and SCB specimens and for the full range of mode mixities.
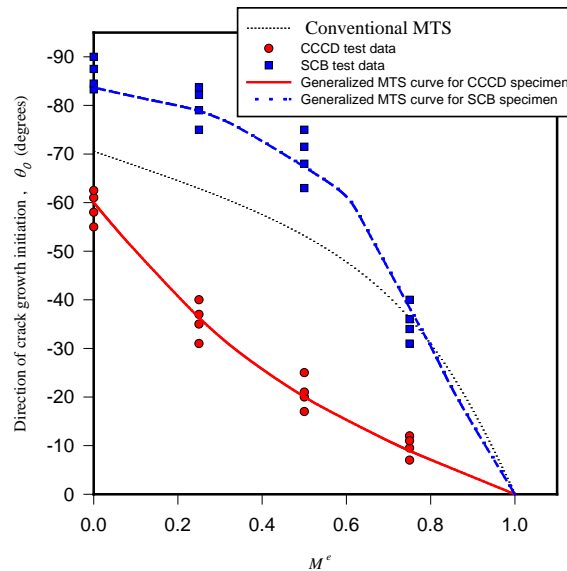


Figure 5. Predictions of the generalized MTS criterion for fracture initiation angle of CCCD and SCB specimens made of Guiting limestone.

There are several methods for predicting the crack growth path (e.g. [10, 11]). The incremental crack growth method is a common technique and involves a large number of small crack extensions in appropriate directions. The direction of crack growth for each increment can be determined by using the generalized MTS criterion. For each increment the fracture parameters of $K_I$, $K_{II}$, $T$ are required using Eq. (3). After calculating the direction of maximum tangential stress, the crack is remodeled with a small extension along the calculated direction and the same procedure is repeated for the next increment to locate the whole fracture path. This technique was applied using the ABAQUS finite element code. The simulated incremental crack growth path in one

of the SCB specimens (i.e. for pure mode II loading) is shown in Fig. 6 for some of the crack growth steps.
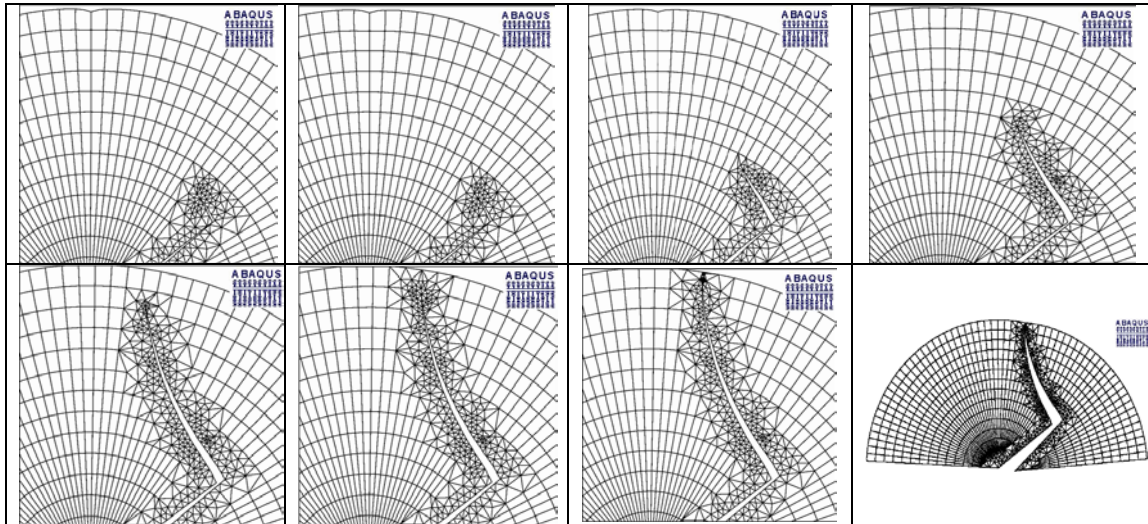


Figure 6. Simulated crack growth path for the SCB specimen using the incremental method.

The procedure for obtaining the fracture path in the CCCD specimen was the same as described for the SCB specimen. It was found that the crack path using the generalized MTS criterion for both CCCD and SCB specimens and for various mode mixities are in very good agreement with the observed fracture trajectory from the broken samples. However, it should be noted that the effect of $T$-stress was only noticeable for the first step in the model (i.e. just for determining the initial direction of crack growth). Based on the generalized MTS criterion the influence of $T$-stress is more pronounced when the $K_{\mathrm{II}}$ component is greater than zero and for dominantly mode I loading conditions (i.e. $K_{\mathrm{II}} \approx 0$) the predictions of the conventional MTS and generalized MTS criteria are identical.

During the crack path simulations it was observed that the magnitude of shear component ($K_{\mathrm{II}}$ value) reduced dramatically and tended towards zero for all increments after the initial kinking of the crack. Under such conditions, the effect of $T$-stress vanishes and the predictions of both criteria for propagation path would be nearly the same. For example the simulated fracture trajectories derived from both criteria for one of the SCB specimens ($M^{\mathrm{e}} = 0.5$) are compared in Fig. 7. Except for the initial crack kinking where the generalized MTS criterion provided better estimates for the crack path, the predicted propagation paths derived from both criteria are nearly the same. Consequently, it is recommended that the conventional MTS criterion is used for predicting the propagation path of cracked geometries subjected to mixed mode loading. Others [12] have also demonstrated that the propagation of growing cracks in brittle materials are governed by the tensile type fracture or mode I loading [12] which is in agreement with the obtained crack growth trajectory for tested CCCD and SCB samples made of limestone.
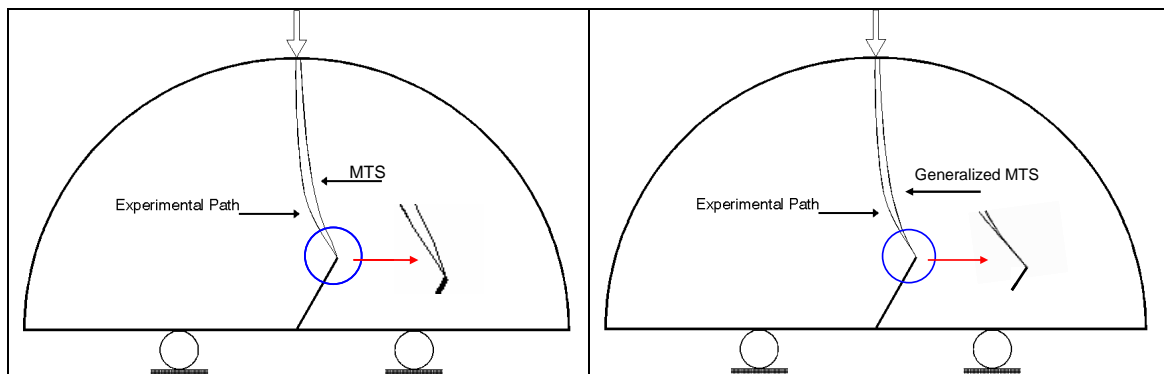
Figure 7. Comparison of conventional and generalized MTS criteria for fracture path
in a SCB specimen.


## CONCLUSIONS

-The fracture initiation angle and the crack path trajectory under mixed mode loading
conditions were not the same for CCCD and SCB specimens. The trajectories depended
on the mode mixity and the specimen type.

-There is a positive *T*-stress in the SCB specimen and this is the main reason why the
crack path deviates more when this specimen is subjected to mixed mode loading
compared to the CCCD specimen which has very large negative *T*-stress.


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