Crack Path Predictions in Ni-based Superalloy Plates Using Coupled Nonlocal Damage-Plasticity

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ABSTRACT. *The present paper introduces a new nonlocal coupled damage-plasticity model aiming at predicting crack paths within plates made of ductile material and subjected to pure tensile loading. This model is inspired by Nguyen's model [1] for quasi-brittle material. It uses the Houlsby and Puzrin [2] framework that makes the model formulation internally consistent by linking the damage and the yield functions via a dissipation potential. However, as in Lemaitre [3], the damage law is explicitly defined in order to ease the implementation. This model has been implemented within a UMAT for the finite element (FE) package Abaqus (implicit). Thin plates of different geometries have been modelled and tested using this model. The model ability to follow the process until total failure without encountering problems such as numerical instabilities, to give mesh-independent results and to predict reasonable crack paths has been demonstrated. Advantages and limitations of the present approach are discussed. Emphasis is placed on the need to calibrate the model parameters so as to achieve the best match with the experimental data.*

INTRODUCTION

The study of crack propagation within ductile material under thermo-mechanical cyclic loading is a key issue for the aeronautical industry mostly for the reasons of improved design and safe operation. Indeed, the widespread use of damage tolerant design principles has made the crack propagation and trajectory analysis absolutely essential. Even though the capability of predicting crack rates and trajectories using the finite element framework (FE) has been greatly advanced over the past decades, efficient implementation remains a challenge even under simple loading conditions, such as pure tensile or pure shear loading. Two different approaches have been explored since the 1980's when the problem began to be addressed. The first one regards the crack as a discontinuity, an interface, while the second considers it as a fully degraded part of the continuum. Both approaches have their advantages and disadvantages. The first approach has the advantage of representing the discontinuous nature of the crack. The second approach (continuum damage mechanics - CDM) captures the softening observed within the material in the crack's immediate neighbourhood, but doesn't provide precise identification of the crack's spatial location. Numerical instabilities arising due to material softening, mesh-dependent results and the model's incapacity to predict size effects are also some of the critical problems known to arise in either of those approaches. Recently, Nguyen [1] proposed a CDM nonlocal model for brittle materials that overcomes successfully all of the previously mentioned problems. In the present study we introduce an adaptation of Nguyen's model to the case of ductile materials. Of course, the micro-mechanics of ductile and brittle fracture are different: ductile fracture is known to be due to voids nucleation, growth and coalescence (Rice and Tracey [4]), while rupture of quasi-brittle materials is associated with distributed micro-cracking, e.g. at the interface between the matrix and the aggregates. However, the two fracture modes also possess some similarities. For example, in the overall load-displacement curves associated with both rupture processes, the early linear elastic stage is followed by a phase for which material's hardening and material's softening are in competition and that is manifested first in a nonlinear increasing curve (when hardening is preponderant) and then by a decreasing curve (when softening becomes paramount). We have implemented our model within a UMAT for the FE package ABAQUS and tested it for the example cases of thin plates (2D plane stress models) subjected to pure tensile loading. The model ability to follow the rupture process to its end, to give mesh-independent results and to predict reasonable crack paths in plates of various geometries has been demonstrated. The needs for an efficient method of model calibration and for a more physically realistic damage law are identified.

MODEL FORMULATION

The model used in this study is an adaptation of Nguyen's model [1] for quasi-brittle materials. It is inspired by Houlsby and Puzrin's [2] thermodynamic framework, but also has some similarities with Lemaitre's [3] approach. Thus, the yield and the damage function are linked by a dissipation potential which makes the model formulation consistent as in Houlsby and Puzrin [2], but the damage is explicitly defined as in Lemaitre [3] so as to ease the implementation. The 1D formulation of this model has already been presented in Belnoue et al. [5], and its full version will be outlined in a forthcoming paper by Nguyen et al. [6].

Stress vs. Effective Stress

As in all CDM models, the stress-strain relationship and the effective stress-strain relationship are expressed as follows:

$$\sigma_{ij} = (1-\alpha_d)\times a_{ijkl}\times(\varepsilon_{ij}-\alpha_{ij}) \tag{1}$$

$$\overline{\sigma}_{ij} = a_{ijkl}\times(\varepsilon_{ij}-\alpha_{ij}) \tag{2}$$

In the above equations, σ_{ij} and $\overline{\sigma}_{ij}$ are respectively the stress and the effective stress tensor, while ε_{ij} and α_{ij} are the total strain and the plastic strain tensors. a_{ijkl} is the elastic stiffness matrix and α_d is the so-called scalar damage variable. Following the effective stress concept (Kachanov [7]) and the strain equivalence principle (Rabotnov [8] and Lemaitre [3]), α_d takes materials degradation into account by decreasing the initial coefficients of a_{ijkl} as the material is progressively damaged. Once α_d reaches unity, the material is thought to be fully damaged and to have lost all its stress-carrying capacity. The scalar nature of α_d implies the isotropic nature of damage. This approximation may be thought to be suitable in our case, since the focus is on damage caused by voiding, rather than micro-cracking.

Plasticity and damage coupling

The Drucker-Prager yield criterion used in Nguyen [1] is replaced, here, by a typical Von Mises yield criterion (see Eqs. 3) that describes better the plastic behaviour of metallic alloys:

$$y_p^* = \sqrt{\frac{3}{2} \times \sigma_{ij}^{'} \sigma_{ij}^{'}} - F_{pi}\left(\varepsilon_p\right) = 0 \tag{3}$$

In (3), y_p^* is the yield function, $\sigma_{ij}^{'}$ is the deviatoric part of the stress tensor σ_{ij}, and $F_{pi}\left(\varepsilon_p\right)$ is the hardening function that depends on the equivalent plastic strain ε_p whose time increment, $\dot{\varepsilon}_p$, is defined in Eq. 4:

$$\dot{\varepsilon}_p = \sqrt{\frac{2}{3} \times \dot{\alpha}_{ij} \times \dot{\alpha}_{ij}} \tag{4}$$

As in Grassl and Jirasek [9], the model nonlocality appears only when the damage criterion is defined (Eq. 5). The coupling between damage and plastic strain is maintained by defining the damage function $G_d\left(\overline{\varepsilon}_p\right)$ as a function of the local-nonlocal equivalent plastic strain $\overline{\varepsilon}_p$ that itself is a function of ε_p and of the nonlocal plastic strain $\tilde{\varepsilon}_p$ (that is also a function of ε_p).

$$y_d = G_d\left(\overline{\varepsilon}_p\right) - F_d\left(\alpha_d\right) = 0 \tag{5}$$

$$\alpha_d = G_d\left(\overline{\varepsilon}_p\right) \tag{6}$$

To ease the implementation, the damage criterion (Eq. 5) is simply used to start the damage process. The dissipation potential, $F_d\left(\alpha_d\right)$, is implicitly defined and taken as the maximum previously reached value of $G_d\left(\overline{\varepsilon}_p\right)$ known explicitly, as in Lemaitre [3],.

Nonlocal plastic strain and local-nonlocal equivalent plastic strain

The increments of the nonlocal equivalent plastic strain $\dot{\tilde{\varepsilon}}_p$ and of the local-nonlocal equivalent plastic strain $\dot{\bar{\varepsilon}}_p$ are respectively defined in (7) and (8) below:

$$\dot{\tilde{\varepsilon}}_p = \frac{1}{G(x)} \times \int_{V_p} g(\|x - y\|) \times \dot{\varepsilon}_p \times dV \tag{7}$$

$$\dot{\bar{\varepsilon}}_p = m \times \dot{\tilde{\varepsilon}}_p + (1 - m) \times \dot{\varepsilon}_p \tag{8}$$

Eq. (7) is the traditional formulation of nonlocal plastic strain, as originally formulated by Pijaudier-Cabot and Bazant [10]. The implication is that the damage state at a certain point x of the continuum depends not only on the plastic strain state at that point, but is also influenced by the strain state of points situated in its close neighbourhood. Moreover, the farther a point, y, is from the point, x, the less its influence is. It is considered that the influence does not extend outside a certain region of radius R, known as the nonlocal radius. Thus, R acts as a material length scale and determines the size of the localization area. All this information is contained within the integral formulation, where V_p below is a volume defined by the sphere of centre x and radius R, and the bell-shaped weight function $g(r)$ (where r denotes the distance between the points x and y) is defined as:

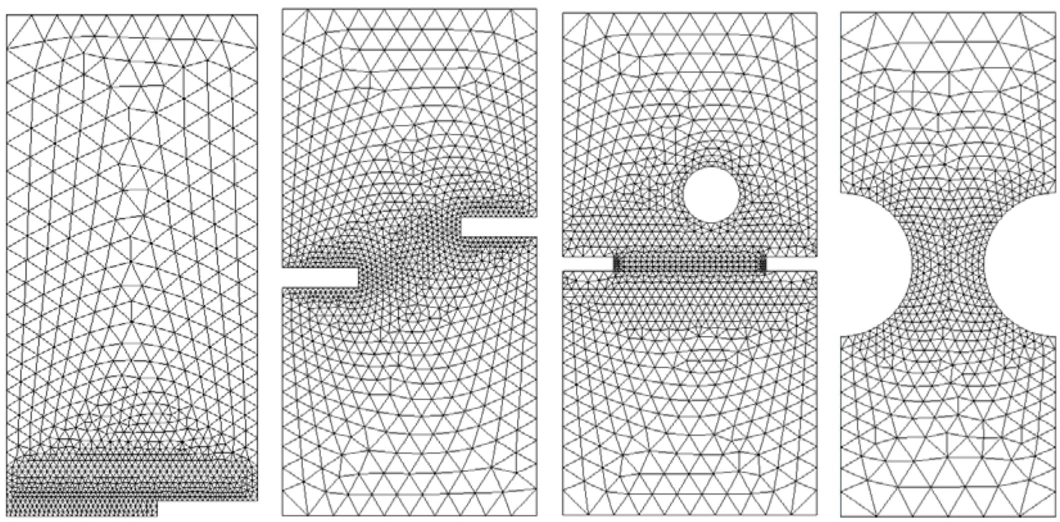
$$g(r) = g(\|\mathbf{y} - \mathbf{x}\|) = \begin{cases} 0 & \text{if} \quad r > R \\ \left[1 - \left(\dfrac{r}{R}\right)^2\right]^2 & \text{if} \quad r \le R \end{cases} \tag{9}$$

Here $G(\mathbf{x}) = \int_{V_p} g(\|\mathbf{y} - \mathbf{x}\|) dV$ is used to normalize the weighting scheme applied to the local equivalent strain.

Grassl and Jirasek [9] have remarked that the coupling between damage and $\tilde{\varepsilon}_p$ is not sufficient to assure the model's mesh independence and then have proposed to couple it with $\bar{\varepsilon}_p$ instead, with the nonlocal ratio, m, being strictly greater than unity. We can remark that purely local formulation and nonlocal formulation of Pijaudier-Cabot [12] and Bazant type are both particular cases of Eq. (8). Taking m equal to zero gives a purely local formulation, while setting m equal to unity restores the nonlocal formulation of Pijaudier-Cabot and Bazant [10].

MODEL IMPLEMENTATION

The model presented above has been implemented in 2D (plane stress) within a UMAT subroutine for the FE package ABAQUS and applied to thin plates of different geometries (see Figure 1) fixed at one end and pulled in tension at the other extremity. Implicit integration of plasticity has been used. Chaboche [11] hardening law and a Lemaitre damage function [12] have bee chosen.



a b c d

Figure 1: Different plate geometries to which the model has been applied: (a) Double edged-notched tensile specimen (DENT) – only a quarter of the actual piece has been meshed due to symmetry; (b) Asymmetric DENT specimen, (c) DENT specimen with crack deviator (hole), and (d) D-notched tensile specimen.

Figure 2 illustrates the model's ability lo follow the deformation process to the end and to give mesh-independent results, as already demonstrated for the 1D version of the model (Belnoue et al. [5]) is preserved also in the 2D version. Hence, the overall load-displacement curves of the DENT specimen clearly show that the process can be followed till almost total loss of the plate's load-carrying capacity (i.e complete rupture). Moreover, plotting this curve for different mesh sizes, one coarser than the other, doesn't show any difference in the model behaviour.

CRACK PATHS PREDICTIONS

Finally, the model's ability to predict crack paths within plates of different geometries has been validated. Figure 3 shows the damage distribution of the plates in Figure 1 once they have been totally broken. As explained previously, cracks can't be clearly identified as a discontinuity in this CDM implementation. Instead, crack paths are represented by broadened de-localised bands of high damage. Thus, crack location and path are only predicted to the accuracy of about one element diameter. However, some improvement of this spatial resolution can be obtained by post-processing the simulation results by analysing the damage profile across the damage band, and associating the crack with the centre of this band in the direction transverse to its extent. In this context the crack line position can be defined as the centre of gravity of the damage profile, or by locating it in the middle position between centres of edge transitions from damage value of 1 within the band and 0 outside it. This approach may help improve the crack location capability of the model to sub-element accuracy, so that spatial resolution of about 1/10th of the element size can be expected.

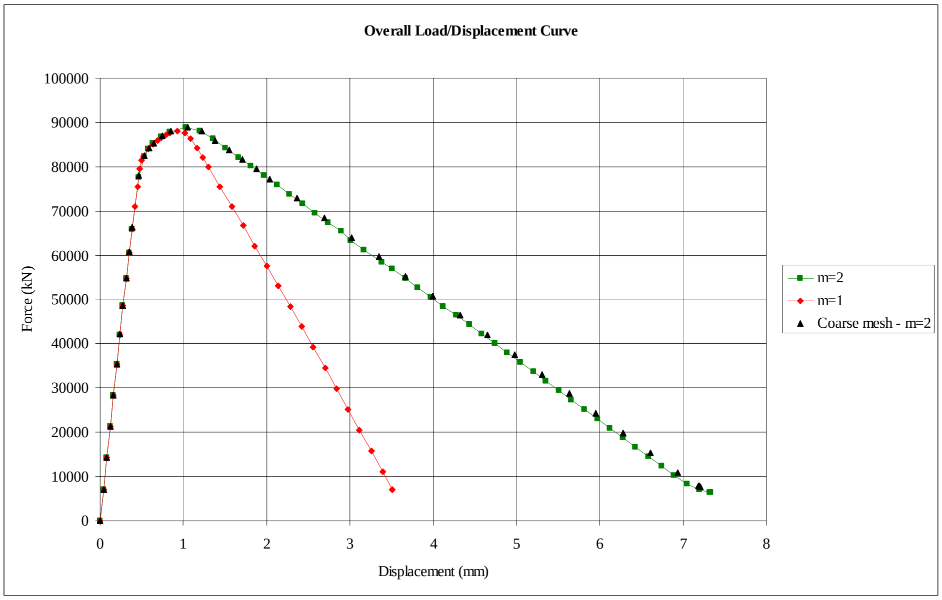


Figure 2: Overall load-displacement curves for DENT specimens under tensile loading for different values of the nonlocal ratio m and different mesh sizes.

Figure 3 shows that the present model is able to predict the crack paths that might be expected for plate specimens shown in Figure 1. For the DENT specimen, the crack initiates at both notches and propagates linearly. The specimen loses its load-carrying capacity entirely once the cracks originated from the opposing notches join up in the middle of the plate. A similar phenomenon is observed in the D-notched tensile specimen. For the asymmetric DENT, however, the shear loading created by the offset of the two notches from the mid-side position forces the growing crack to deviate from a linear path. A diagonal crack joining the two notches is observed instead. Finally, Figure 3(c)/ shows the crack-deflecting effect of a hole placed off the sample line of symmetry and displaced towards one of the notches in the transverse direction.

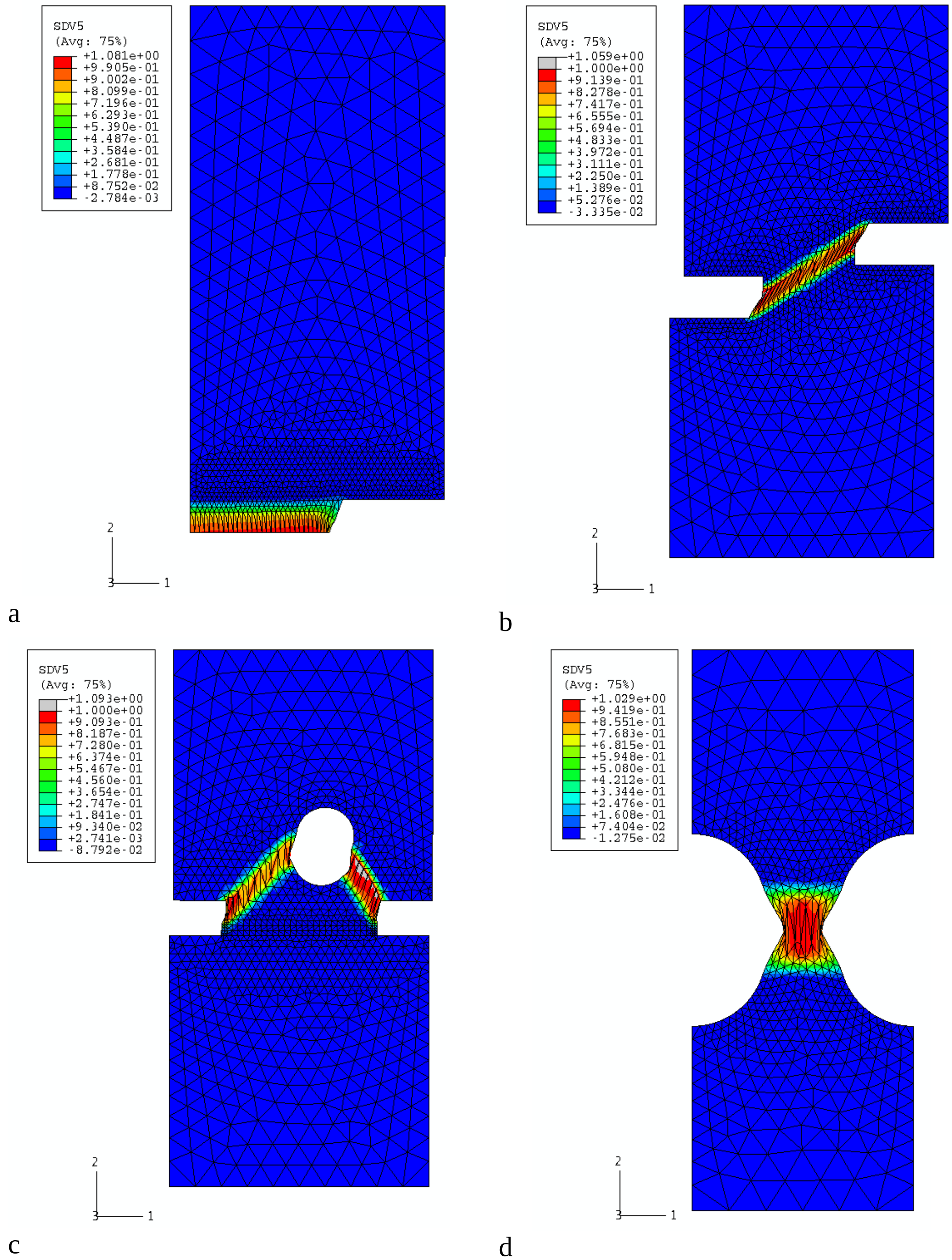


Figure 3: Damage distribution in the plates presented in Figure 1 at the end of the damage process.

DISCUSSION AND CONCLUDING REMARKS

The present paper introduced a new nonlocal model for ductile materials. The model's ability to follow the tearing process in thin plates under pure tensile loading until complete failure without encountering problems related to numerical instabilities, the capability to give mesh-independent solutions and to predict reasonable crack paths through plates of different geometries have been demonstrated.

However, as shown Figure 2, the choice of parameters such as the nonlocal ratio m influences the overall mechanical response of the plates under tearing. The value of the nonlocal radius R is also known to change the material's behaviour significantly. The development of an efficient calibration method for the present model thus appears to be an essential requirement for being able to predict the behaviour of real components.

Further developments aiming at incorporating creep and fatigue effects within the model formulation are being undertaken.

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