Accounting for cohesive process zones in discrete crack path prediction

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ABSTRACT. *This contribution presents a novel approach of the cohesive finite element method based on an initially rigid traction separation law which allows to insert the cohesive elements during the simulation depending on a crack growth criterion. In order to represent arbitrary crack patterns, this procedure is combined with an adaptive modification regarding the nodal coordinates and element boundaries of the initial discretization. In addition to the description of the formulation and algorithmic implementation of both the discrete crack model and the adaptive system modification, the influence of the traction-separation-dependencies on the global structural response in comparison with traction-free crack propagation models are investigated. The application of the proposed model to different analytical and experimental problems confirms its capabilities regarding the simulation of arbitrary discrete crack growth.*

INTRODUCTION

Fracture mechanical investigations are of special importance for all material classes. In order to predict the safety and durability of a component by a finite element simulation, the fracture mechanical sensitivity as well as the potential crack path have to be investigated. Common approaches to simulate the propagation of cracks include the application of softening material formulations to continuum elements leading to a smeared representation of the crack path or the application of adaptive crack propagation algorithms. However, these strategies are not able to represent the process of crack growth within the process zone. In contrast, the implementation of cohesive surfaces between the continuum finite elements in order to model discrete cracks provides a mesh independent framework to represent failure processes.

The discrete crack model on basis of the cohesive finite element dates back to investigations on steel sheets by Dugdale [1] and theoretical studies on an atomistic scale by Barenblatt [2]. First numerical implementations of cohesive process zones by Hillerborg et al. [3] featured a staggered substitution of the symmetric supports with equilibrium forces related to the crack opening displacement to simulate the localized failure of the structure. A first representation of a crack and interface delamination in the framework of the finite element method was presented by Needleman [4] who

introduced a formulation with coincident nodal points in the initial configuration and who furthermore stated the now common representation of the cohesive constitutive relations in terms of a traction separation law. This separate description of the cohesive zone and the spatial bulk material, which is represented by stress-strain-dependencies, allows to account for a realistic modelling of the crack opening process zone.

However, the conventional method to integrate a priori considered cohesive surfaces in the finite element mesh suffers from a major drawback: Since the cohesive elements can only be located at the bulk elements' boundaries, the crack path has to be known in advance, e.g. in case of the delamination of composite materials or glued structures. For computations with unknown crack paths, cohesive surfaces must be provided between all internal continuum element boundaries, as shown in Xu and Needleman [5] as well as in Tijssens et al. [6]. The second technique suffers from two main disadvantages: Firstly, it leads to an exorbitant increase of the system's degrees of freedom and, secondly, the effective stiffness of the structure is seriously decreased. In case of a one-dimensional analysis, the effective stiffness yields

$$E_{eff} = E_0 - \frac{E_0}{1 + K_0 h_e / E_0} = \frac{E_0 K_0}{E_0 n_e + K_0}, \tag{1}$$

depending on the bulk material's modulus E_0, the initial stiffness of the traction separation law K_0, and the uniform cohesive element spacing h_e or the number of surfaces n_e, respectively.

This contribution concentrates on a new approach which does not rely on an initial implementation of cohesive surfaces but uses instead an adaptive insertion of these elements in dependence on a crack growth criterion.

THE INITIALLY RIGID COHESIVE ZONE MODEL

The conventional method described above, which is also referred to as an intrinsic model (cf. Kubair and Geubelle [7]) due to the failure criterion as an inherent component of the cohesive phase, features an initially elastic traction separation law, as shown in Fig. 1a.

In contrast, the proposed approach is based on an initially rigid traction separation law (Fig. 1b). Such initially rigid descriptions of the cohesive constitutive relations have been used for example by Hillerborg et al. [3] or Carpinteri and Colombo [8] who proposed an algorithm to model the prescribed state of a certain crack extension with the help of an equilibrium iteration based on the crack tip opening δ and the corresponding cohesive forces F_c. Camacho and Ortiz [9] presented the first application of an initially rigid traction separation law in the context of a general finite element framework. Recent publications covering three-dimensional investigations (Pandolfi and Ortiz [10], Pandolfi and Ortiz [11]) as well as several applications (e.g. Pandolfi et al. [12]. Ruiz et al. [13], Ruiz et al. [14]) are so far only limited to short time dynamics in an explicit time integration scheme.

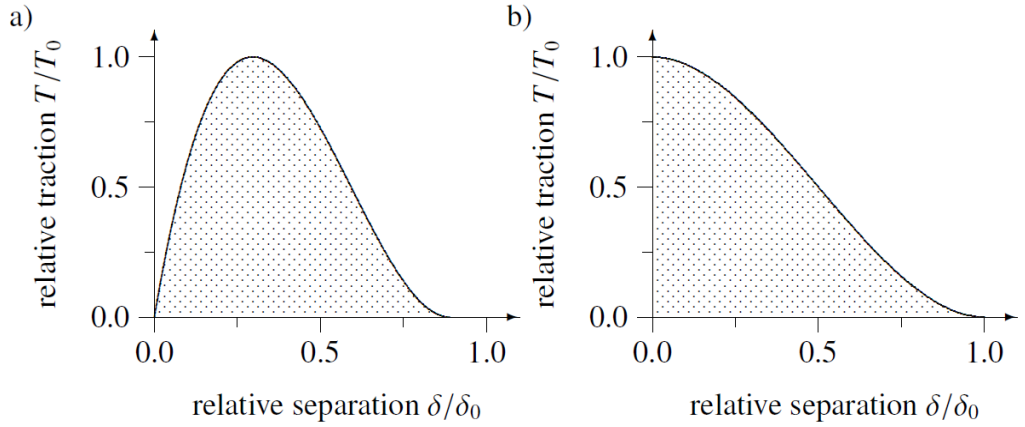

Figure 1. (a) Initially elastic and (b) initially rigid traction separation law for equivalent values of G_c

However, in case of most real fracture processes, the crack will evolve slowly and stably. The initially rigid description of the cohesive zones was therefore derived and implemented for a quasi-static implicit finite element code.

While procedures basing on an initially elastic traction separation law involve only a cohesive element formulation at an appropriate programming interface, the initially rigid approach requires an additional consistent modification of the global data structure in every time step associated with crack growth. Based on the boundary representation update procedure proposed by Pandolfi and Ortiz [10, 11] for tetrahedral elements with quadratic interpolation for explicit dynamical applications, a model adaptive discrete fracture simulation on basis of hexahedron elements with linear interpolation was derived for an implicit finite element framework.

The proposed finite element simulation of discrete crack growth consists of a constitutive analysis of the structure, where the non-linear system of equations has to be solved by a Newton iteration accounting for the residual vector and the stiffness matrix, and a modification of the finite element system depending on the extrinsic crack growth criterion. If the value of the particular failure criterion at one element point exceeds the critical value (encircled nodes in Fig. 2a), a separate boundary update routine is applied. According to the anticipated crack propagation direction suggested by the failure criterion, the relevant corresponding surface is selected for further system modification. In this surface, all nodes which exceed the critical value are duplicated and new cohesive faces are created between them (Fig. 2b). The nodal connectivity is modified for one of the two associated volume elements. In further crack propagation steps, an additional modification of the nodal connectivity for the preceding cohesive elements is required (Fig. 2c).

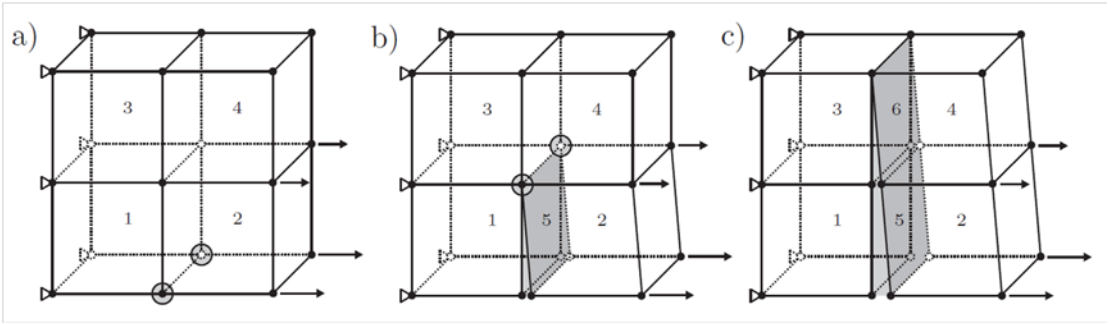

Figure 2. Boundary update scheme

In order to ensure robustness and convergence of the proposed solution procedure, the equilibrium state of a duplicated nodal point before and after the boundary update has to be preserved, i.e. the initial traction in the cohesive elements has to adopt the forces released by the separation of the bulk elements.

With respect to the equilibrium state of the assembled structure

$$\mathbf{K}_{ji}^{E1}\mathbf{u}_i + \mathbf{K}_{ji}^{E2}\mathbf{u}_i = \mathbf{F}_1 + \mathbf{F}_2 = 0 \tag{2}$$

where $\mathbf{u}_i$ denotes the current deformations and $\mathbf{K}_{ji}$ the corresponding element stiffness contributions, the released nodal forces $\mathbf{F}_1$ and $\mathbf{F}_2$ must represent the affine initial traction of the particular traction separation law. A detailed discussion on the time continuity requirement of initially rigid implementations can be found in the publications of Papoulia and Vavasis [15] and Sam et al. [16]. The correct initial traction vector $\mathbf{T}$ can be computed from the equation of the resultant nodal forces of the volume element and the cohesive element at the time of node duplication

$$\mathbf{F}_1 = \int_{\Gamma_c} N^T \mathbf{T} d\Gamma_c = \mathbf{K}_{ji}^{E}\mathbf{u}_i = \int_{\Omega} \sigma \frac{\partial u}{dx} d\Omega \tag{3}$$

which is derived from the decomposed stiffness $\mathbf{K}$ and the displacement $\mathbf{u}$ of node i or by an integration of the stresses over the element domain Ω.

Thus, the initial state of the resulting traction separation law is not traction-free. To ensure the continuity of the computation with respect to time for each node duplication process independently from the composition of the nodal force vector, the initial traction has to be determined individually. This results in individual parameters for each traction separation law, which is usually referred to as “encoding” in the cited literature.

The resultant material description of one cohesive surface consists of different material models for each node. In this context, a numerical integration scheme of the Newton-Cotes type for element matrix computation is used. A more detailed investigation of the time continuity statement and the resulting requirements can be found in the publication of Papoulia and Vavasis [15].

ADAPTIVE MESH MODIFICATION

In order to represent arbitrary crack paths, an adaptive modification of the initial disretization with respect to the location of nodes and element boundaries on basis of the anticipated crack propagation direction suggested by the failure criterion is additionally required.

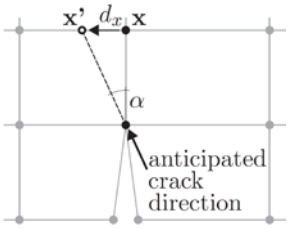

Figure 3. Anticipated crack propagation angle α

The relocation of the new crack tip *dx* from the original to the modified nodal coordinates *x* and *x'*, respectively, is obtained from the interpretation of the crack growth criterion (cf. Fig. 3). The remaining mesh is then modified subsequently for each crack propagation step. Considering constant nodal locations in normal direction, the external boundaries of the numerical model as well as restrictions regarding the shape of the continuum elements are taken into account by weight functions N_x and N_y set for each individual relocation of a node in *x*- and *y*-direction depending on the distance from the crack tip.

THREE POINT BENDING TEST

The proposed algorithm is applied to the three-dimensional model of a symmetric three point bending beam. This kind of numerical example is used in a number of investigations related to brittle crack propagation, e.g. Carpinteri [8]. More recent investigations with the extended finite element method (Moës and Belytschko [17]) and a staggered energy minimisation algorithm (Miehe and Gürses [18]) can be found in the literature.

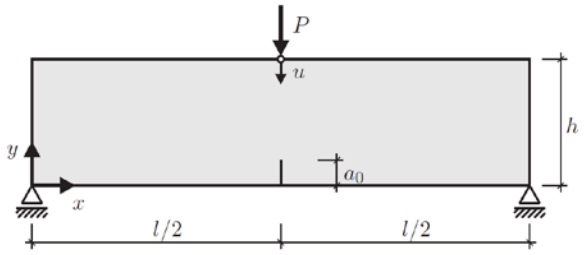

Figure 4. Three point bending test specimen: geometry, dimensions, loading

The geometry of the used beam is shown in Fig. 4. A span width of l = 0.6 m, height and depth $h = t = l/4$ = 0.15 m and a_0 = 0.03 m as the initial crack length are used as geometric parameters. The material is specified by a Poisson's ratio of ν = 0.1 and an elastic modulus of E = 36.500 MPa. The fracture process zone is characterised by the fracture energy Γ_0 = 0.05 N/mm and a polynomial decreasing traction separation law. The effective traction criterion is applied to drive the fracture process and the strength of the material at pure tension is assumed as f_T = 3.19 MPa. To prevent rigid body motion, the point of loading is fixed in x-direction. Due to the symmetry of the problem, the straight line of crack propagation is known in advance. The spatial discretization, the crack state at maximum applied displacement and the qualitative σ_{11} stress state are given in Fig. 5.

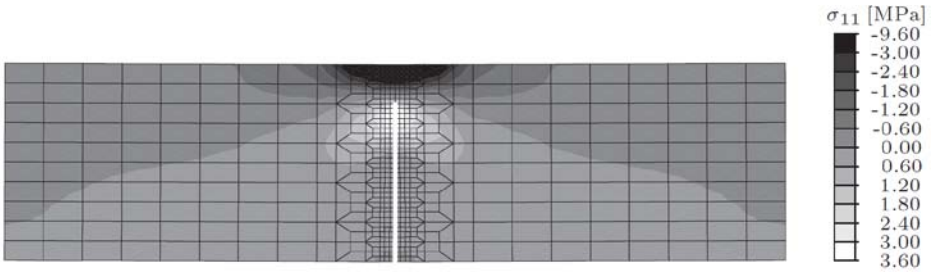

Figure 5. Three point bending test specimen: finite element discretization, final crack state and σ_{11} stress field

To evaluate the resultant load-displacement-dependency of the initially rigid cohesive zone formulation, a comparison to an equivalent initially elastic computation is carried out. Here, a polynomial traction separation law with a maximum normal traction of T_0 = 3.19 MPa and a crack opening separation of δ_0 = 0.0139 mm is used to obtain an identical fracture energy.

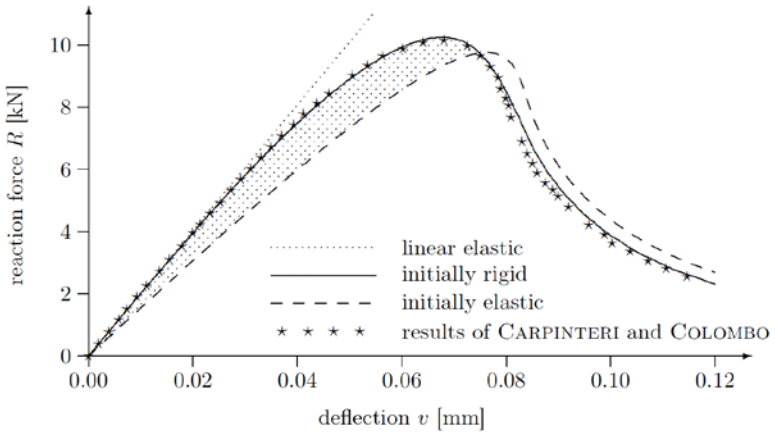

Figure 6. Three point bending test specimen: comparison of numerical results

As depicted in Fig. 6, the initially rigid approach does not deviate from the linear elastic response, which represents the stiffness of the beam without any damage or crack propagation, until first crack initiation at a vertical deflection of $v \approx 0.02$ mm. The applied load at this stage is approx. 40% of the ultimate load for the specified example, showing the load increasing capability after first failure. In contrast, the initially elastic computation shows a weaker response from the beginning for equivalent parameters due to the surface opening before crack initiation.

The influence of the traction-separation-dependencies on the global structural response in comparison with traction free crack propagation models are investigated by relating the proposed method to an algorithm for configurational force driven brittle crack propagation presented by Miehe and Gürses [18] and Gürses [19]. This displacement driven implementation solves the problem of snap back instabilities caused by crack propagation with the help of a separation of crack growth and load increase in terms of a staggered procedure. To modify the proposed initially rigid cohesive crack propagation algorithm with regard to the Miehe and Gürses model, the cohesive process zone is removed and the system modification is restricted to one node duplication procedure within one time step. Moreover, the internal time value t related to the linear increasing load function $u(t)$ is reduced by the increment Δt for the case of a changing boundary representation. The related application of $t_n \rightarrow t_{n+1}$ leads to a constant displacement value for the subsequent equilibrium iteration and enables a further system modification procedure for equivalent boundary conditions.

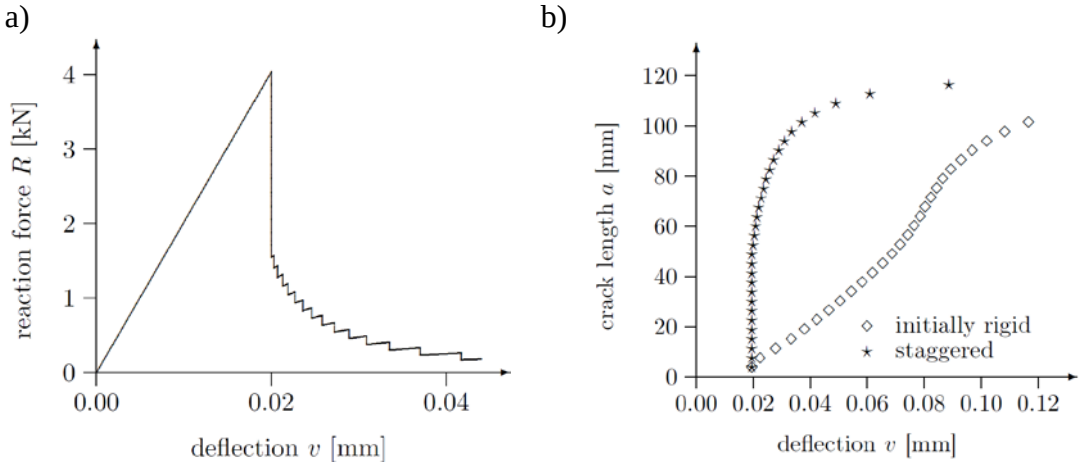

Figure 7. Three point bending test specimen: (a) numerical results for staggered energy minimization approach; (b) crack length depending on the applied displacement

The application to the three point bending beam model with equivalent boundary conditions and material parameters leads to a global response, which shows the characteristic branch of the relevant computation presented in [18] and [19]. The global relation between applied vertical displacement v and the related reaction force R is shown in Fig. 7a. If the maximum load bearing capacity is reached, a significant

decrease of the associated load occurs resulting from the sudden crack growth. For the next stable state, associated with stresses below the critical condition at the current crack tip, an increase of the global displacement value leads to a higher reaction force. This leads to the characteristic sequence of zigzag sections in the global response for that approach.

Figure 7b shows the completely different quality of crack propagation prediction for both models. While the staggered algorithm shows a significant crack elongation in a very early stage, the initially rigid approach exhibits a more distributed occurrence of crack propagation and total failure at a higher displacement level.

SUMMARY

A unique implementation of the cohesive zone model within the finite element method was presented. Based on an adaptive system modification and the evaluation of the preferred crack direction, the model also allows the representation of arbitrary curvilinear crack propagation independent of the initial discretization. The simulation of a three point bending beam in comparison with other current approaches showed that only the consideration of an appropriate process zone model allows a realistic simulation of local and global crack growth phenomena.

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