A weight function method to predict mode I stress intensity factors of multiple cracks

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ABSTRACT. *It is known that a powerful feature of a weight function approach is the ability to determine stress intensity factor (SIF) solutions for an arbitrary applied stress. The weight function is a universal function of a cracked body for any given geometry. Weight function methods have been applied extensively to problems concerning a single crack. So far, no attempt has been made to use a weight function method to determine the Mode I crack tip SIF of multiple cracks. This paper discusses the development of a novel weight function method in order to predict Mode I SIFs of two edge cracks in a finite body under uniform tension. The crack interaction effect was established using a non-uniform stress distribution along the potential crack plane to simulate the presence of an additional edge crack. The FE modelling technique used in this paper is also briefly discussed. Overall results obtained from the weight function approach are encouraging as they display the general expected trend and compare well to the FEA results. The results demonstrate that the weight function method can be used to determine SIFs for multiple cracks provided that the stress distribution along the potential crack plane is known.*

FE MODELLING TO DETERMINE SIFS OF TWO EDGE CRACKS

As a base-line study, two-dimensional FE models were constructed to calculate SIFs of two parallel edge cracks in a sheet under uniform tension. The FE software package that was used for FEA in this paper is ABAQUS [1]. A full FE model was constructed to model a finite strip with length ten-times times longer than its width. This ensured that there was no strip length edge effect on local stress distribution near the crack tip area. To model uniform tension, nodes at the end of the strip were constrained and nodes at the opposite end were applied with a point load as shown in Fig. 1. The FE models were prepared using a mesh generation program coded in Visual Fortran [2]. The program produced FE pre-processing information in a format compatible with an input file required by ABAQUS [1].

The mesh generator for two cracks was modified from the work of Love [3] which was used to generate a mesh for a single crack in various geometries. Teh *et al*. [4]

employed the same modelling technique to conduct extensive validation of SIFs for a single edge crack by comparison with published solutions. FE models were partitioned into a number of quadrilateral areas for meshing. Two-dimensional isoparametric continuum elements were used throughout the mesh. The elements used were plane stress elements having eight nodes. They are termed bi-quadratic or second order elements and are denoted within ABAQUS as CPS8R. The elements surrounding the crack tip were formed from the same bi-quadratic elements used throughout the model but are collapsed into a triangular element. The notation used for the FE modelling is shown in Fig. 1.

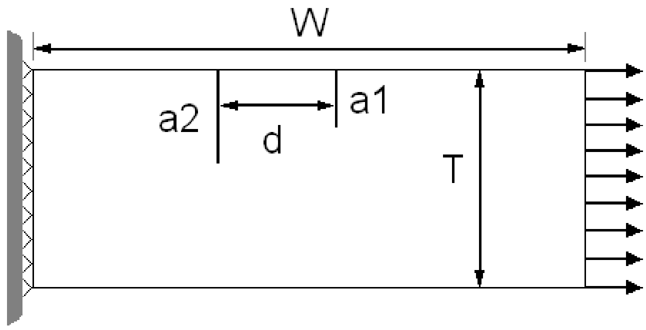

Fig. 1. Notation used in FE modelling.

The SIF of the FEA results were normalised in the form below:

$$Y = \frac{K}{\sigma_0 \sqrt{\pi a}} \quad (1)$$

where σ_0 is the remotely applied stress.

There are no closed-form analytical solutions available in the literature for two edge cracks with unequal crack lengths in a finite strip under uniform tension. Jiang *et al.* [5] conducted a FEA to calculate SIFs of two edge cracks with unequal lengths. The results of their FEA SIFs which are tabulated for different crack geometries were used as a comparison with the FEA results obtained in this paper. An example of this comparison is shown in Fig. 2. This shows the results of a normalised SIF for crack 2, Y_{a2} using three a1/T values equal to 0.05, 0.30 and 0.45 and d/T equal to 0.4.

Overall results show good correlation with the published values which were all within 1% of each other.

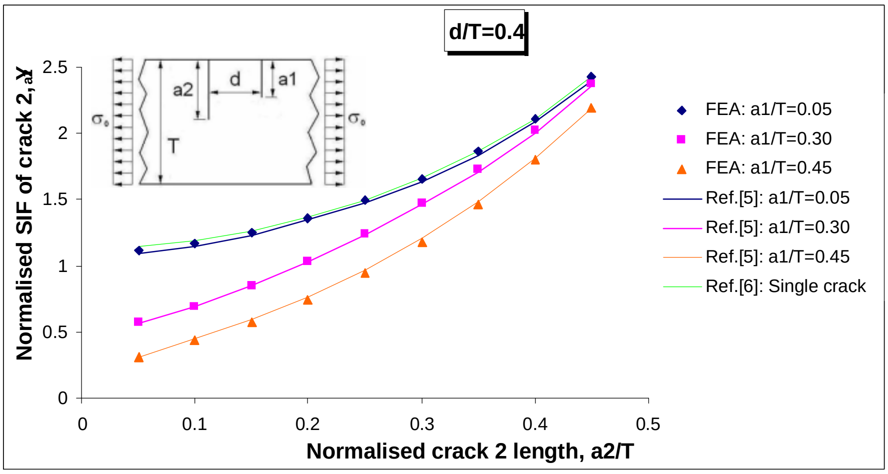

Fig. 2. Comparison of FE results with Reference 5.

STRESS DISTRIBUTION ALONG THE POTENTIAL CRACK LINE

FE modelling was also used to study non-uniform stress distribution along the potential crack line. The objective of the study was to examine the relationship between the crack-line stresses and the interaction effect between cracks. Only stress in the longitudinal direction was measured as the SIF solutions evaluated in this work are for crack opening mode (mode I) only.

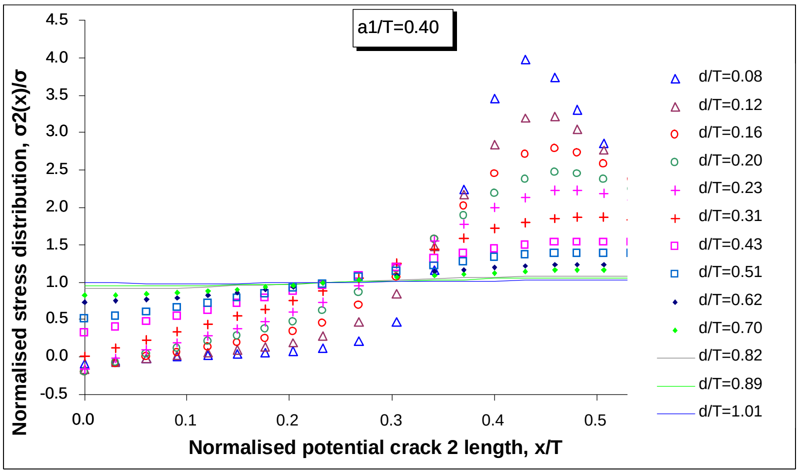

Fig. 3. Results of stress distributions along x/T with a1/T = 0.40.

A succession of models containing varying crack 1 lengths was generated using the mesh generator program. For each fixed value of crack 1, the y-component of stress, σ2(x) was measured along the potential crack 2 plane, x at crack separation d. Stress measurements were taken from x/T equal to 0.0 to approximately 0.50. The values of $\sigma 2(x)$ were normalised against the stress remote from the crack plane, σ_0. Using the same FE model, the measurements of $\sigma 2(x)$ were repeated at other d values. A total of nine FE models of varying crack 1 lengths were used for $\sigma 2(x)/\sigma_0$ measurements. A FE mesh with a1/T equal to 0.40 is shown in Fig. 3.

Complete results of $\sigma 2(x)/\sigma_0$ for all geometric parameters can be found in Ref. [7].

RELATIONSHIP BETWEEN THE NON-UNIFORM STRESS DISTRIBUTIONS AND THE INTERACTION EFFECT BETWEEN CRACKS

Results of stress distributions along the potential crack 2 plane shows that their variation is similar to the characteristics of the interaction effects as illustrated by the SIF results shown in Fig. 4. Variation of stress distribution along the potential crack plane and the interaction effect on SIF depend on the neighbouring crack length. A longer neighbouring crack length produces a stronger interaction and larger variation of stress distribution. A longer neighbouring crack needs larger crack separation in order to eliminate the interaction effect and stress distribution variation. The stress distribution study suggests that the interaction effect between two cracks is governed by the non-uniform stress distribution along the potential crack plane.

The calculation of the SIF using a weight function method requires the knowledge of the stress distribution along the potential crack plane. Under uniform applied stress, the stress distribution at the potential crack plane for a single edge crack will be uniform but for two edge cracks it will be non-uniform because of the crack interaction. It is proposed that by including this non-uniform stress distribution the SIFs of multiple cracks could be determined. In order to investigate this further it was necessary to establish the interaction effect in a general form so that it could be used directly with the SIF weight function.

A WEIGHT FUNCTION METHOD FOR THE CALCULATION OF SIFS FOR INTERACTING CRACKS

In the absence of any geometric discontinuities, the stress distribution in the potential crack plane, $\sigma(x)$ is the same as the nominal stress distribution. For example Fig. 4 (a) shows remotely applied uniform tension and therefore the stress distribution in the potential crack plane is also uniform. In order to calculate crack tip SIFs when two cracks are present the SIF weight function equation can be written as:

$$K_{a1} = \int_0^{a1} \sigma 1(x) m(a1, x) dx \quad (\mathrm{MN/m^{3/2}}) \tag{2}$$

and $$K_{a2} = \int_0^{a2} \sigma 2(x) m(a2,x) dx \quad (MN/m^{3/2}) \tag{3}$$

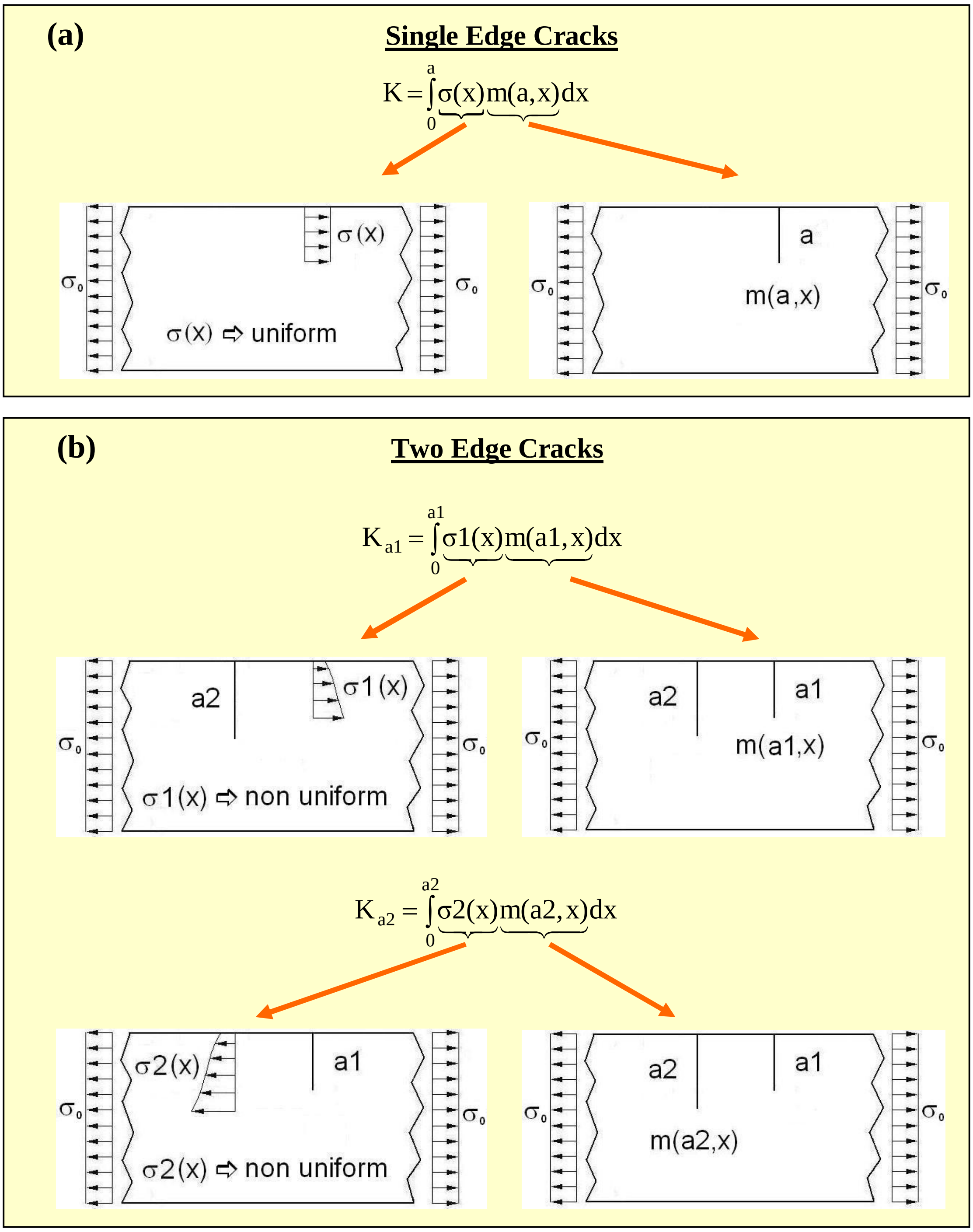

Fig. 4. (a) SIF weight function equation used for a single edge crack. (b) Modifications made to predict SIF weight function of two edge cracks.

Figure 4 above shows weight function m(a1,x) and m(a2,x) are both single edge crack weight functions, however, stress distributions σ1(x) and σ2(x) are not equal to the nominal stress due to the geometric discontinuity arising from the presence of the adjacent crack. These non-uniform stress distributions are used to model the interaction effect in solutions for K_{a1} and K_{a2}, the SIFs of crack 1 and 2 respectively.

VALIDATION OF THE MODIFIED WEIGHT FUNCTION METHOD

Fig. 5 below shows comparison of the Y_{a2} values obtained by the weight function method and FEA for different values of d/T and of a1/T = 0.125. Many other cases were compared and are detailed in reference 5. Overall results using the weight function method show good agreement with the FEA results especially for large crack separation. At very small crack separation the errors are greatest when the two cracks are approximately the same length. Most of the Y_{a2} values at very small crack separation underestimate the FEA values.

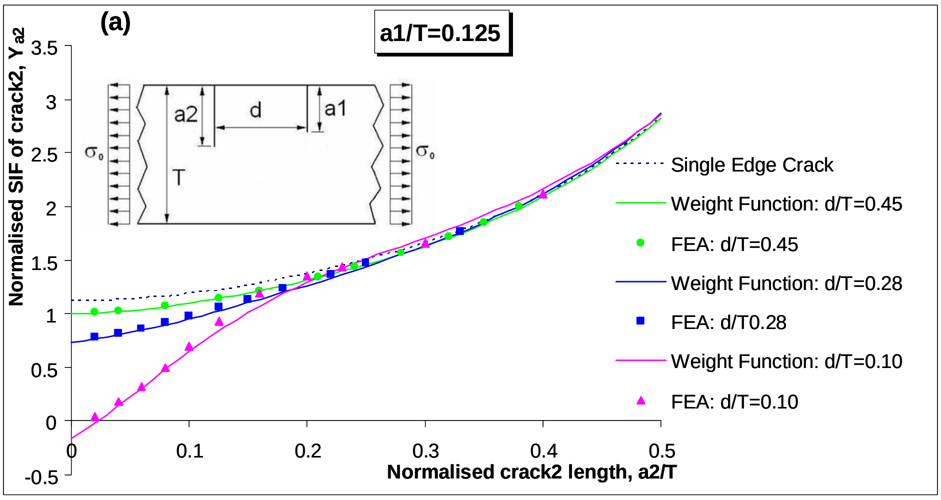

Fig. 5 Normalised SIF of crack 2 with different crack separation d/T and with a1/T = 0.125.

When compared to the FEA results, the maximum error of the weight function results for a1/T equal to 0.125, 0.25 and 0.375 were calculated to be 10.6%, 20.4% and 11.3% respectively. These maximum errors occurred at the smallest crack separation investigated for each crack length a1/T. It is likely that the maximum error for a1/T equal to 0.375 would be larger than 11.3% had the smallest d/T been used instead of 0.15. The error increases as a1/T increases. The weight function results show a very good correlation with FEA results for d/T value more than 0.50 with a maximum error less than 1%.

Normalised SIF values, for a single edge crack are also plotted in Fig. 5 as a comparison with Y_{a2} values. Values of Y_{a2} are similar to Y for cases where there is little interaction between cracks. For cases where there is no interaction between cracks both FEA and weight function results would be equal to the single crack results. For situations where an interaction between cracks exists, the results would be expected to be lower than the single crack solution due to a shielding effect. As crack 2 increases in length the shielding due to crack 1 is reduced, thereby reducing the interaction and the SIF solution would be expected to converge upon the single crack solution.

CONCLUSIONS

The non-uniform stress distributions due to the presence of an additional edge crack in a finite body under uniform tension can be used to establish the mode I crack interaction in a general form. With this crack interaction the traditional weight function method can be applied to predict the mode I SIFs of two edge cracks in a finite body under uniform tension. The weight function method has been shown to give reliable solutions for a wide range of geometric parameters.

Generally the accuracy of the modified weight function method is very good compared to FEA results. For small crack separations generally for d/T, less than 0.30, small disparities between weight function calculations and FEA results can be observed especially where cracks are of the same length. The most likely sources of error are due to the use of a single crack weight function and high stress gradients used to calculate SIFs. Errors were observed to be small for realistic crack situations. If two very short edge cracks were to initiate very close together, it is inevitable that one crack will become dominant and continue to grow while the other will arrest. The procedure therefore provides a valid method for the calculation of SIFs of high accuracy for problems concerning multiple cracks without the need for extensive finite elements computations.

Although the work contained in this paper is based on two edge cracks in a finite body under uniform tension, the results demonstrate that the weight function method can be used to determine mode I SIFs for multiple cracks provided that the stress distribution at the potential crack plane is known.

ACKNOWLEDGEMENTS

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