Crack Growth Trajectories under Mixed Mode and Biaxial Fracture

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ABSTRACT. *Two approaches are developed for geometrical modeling of crack growth trajectories for the inclined through thickness central cracks and the part-through surface flaw respectively. The principal feature of such modeling is the determination of crack growth direction and the definition of crack length increment in this direction. The damage process zone size concept is employed for calculations of mixed-mode crack growth trajectories and surface flaw shape and positions. Crack front behavior for a straight-fronted edge crack in an elastic bar of circular cross-section is studied through experiments and computations under axial tension loading. The elaborated theoretical model is applied for fatigue crack shape simulation of part-through cracks in a hollow thick- and thin-walled cylinders under different biaxial loading conditions. Suggested approach of crack paths modeling is used for an analysis and prevention of operation failures of existing in-service aircraft gas-turbine engine rotating components.*

INTRODUCTION

Main feature of mixed-mode fracture is that the crack growth would no longer take place in a self-similar manner and does not follow a universal trajectory that is it will grow on a curvilinear path. It is known that a “bent” crack does not propagate in its initial orientation direction. A mixed mode crack propagates along a definite trajectory which is determined by the stress state, the previous crack orientation angle and the material properties. For mixed mode crack propagation, the crack front is continuously changing shape and direction with each loading cycle. Under these conditions, in order to predict the fatigue life and crack propagation rate, it is necessary to determine crack paths on the base of experiments and calculations.

The assessment of both the form and size changes of the surface crack during propagation is an essential element for structural integrity prediction of the pressured vessels and existing in-service gas-turbine components in the presence of initial and accumulated operation damages. Therefore in the present work attention is paid on the mixed mode crack and the surface flaw behavior under different loading conditions.

CRACK PATHS UNDER MIXED-MODE LOADING

Two approaches are developed for geometrical modeling of crack growth trajectories for the inclined through thickness central cracks and the part-through surface flaw respectively. The principal feature of such modeling is the determination of crack growth direction and the definition of crack length increment in this direction.

Fracture damage zone concept

The crack growth from an inclined crack illustrates mixed-mode crack behavior on the initial crack. As follows from experimental data for materials of different properties, the angle of crack propagation θ^* lies in the range between the curves corresponding to the $\sigma_{\theta_{max}}$ maximum normal stress and the σ_e minimum effective stress criteria. Therefore the most general empirical criterion is obtained by Shlyannikov [1] on the basis of the limiting state theory of Pisarenko and Lebedev [2] and the fracture damage zone size

$$\chi\theta_1^*\left(\sigma_e\right)+\left(1-\chi\right)\theta_2^*\left(\sigma_{\theta_{max}}\right)=\theta^* \qquad (1)$$

in which $\chi=\sigma_t/\sigma_c$ is the experimental constant and σ_t is the tension static strength, σ_c is the compression static strength, $\theta_1^*(\sigma_e)$ and $\theta_2^*(\sigma_{\theta_{max}})$ - are the crack growth directions in accordance to the $\sigma_{\theta_{max}}$ and the σ_e criteria respectively. For brittle materials χ=0, while for plastic materials χ=1.

When applying any of fracture criteria to predict crack propagation, the point of view being that the stress-strain characteristics are not determined at the crack tip itself, but at some distance r_c from it. To take advantage of equation (1), it is necessary to define the sense of the radial distance r_c. Many of the fracture mechanics theories are based on a critical distance local to the crack tip. In the present work the critical distance r_c ahead of the crack tip is assumed to be located where the stress strain state in the element reaches a certain critical value that can be measured from a uniaxial test. Both relative fracture damage zone size (FDZ) $\overline{\delta}_c=r_c/l$ and crack growth rate model were introduced by Shlyannikov [1]

$$\overline{\delta}_c=\left\{\frac{\overline{S}_2\pm\left[\overline{S}_2^2-4\left(\overline{W}_c^*-\overline{S}_3\right)\left(\overline{S}_1+\overline{S}_p\right)\right]}{2\left(\overline{W}_c^*-\overline{S}_3\right)}\right\}^2,\quad \frac{dl}{dN}=2\overline{\delta}l\left(\frac{\sigma_n^2\overline{K}_f^2-\sigma_{th}^2\Delta\overline{K}_{th}^2}{4\sigma_f^*\varepsilon_f^*E\overline{\delta}}\right)^{\frac{1}{m}} \qquad (2)$$

$$\text{static loading}\quad \overline{W}_c^*=\left(\frac{\sigma_0}{\sigma_{yn}}\right)^2\left[\frac{1}{2}\overline{\sigma}_f^2+\frac{\alpha n}{n+1}\overline{\sigma}_f^{n+1}\right];\quad \text{cyclic loading}\quad \left(\overline{W}\right)_c^*=4\sigma_f^*\varepsilon_f^*\left(2N_f\right)^{-m}$$

In these equations σ_0 is the yield stress, σ_f is the true ultimate tensile stress, E is modulus of elasticity, ε_f^* is the fatigue ductility, σ_f^* is the fatigue strength coefficient,

n is the strain hardening exponent, $\Delta\overline{K}_{th}$ is the threshold value of stress intensity range, N_f is the fatigue life. In equation (2) $\overline{S}_i\left(i = 1,2,3\right)$ and $\overline{S}_p$ are elastic and plastic coefficients respectively. The work [1] contains more details about the determination of $\overline{S}_i = \overline{S}_i\left(\theta, \kappa, \beta, \eta, Y_I, Y_{II}\right)$ and $\overline{S}_p = \overline{S}_p\left(n, \nu, I_n, M_p, \widetilde{\sigma}_e, Y_I, Y_{II}\right)$ for the general case of mixed-mode elastic-plastic fracture.

Geometrical modeling of crack trajectories

Criterion (1) was applied for the crack path prediction for the two geometric configurations containing the single-edge and the central initial cracks of length a_0 and obliqueness β_0 as shown in Fig.1 (a) and (b). Crack path prediction for the mixed modes I and II initial crack involves replacing a bent crack with a staightline crack approximation, as shown in Fig.1. The principal feature such modelling is determination of the crack growth direction and definition of crack length increment in this direction.

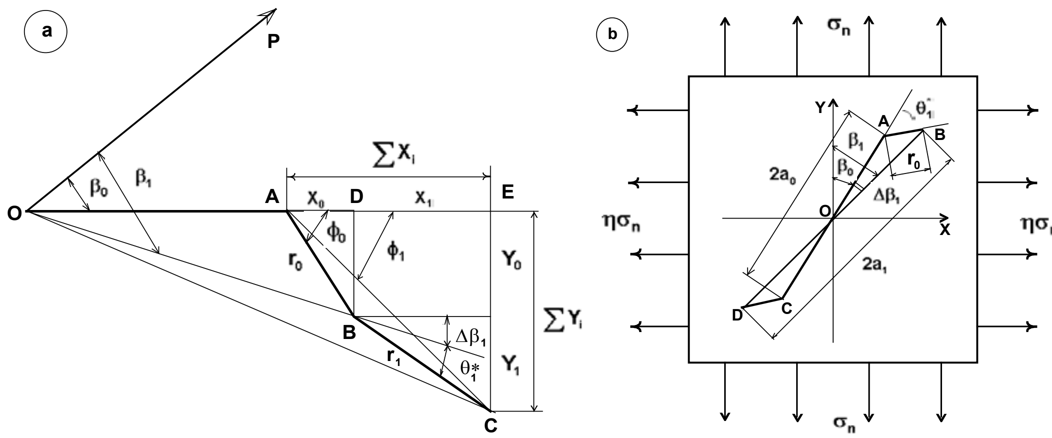


Figure 1. Crack growth trajectory approximation by fracture damage zone size, (a) single-edge crack geometry, (b) central notched biaxially loaded crack geometry.

Crack may be assumed to grow in a number of discrete steps. After each increment of crack growth, the crack angle changes from the original angle β_0 and so does the effective length of the crack. For the next increment of crack growth, one has to consider the new crack length a_1 and crack angle β_1. As shown in Fig. 1, OA is the initial crack length a_0 oriented at an angle β_0. Let r_0=AB be the crack growth increment for the first growth step. It would correspond to the FDZ size. Making use of equation (1), $\overline{\delta}$ and hence $r_0 = \overline{\delta} a_0$ can be computed. The value r_0 is then extended along AB with the angle θ_0^*. For the single-edge crack geometry (Fig.1,a) the first step of crack growth obtained as $\phi_0 = \theta_0^*$ and $x_0 = r_0 \cos\theta_0^*$, $y_0 = r_o \sin\theta_0^*$. The next step plotting r_1 along BC oriented at the angle θ_1^*, while AC=$\sqrt{\sum x^2 + \sum y^2}$, $\phi_1 = \tan^{-1}\left(\sum y / \sum x\right)$ and $x_1 = r_1 \cos\gamma_1$, $y_1 = r_1 \sin\gamma_1$, where $\gamma_1 = \Delta\beta_1 + \theta_1^*$,

$\sum x = x_0 + x_1$, $\sum y = y_0 + y_1$ and so on. For the central crack geometry subjected to biaxial loads (Fig.1,b) the crack path can be determined using the formulae [1]

$$\left\{ a_1 = \left[a_0^2 + r_0^2 - 2a_0 r_0 \cos\left(\pi - \theta_0^*\right)\right]^{1/2}, \quad \beta_1 = \beta_0 \arcsin \frac{r_0 \sin\left(\pi - \theta_0^*\right)}{a_1}. \right. \qquad (3)$$

Experimental data

Compact tension shear specimens (CTS) are made from a 30Cr steel and used for mixed-mode fracture test with the loading direction to an angle $\beta_0 = 0^\circ \div 90^\circ$ to the initial crack plane. Eight-petal specimens (EPS) are made from aluminum alloys and used for mixed-mode fracture test under biaxial stress ratio $\eta = 0.5$. The inclined angle of initial crack, β_0, was varied from 0° up to 90°.

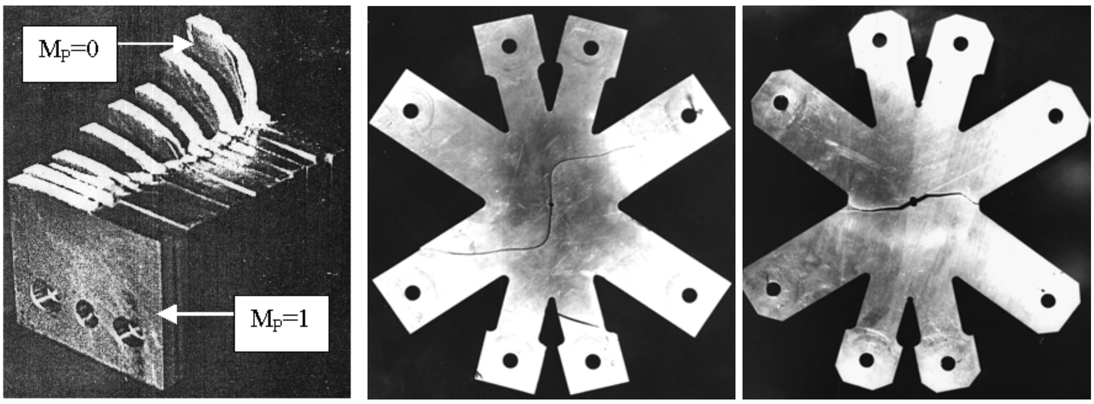


Figure 2. Experimental crack paths for compact tension shear and eight-petal specimens

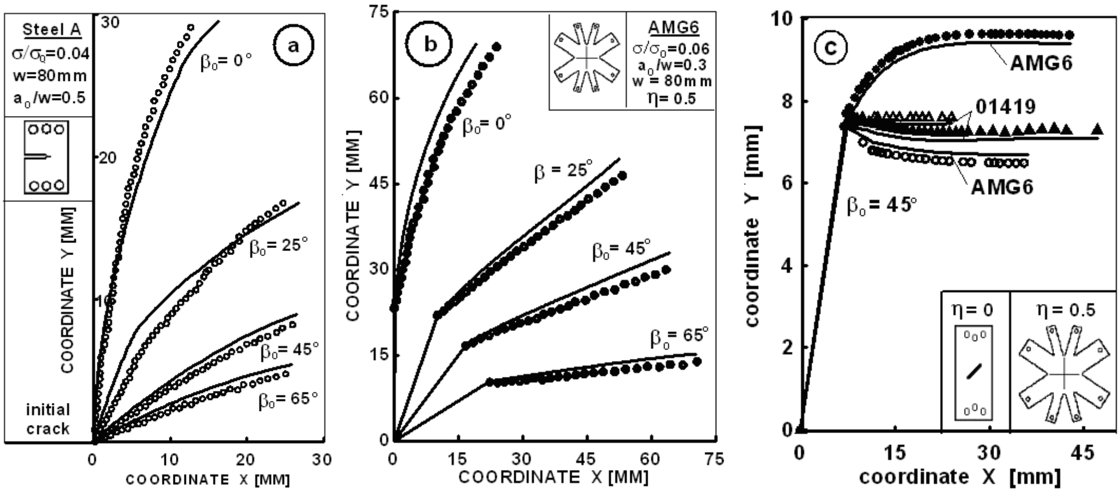


Figure 3. Comparison theoretical (curves) and experimental (points) crack paths for mixed mode loading (a) compact tension shear specimen, (b,c) eight-petal specimen.

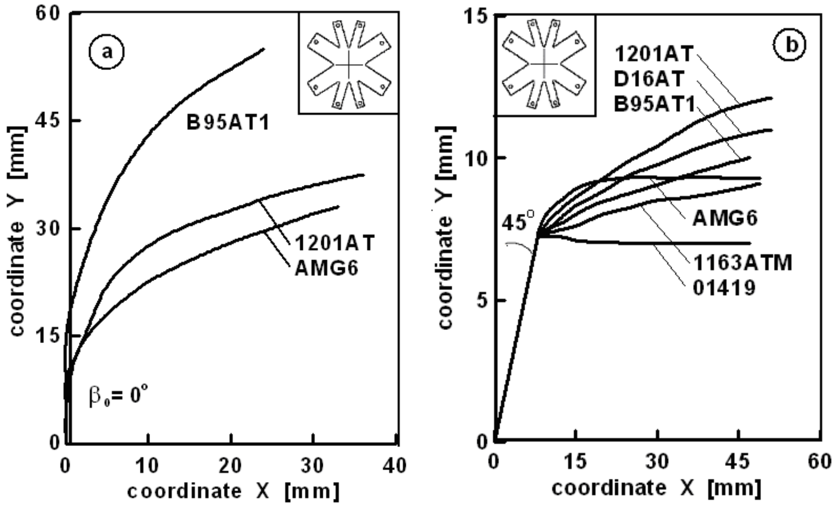


Figure 4. Influence of material properties on mixed mode crack growth trajectories under biaxial loading (a) η=0.5, β_0=0°, (b) η=0.5, β_0=45°

A typical experimental crack paths for specimen geometry considered are shown in Fig.2. Equation (3) is applied for analyzing the fatigue crack growth trajectories in specimens the above geometries. Figure 3 presents a comparison of both computational and experimental fatigue crack growth trajectories for 30Cr steel and for aluminium alloy. Their conformity suggests the validity of the fracture damage zone concept and hence equations (3) may be used in fatigue crack path calculations. Note that for eight-petal specimens under biaxial loading the amount of crack path curvature is a function of the main mechanical properties of the aluminum alloys (Fig. 4). A characteristic feature of equations (3) is the fact that they take into account an influence of both the material properties and nominal stress σ_{yn} on the crack growth trajectory via the angle of crack propagation θ^* defined by the equation (1).

SURFACE FLAW

Hollow cylinder

The elaborated theoretical model (Eq.2) [1] is used for fatigue crack shape simulation of part-through cracks in a hollow thick- and thin-walled cylinders under different biaxial loading conditions (Fig. 5). The initial defect is assumed to have an elliptical-arc shape. The propagation path of the surface flaw is obtained as a diagram of aspect ratio against relative depth. The numerical procedure calculates the local growth increments at a set of points defining a crack front by employing a fracture damage zone size model (Eq.2), that can directly predict the shape development of propagating cracks.

To substantiate the proposed model (Eq. 2), a comparison between the numerical and experimental results has been made for aspect ratio change. The experimental data

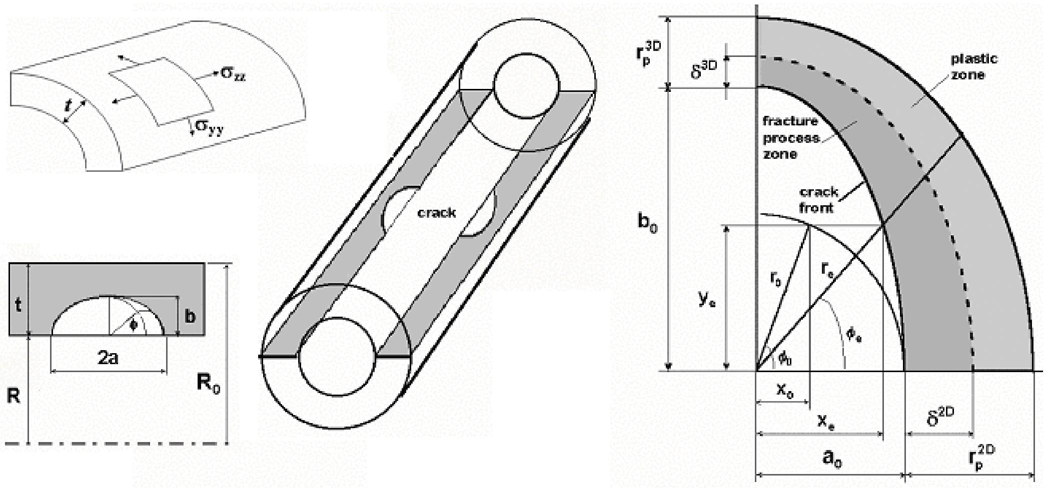


Figure 5. Basic geometry of problem. Simulating fatigue crack growth of crack front with varying defect sizes

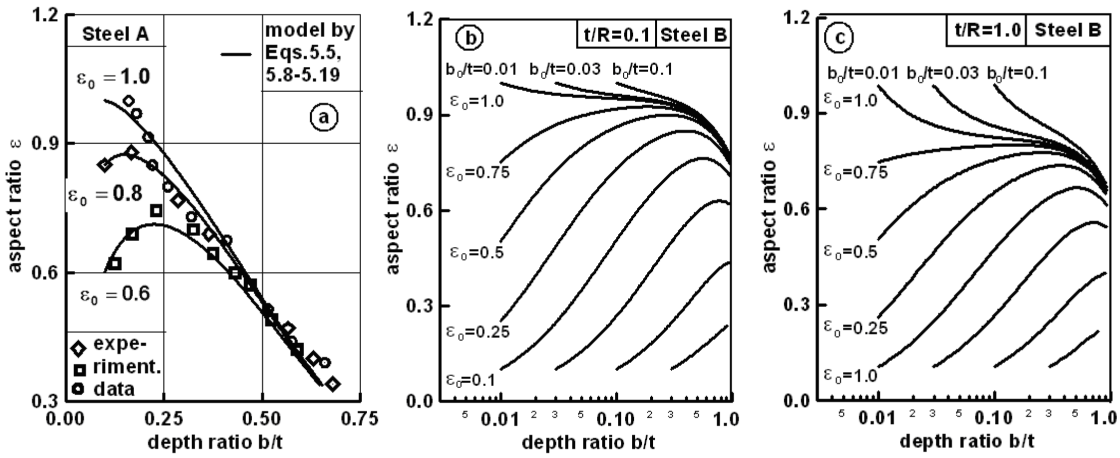


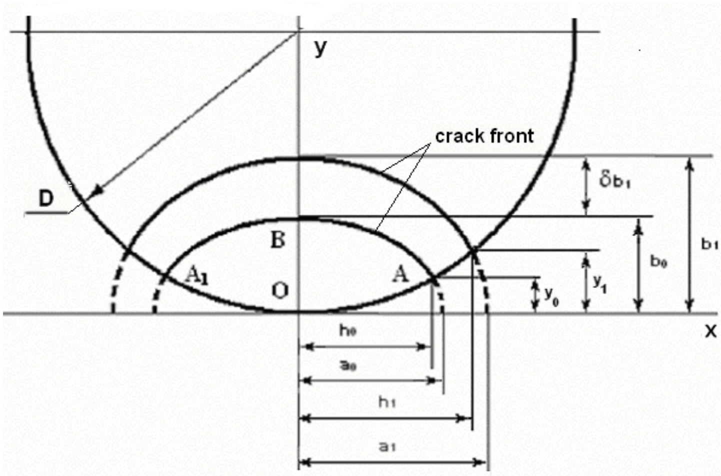
Figure 6. Predicted aspect ratio changes for thin- and thick-walled cylinders

reported in work [3] for a pressure vessel of cylindrical type made of steel has been compared with the simulation results obtained with the present model, which is shown in Fig.6,a. The flaw propagation paths determined for both t/R=0.1 and t/R=1.0 under cyclic loading of the steel B thin- and thick-walled cylinders are displayed in Figs 6,b and 6,c. The diagrams in Fig.6 show the advance of the crack front during the early stages up to the point of breakthrough. The crack behavior in thin-walled cylinder (Fig.6,b) and thick-walled cylinder (Fig.6,c) is different. The most remarkable feature of the early stage of growth is that the crack shape change is strongly dependent on the initial shape. It is shown that the surface defects in pipes tend to follow preferred fatigue propagation paths, that is, the flaw aspect ratio is a function of the relative crack depth. Also the influence of the elastic-plastic characteristics of the steels' and biaxial loading conditions on the aspect ratio variation is investigated.

An elastic bar of circular cross-section

Fatigue crack growth for a straight-fronted edge crack in a bar of circular cross-section is studied through experiments and computations under axial tension loading (Fig. 7). The aspect ratio during the crack growth process was predicted using the fracture damage zone size. The variation of aspect ratio is described by the following equation

$$\left(\frac{b}{a}\right)_i = \frac{\left[(b_{i-1}+\delta b_i)^2 - \left(\sqrt{(\delta d_i)^2-(h_i-h_{i-1})^2}+y_{i-1}\right)\right]^{1/2}}{R\sin(\theta_{i-1}+\theta_i)} \tag{4}$$



$$d_i = \sqrt{h_i^2 + y_i^2}\ ;\ \ b_i = b_{i-1} + \delta b_i$$

$$h_i = R\sin(\theta_{i-1}+\theta_i),\ R = D/2\,;$$

$$y_i = \sqrt{(\delta d_i)^2 - (h_i - h_{i-1})^2} + y_{i-1}$$

$$\theta_{i-1} = \arcsin(2h_{i-1}/D)$$

$$\theta_i = \arcsin(\delta d_i / D)$$

Figure 7. Basic configuration for edge crack in bar of circular cross section

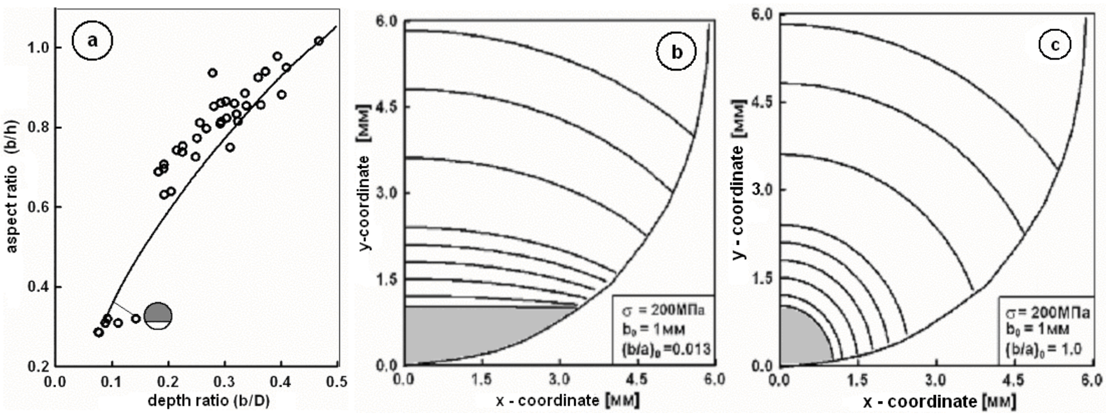
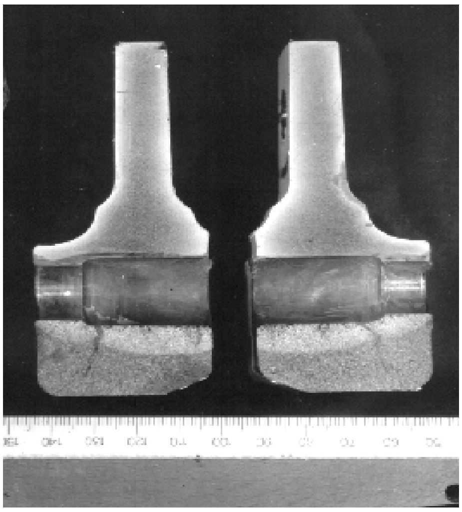


Figure 8. Propagation paths in circular bar for different initial flaw configuration

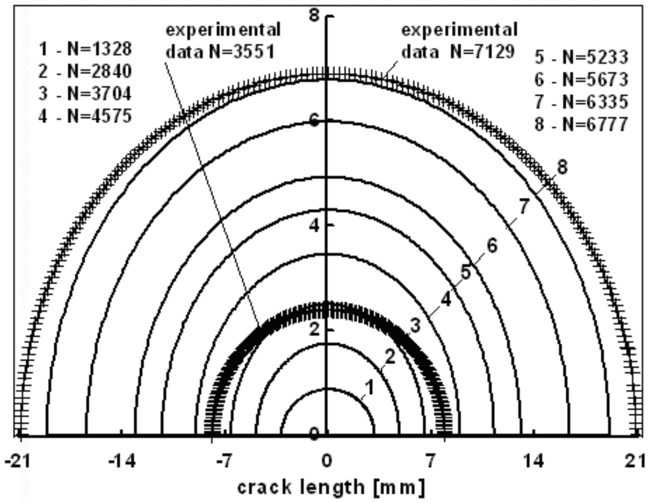
The relations between aspect ratio (b/c) and relative crack depth (b/D) are obtained and compared with experimental data (Fig. 8,a). It is shown that there is great difference in the growth of cracks with different front shapes and initial notch depths (Figs. 8,b and 8,c). Using the relations, predictions are made of the crack front shape and crack growth rate in the depth direction.

Practical applications

Suggested approach of crack paths modeling (Eqs.2) is used for an analysis and prevention of operation failures of existing in-service aircraft gas-turbine engine rotating components. Initiation and growth of surface flaws for compressor disk with defect in a disk and blade attachment and for turbine disk with damage on the inner surface of hole in a hub of wheel are considered.



a)



b)

Figure 9. Predicted and experimental fatigue flaw shape for turbine disk

Numerical crack growth modeling was performed to predict fatigue life, cyclic crack size and aspect ratio as a function of various stress distributions near the bolt hole in turbine disk and the slot fillet under blade for compressor disk. Equations 2 were used to calculate the fatigue life curves and crack front position in turbine disk. A comparison of the actual and predicted crack shape for the stage of crack growth is given by Fig.9. This figure shows the actual flaw shape on the fracture surface of the one of the bolt-hole specimens resulting from the application of the "marker" loads and the result of fitting a semi-elliptical curve through two readings observed on the surfaces – *a* and *b*. Figure 9 indicates accuracy of the description growing crack size and shape. It should be pointed out that any life estimate is strongly dependent upon the assumed both initial crack size and form.

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