Spiral Crack Path in Thin Sheets

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ABSTRACT. *Thin layers are commonly used in a wide kind of industrial products (from everyday packaging to airplanes) and are also frequently found in biological systems. The mechanics of thin sheets is rich and complex, with strong geometrical non-linearities leading for example to the intricate folds and singularities that we can observe in a crumpled sheet of paper. But here we show that the fracture path in thin sheets can follow remarkably regular geometrical path. We have observed crack path that evolved from an initial notch a few millimeter wide into a logarithmic spiral crack path that reached a meter in diameter. We present a model that explains the impressive regularity of this crack path.*

INTRODUCTION

Thin sheets and slender bodies are ubiquitous in industrial applications, which often try to reduce material weight. The study of their strength and rupture mechanism is therefore very important, and involves the coupling of out-of- plane bending (strong geometrical non-linearities) with crack propagation. Ductile materials are often chosen when thin plates constitute part of the mechanical strength of the structure. The rupture of such plates by a blunt tool studied in the case of ship grounding leads to interesting diverging crack path morphologies (concertina tears) [1].

Here we focus on the case of brittle materials, which are commonly used for packaging, since the opening process has to be easy. Using such material, we show that when a blunt object is pushed against the same fracture lip, the crack propagates in a very robust and reproducible spiral path. Indeed the shape is independent of the object shape, speed, or precise movement, as long as it always pushes on the same lip. Others spiral fracture path have been observed in the very different context of drying-induced crack propagation [2–4]. We characterize the spiral and show how a model developed in [5] for the case of a rectilinear displacement of the blunt ob ject (leading to oscillatory crack path) explains this surprising behavior and predicts the spiral shape.

THE EXPERIMENT

In our experiments, a brittle thin sheet (Bi-Oriented PolyproPylene, thickness t from 30 to 90[μm]) is clamped at its edges and a small (5[mm]) straight incision is made far from its boundaries. A blunt object, our tool, is placed inside the incision perpendicular to the sheet. With this object, we start to push on one edge of the incision UT (see fig. 1-a) and as the loading increases the crack eventually starts to propagate. We then displace the tool with the single following rule: the tool always pushes on the same lip (see fig.1-b). A curved path sketched progressively develops. In figure 1-c) is presented a picture of the final crack path obtained: a spiral that reached up to a meter in diameter in only about 2.5 turns. We stress the fact that in the experiment we do not specify the exact object displacement, as long it is continuously pushed against the same edge (see fig. 1a) and b). Despite this loose control procedure, the final spiral crack is impressively smooth and reproducible. What sets the final shape?

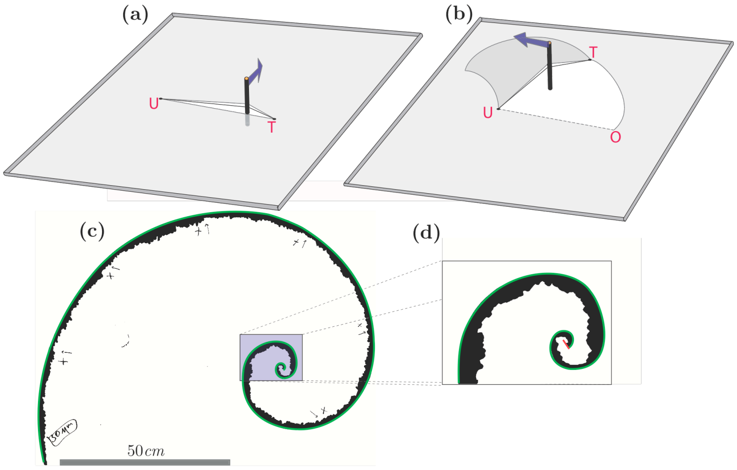

FIG. 1: Diagrams of the experiment in initial (a) and intermediate (b) stage. (c) Picture of the final spiral crack path (30μm thickness BOPP sheet). The edge is painted in black for contrast reasons, and a green line is superposed in order to present the path of the crack in the picture. (d) Zoom of the initial part of the spiral, the red line represents the initial notch.

MODEL

We follow here the approach developed in the study of a oscillatory crack path made by a blunt object [5, 6]. Although it is based on classical fracture mechanics theory and thin sheet elasticity, it surprisingly gives geometrical rules for the prediction of the crack propagation.

Elastic energy in thin sheet is dominated by in-plane stretching energy (scaling as the thickness t) since the bending elastic energy only scales as t^3 and will not contribute to crack propagation. We then define a soft region as the convex hull of the crack path (white area in fig. 2). This area is allowed to bend out of plane without generating stretching: when placed there, the tool will only produce out of plane deformation with negligible elastic energy. However if the tool moves out of this region, in-plane strain appears. For example in fig 2 the edge segment UT is being stretched by the tool being outside the white soft zone.

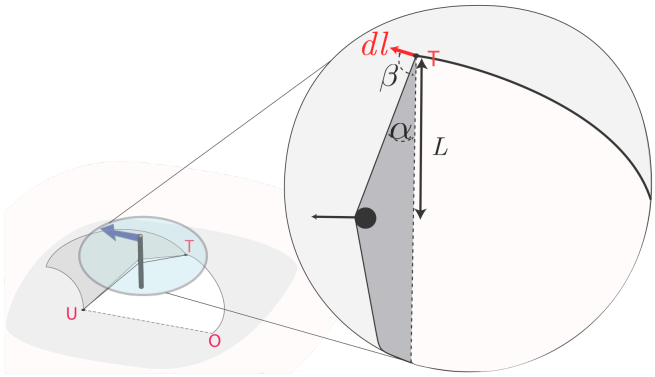

FIG. 2: Geometry of the fracture process. The white zone in the right is free to bend out-of plane and does not contribute to stretching elastic energy in the system.

The elastic energy can then be estimated by $\mathcal{E} \propto E\epsilon^2 At$, where E is Young's modulus of the sheet, ϵ is a typical strain, and A is the area of the stretched zone (roughly the dark gray in fig. 2). The strain ϵ is related with the change in the length of the segment *UT* and goes like $\epsilon \sim \alpha^2$ for $\alpha \ll 1$ (see fig.2). Finally, the elastic energy reads $\mathcal{E} \sim EtL^2\alpha^5$, since $A = L^2\alpha$. If we push more, both the angle α and the energy will increase, up to the point when it is more energetically favorable to propagate the crack tip *T* (Griffith criterion). Considering a crack advance by dl in the direction given by the angle β (see fig. 2) the soft zone advances, α decreases by $d\alpha = \cos^2\alpha(\tan\alpha\cos\beta - \sin\beta)dl / L$. Propagation takes place if the elastic energy release rate $d\mathcal{E} \sim EL^2t\alpha^4 d\alpha$ compensates the crack energy $\Gamma t dl$, i.e. when α reaches a critical value α_c. A second equation comes from the maximum energy release rate criterion that specifies that the crack propagate in the direction defined by $\frac{\partial}{\partial\beta}\frac{d\alpha}{dl} = \tan\alpha + 1/\tan\beta = 0$. Finally, propagation takes place when:

$$\alpha = \alpha_c, \quad \alpha_c \sim \left[\Gamma/EL\right]^{1/4} \tag{1}$$

$$\text{in direction } \beta = \alpha + \frac{\pi}{2}. \tag{2}$$

This set of equation is remarkable since it would predict crack propagation using only geometric quantities (the angles α, β), although it is based on fracture mechanics. Since the elastic energies are localized (grey area in figure 1-a), the shape of boundary conditions plays no role. A second surprise is that equation (2) establishes that crack propagates in a well-defined direction β with respect to the unstressed edge *UT*, *independently of the shape or movement of the pushing tool*. Indeed the changes in L modify the critical angle α_c with such a low exponent that we consider the value α_c to be constant. This constant-angle condition explains the final spiral shape.

To understand the crack path we can identify three stages of the cutting process, where the geometry of the soft-zone, are different. (*i*) In a first initial stage (fig. 3-a and b) the soft zone ends on a line containing a fixed point *O*. The crack tip *T* then propagates with a constant direction with respect to the radius *OT*. In polar coordinates centered in O, the radius $r = r_0 e^{-\cot(\beta)\theta}$. This is a logarithmic spiral with center *O*. (*ii*) After half a turn, the soft zone, changes morphology (fig. 3 -c) and the crack now propagates around another fixed point, the other end of the initial notch. The model thus predicts another logarithmic spiral, with the same pitch but another center. (*iii*) Finally after another half turn (see fig. 3 -d), the edge of the soft zone does not stop on fixed point, but constantly has a tangent contact with previous part of the curve. The crack path develops around itself in a complex way.

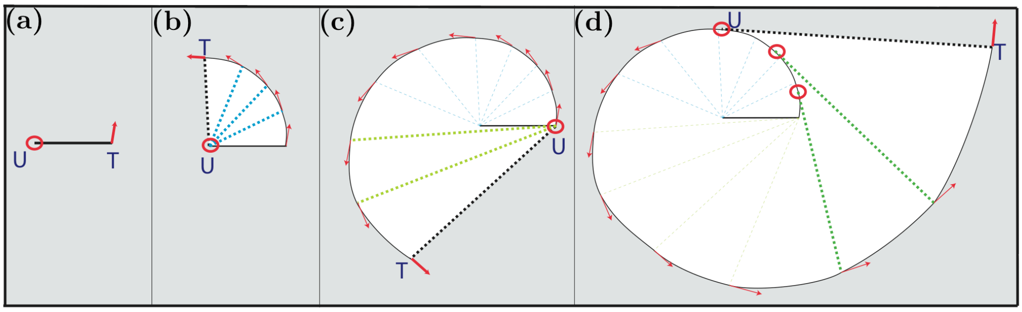

FIG. 3: Different stages in the evolutions of the spiral. (a) and (b): First stage where the fracture is growing around a center point *U*. (c) Second stage defined by the path growing around another single point. (d) Third stage defined by the path growing around a point that develops onto itself.

EXPERIMENTAL RESULTS

We now turn to experimental test of the predictions. The first two stages are predicted to be logarithmic spirals, but they don't span a large radius difference and the prediction is not easy to test. We focus on the last stage (which governs most of the spiral) and show that it leads to spiral is again scale-less, logarithmic spiral with a different pitch.

The spiral shape was digitalized, and assuming a logarithmic shape, a center point was defined by the following procedure. If two points in the spiral have parallel tangents, then the center must be lie in the line that joins them. If we repeat this procedure again and find two another points, then the center is in the interception of the two lines. This procedure is sketched in figure 4-a).

In the semi-log plot in fig. 4 we show the distance to the "center" as a function of the angle for all points of three different spirals. Two of them are initiated with a notch in the same direction in order to show reproducibility of the process, and the third one is initiated with a notch in a perpendicular direction. The crack path is highly reproducible as we can see from the two spirals initiated with the same conditions.

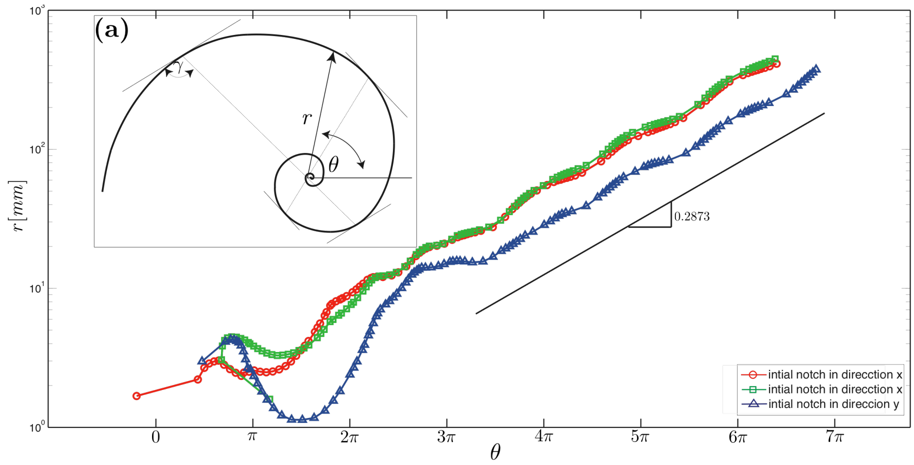

FIG. 4: logarithm of distance of points to the center as a function of angle for three different spirals (□ and ○ : same experimental conditions; △ : initial notch in a perpendicular direction). Inset: geometrical construction used to find the center of the spirals.

In all spirals, after a little more than one turn the distance grows exponentially with the angle: the spiral starts to behave as a logarithmic spiral. In fact it makes sense that the beginning of the plot is not an exponential, because these points are from the firsts stages of the spirals, which are describe from another center. Although the behavior is very close to a logarithmic spiral (linear plot in fig.4), we can observe some oscillations. To better understand this feature, we study the fracture direction at each point from the final in figure 5.

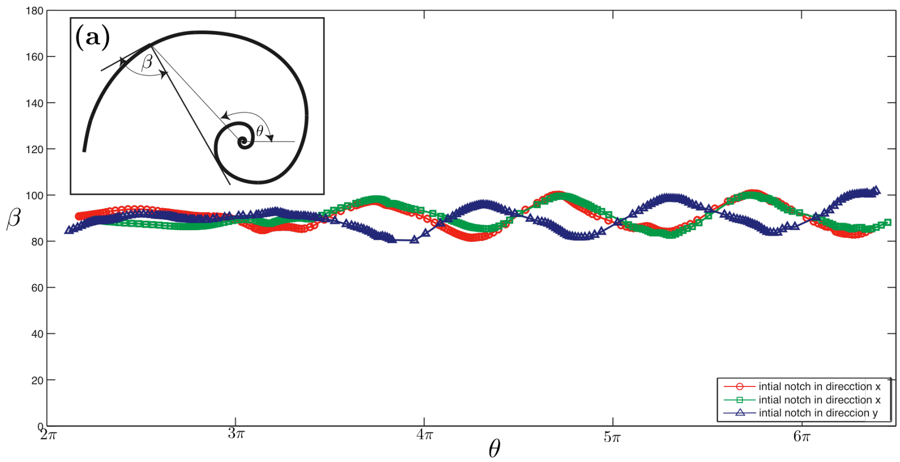

FIG. 5: Fracture angle β measured on the three spirals path as a function of the orientation in the sheet. Same symbols as in figure 4.

From the model, one expects a value of β larger than $\pi/2$ but the measurement show that the actual value fluctuates around $\pi/2$. The values of β have a π-periodicity with respect to the angle θ. We also note that the two spiral initiated in the same direction are out of phase by $\pi/2$ with the other one. This is coherent with the condition used to initiate this third spiral, a notch in a perpendicular direction if we interpret the variations of β as being anisotropy of the fracture properties of the material. As a result, the study of the shape of the spiral obtained very easily gives a measurement of the anisotropy in fracture properties.

CONCLUSIONS

In this paper we have presented a highly reproducible spiral crack path that we interpret using fracture theory arguments that showed that the propagation of cracks in brittle thin sheets could be predicted using geometry.

In addition, a very interesting result from this work is that the regularity in the observed oscillation for the fracture angle allows us to test with a single experiment the anisotropy of the material.

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