The Tearing of Thin Sheets

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ABSTRACT. *We performed an experimental study in order to investigate the stability of crack propagation in a thin elastic sheet under an out-of-plane shear mode (mode III). We find that a single propagating crack always follows a straight path, while two simultaneously propagating cracks interact: their paths merge, forming a tongue-like shape. Moreover, the experimental setup makes it possible to understand how energy introduced at a large scale is focused at the crack tip. We find that, although the loading induces out-of-plane deformations, the material is locally broken in an opening mode (mode I).*

INTRODUCTION

The stability of crack paths under in-plane deformations is now a classical topic [1]. In contrast, there are few experimental observations on the stability of crack propagation under out-of-plane shear deformations (mode III) [2, 3]. In the case of thin films, such a configuration is illustrated by the tearing of a sheet of paper. The interaction between two cracks was investigated in the case of thin films adhering to a hard substrate [4, 5]: when the film is pulled from between two initial notches, two straight, non parallel cracks propagate and merge.

In order to understand mode III crack propagation in thin films, we built a controlled setup enabling the study of both the stability of a single crack path and the interaction between two propagating cracks. This experimental setup also allowed the description of the focusing of the bending elastic energy from large scales to the crack tip, serving to create new surfaces.

EXPERIMENTS

Experimental Setup

We built an apparatus to control crack propagation at constant velocity in a thin elastic sheet, in an out-of-plane shear configuration. The apparatus uses four parallel Plexiglas cylinders, of radius 8 mm and length 20 cm, set between two parallel metallic plates (Fig.1). The two upper cylinders are free to rotate, while the two lower Plexiglas cylinders are driven at the same constant velocity by a motor, so that the velocity at the surface of the cylinders is in the range 0.05 to 1.5 m.s^{-1}. The elastic films are made of polypropylene and have isotropic mechanical properties. In order to study the propagation of a single crack, we initiate a centered cut in a long rectangular sample that we insert between the free cylinders. One side of the cut is fixed on one of the driven cylinders (after passing above the corresponding free cylinder) while the other side of the cut is fixed on the other driven cylinder. The two upper cylinders impose the free crack free length, replacing the action of fingers. The lower cylinders collect the torn parts of the sheet. The crack tip is generally located midway between the upper cylinders (Fig.1). In order to study the propagation of two cracks, we use the same procedure: we start with two cuts that are symmetric with respect to the central axis of the sheet, determining three parts in the sheet; the two external parts are fixed on one of the driven cylinders while the middle part is fixed on the opposite cylinder.

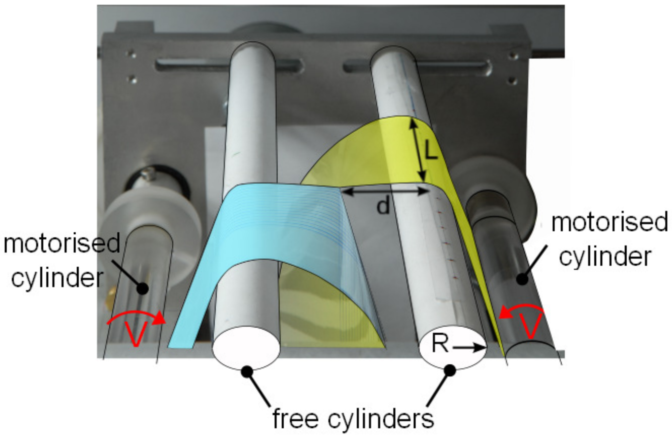


Figure 1. The tearing machine was designed for an out-of-plane shear loading at a constant velocity. A thin film of width $2L$, in which we initially make a centered cut, is introduced between the two free (upper) cylinders. One side of the cut (in front) is coloured in blue while the other (back) is coloured in yellow. Each side of the cut is fixed to one of two (lower) cylinders, which are driven at the same constant velocity. The crack tip is located between the free cylinders, roughly at the same height as the top of cylinders. The horizontal projection of the free part of the crack has length d.

Parameters

We control the following parameters: the distance $2d$ between the two upper cylinders (d is the length of the free part of the crack), the crack propagation velocity, the sample width L and the thickness h of the sheet. We use films of bidirectional polypropylene of thickness 15, 30, 50 and 90 µm and Young's modulus $2.2 \pm 0.4\,GPa$. At ambient temperature and in our range of tearing velocities, the fracture process is brittle for this material. We varied the width of the film from 2 to 16 cm and the crack free length d from 1.8 to 5 cm. This experiment requires a careful adjustment of the parallelism of the cylinders' axes in order to keep a symmetric loading.

Material toughness

We measured the critical toughness K_c of polypropylene films using the approximation of the infinite plane, given by $K_c = \sigma\sqrt{\frac{\pi}{2}l}$ where l is the length of the pre-crack and σ is the magnitude of the far-field stress. We force the crack in opening mode by applying a constant displacement on the edges of a rectangular sample of dimensions 22cm x 10cm x 30 μm in which we initialized a centered notch of length l. We measure the value of the applied force when the crack begins to be propagated. Initial values of crack length l range from 0.4 to 6 cm. We obtained $K_c = 2.6 \pm 0.3\ MPa.m^{1/2}$.

Stability of the Crack Path

Single Crack

In a first experiment, we studied the stability of the crack path when a perturbation is applied. We used a film of width 8 cm with an initially centered crack, and with three incisions of 1.4 cm at an angle of 45° to the axis. These incisions are used to perturb the path.

The initial crack path is straight. When the crack meets an incision, the configuration becomes asymmetric; nevertheless the crack keeps a straight path (Fig. 2) that is roughly parallel to the axis of the sample. The crack path is thus stable to perturbations. It is also stable when the crack is off-centered. This makes the behaviour of the crack path fundamentally different from the case of in-plane propagation under opening mode for which the crack either remains centered or is unstable.

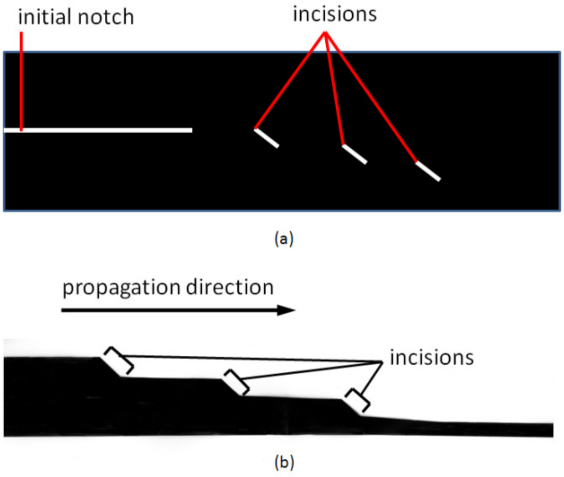


Figure 2. Stability of the crack path. (a) Schematic of the film before tearing: we make a centered cut in the film and prepare inclined incisions in order to perturb the crack path. (b) Scan of the film after tearing: the path appears to be stable under a perturbation, in an asymmetric configuration or when close to a boundary.

Two Cracks

We force two cracks to propagate simultaneously in the sample. We control the distance between the two cracks, which are initiated symmetrically around the central axis, the width of the sample and the crack free length. When the ratio of the width of the central part with the width of the sample is small enough $(l/L < 0.1)$, the central piece of the sample detaches itself from the rest: crack trajectories are curved toward the central axis. The torn part of the sheet has a symmetric tongue-like shape (Fig.3); experiments are reproducible. If the middle part is too wide $(l/L > 0.1)$, experiments are less reproducible and the shape of the tear is not always symmetrical.

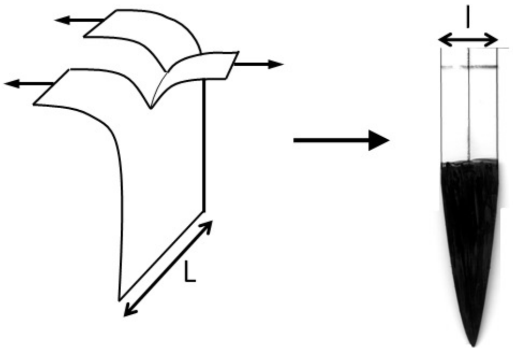


Figure 3. Two cracks propagating simultaneously. The middle part of the film forms a tongue-like shape and detaches itself.

We digitized the tongue-like shapes and measured their width as a function of the distance to their tip. At a given sheet thickness, we observe that profiles can be superposed for different values of the initial width *l*; these profiles are well described by a power law (Fig.4), of exponent between 2/3 and 3/4. We checked that the crack free length does not influence this shape.

Three-dimensional shape of the film

In this second set of experiments, we considered the three-dimensional geometry of the film in the course of tearing: three regions can be distinguished (see Fig.1). A first region, far from the crack, is bent by gravity and it does not seem to feel the presence of the crack (region 1 in Fig.5). A second region hangs under the apparatus and is tilted with respect to the cylinders axes. In a third part, in the neighbourhood of the crack tip (region 3 on Fig.5), the film is flat; this region has a shape close to a triangle with a vertex at the crack tip. The flatness can be interpreted as a sign that this region is stretched, containing most of the elastic energy that can be used to open the crack. Therefore we investigated the geometry of this third region.

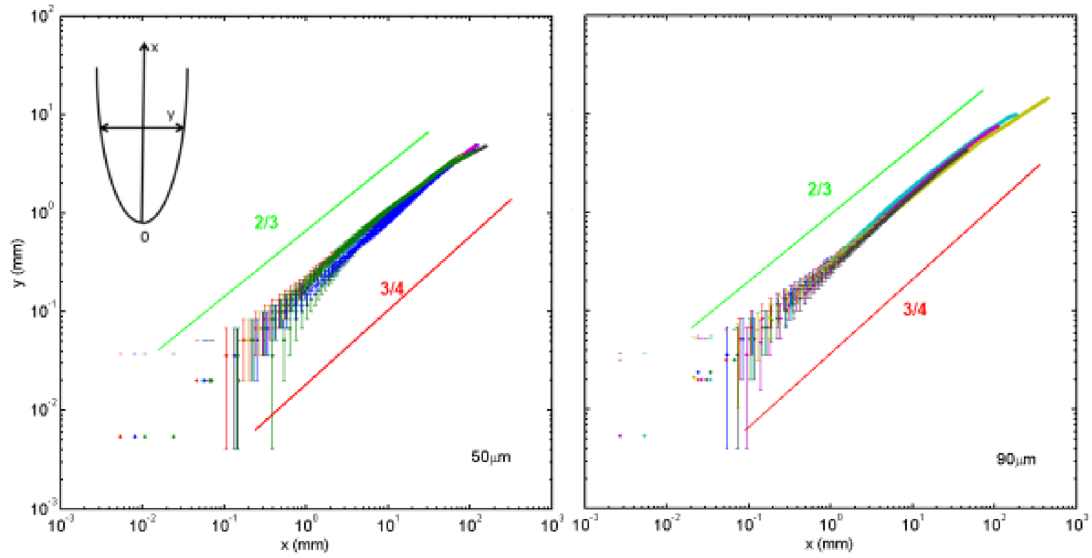


Figure 4. Width of tears as a function of the distance from their tips for two sets of experiments: film thickness is 50 (right) or 90 μm (left); the ratio varies beetween 0.025 and 0.1. The crack free length was kept at *d*=2.6 cm.

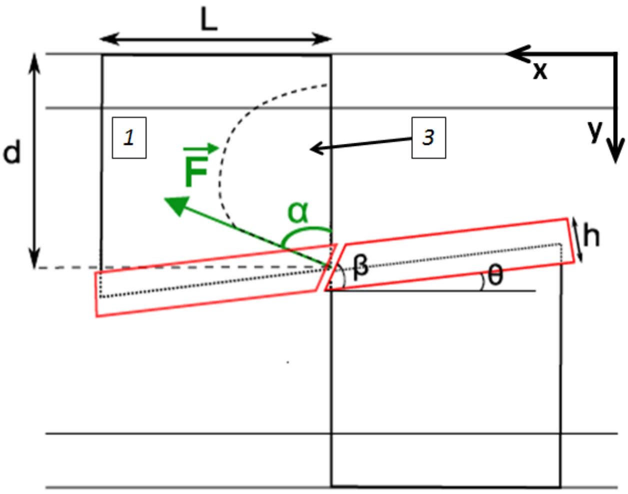


Figure 5. Schematic of a top view of the experimental setup: the red rectangle shows a horizontal cut across the hanging vertical part of the film, with a magnified thickness. The opening angle of region 3 at the crack tip is α, the bevel angle across the thickness of the torn film is β, while θ is the angle between the hanging part and the axes of the cylinders.

We used image analysis to measure the three-dimensional shape of the film as explained hereafter. We note that the curvature in the *x*-direction is very small in comparison with the curvature in the *y*-direction, so that we neglect it. To visualize the film curvature, we draw on the film parallel lines in the y-direction, separated by 2 mm: these lines are straight only in region 3. We used degree-2 polynomial fits to measure the curvatures of these lines. We find that the flat region (3) is bounded by the crack and a straight line, yielding a characteristic angle α (Fig.5). We measured α for an extensive range of experimental parameters (Fig.7a); we found an almost constant value $\alpha = 58° \pm 4°$ that is insensitive neither to the aspect ratio L/d nor to the film thickness; in particular, it appears to be uncorrelated to the angle of tangent L/d determined by the crack and the line going from the crack tip to the intersection of the free edge of the sheet with the top cylinder. (These two angles are clearly different except when they become too close.)

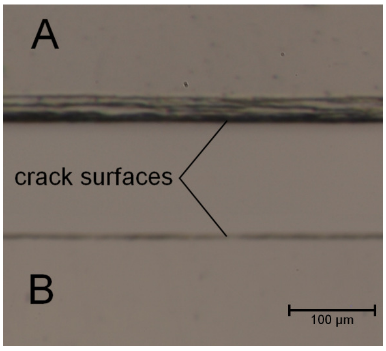


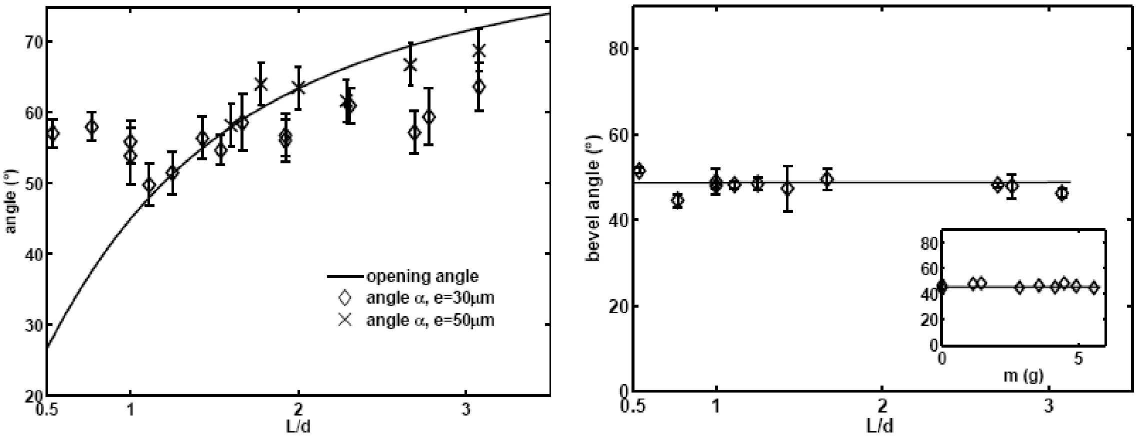
Figure 6. Magnified view of the crack surface, as seen with a microscope. **A**: Crack propagated in out-of-plane shear mode. Crack surface is darker because of the rugosity of the new surface. The width of the dark region shows that the crack surface is tilted, defining a bevel across the thickness of the film. **B**: Crack propagated in opening mode. The new surface appears to be sharp, showing that it is normal to the surface of the sheet.

Post-mortem, we observe a bevel in the thickness of the sample (Fig.6). Using a top view of the crack surface, we deduce the bevel angle, named β (Fig.5) from the thickness of the film and the apparent width of the crack surface. We find that it is independent of the experimental parameters (L/d and the thickness of the film) and has a value of $45° \pm 3°$ for bidirectional polypropylene (Fig.7b).

DISCUSSION

Force Direction

We interpret the limit of the flat region as a line of tension in the sheet, which would correspond to a large scale force pulling on the crack tip. As, we found that $\alpha = \beta + \theta$, this means that this force is perpendicular to the crack surfaces (Fig.5). In other words, the crack propagates in a pure opening mode (mode I), although a large scale out-of-plane shear mode is imposed.



(a) (b)

Figure 7. (a) Vertex angle α of the "triangular" flat region as a function of the aspect ratio L/d (the opening angle is defined by the crack and the line joining the crack tip to the contact point of the film edge with the top cylinder). (b) Bevel angle β of the crack surface, as a function of the parameter L/d. The film had a thickness $h = 30\ \mu m$, while $d = 2.6\ cm$ and $L = 4cm$.

An open question is whether the bevel angle of 45° is preffered because of a structural reason or the loading imposes the force direction and the crack plane has to be perpendicular to it.

Force Intensity

We elaborated a method to measure the intensity of the resultant force. We performed experiments where we increased the mass of the film by hanging weights at the edge of the sample that is below the crack tip. The masses ranged from 1.45 to 7 g (the film mass is 0.8 g). A first observation is that the angle α is not affected by the mass of the film. However, the crack tip departs from the plane determined by the top of the free cylinders (Fig.8): the larger the hanged mass m, the lower the crack tip. We measured the height of the crack tip z, corresponding to an angle δ of the crack edge with the horizontal direction (see Fig.8). As the bevel angle does not vary, the area of the new surfaces is not affected by the weights, so that the necessary surface energy does not change. As a consequence, we may posit that the intensity of the force is independent of the experimental geometrical parameters:

$$\boldsymbol{F} = F_0\left(sin\alpha\ \boldsymbol{u_x} - cos\alpha\ sin\delta\ \boldsymbol{u_y} + cos\alpha\ cos\delta\ \boldsymbol{u_z}\right) \qquad (1)$$

with $tan\delta = \frac{d}{z}$. Then, after projecting force balance on the z-axis, we obtain:

$$z = \frac{gd}{2F_0 cos\alpha} m \text{ for } \frac{z}{d} \ll 1 \qquad (2)$$

Indeed, we observe in experiments that z varies linearly with d (Fig.8) when $0.1 < \frac{z}{d} < 0.3$. We could not investigate a wider range of masses because, when m becomes too large, a stick-slip-like phenomenon appears, such that crack propagation is not quasi-static anymore. The force intensity is obtained with a linear fit as $F_0 = 0.25 \pm 0.05\ N$. We compare this intensity with the minimal force necessary to propagate a crack in this material, as given by the critical toughness K_c. In the experiment, we observed that a force is applied in the horizontal plane far from crack tip. The typical distance l between the effective application point of this force and the crack tip is of the order of the thickness. The necessary force to propagate the crack is given by $F_{th} = K_c h \sqrt{\frac{2l}{\pi}}$. For the $30\mu m$ thickness film, we find $F_{th} = 0.3N$ which coincides with the measured value $0.25N$ within the experimental uncertainty.

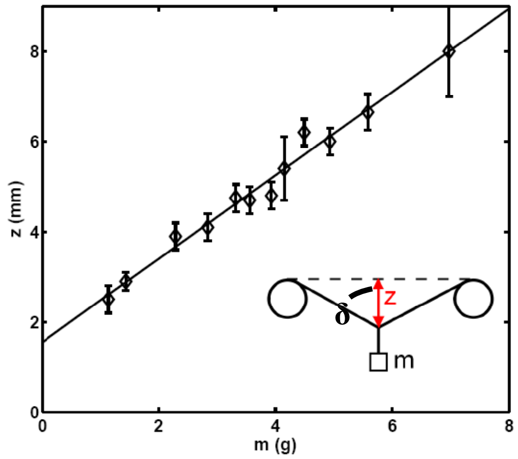


Figure 8. The height of the crack tip z as a function of the added mass m at the bottom of the film, with $h = 30\ \mu m$, $d = 2.6\ cm$ and $L = 4cm$.

CONCLUSION

This experimental study suggests that an isotropic, homogeneous material locally breaks in mode I, whatever the imposed mode (I, II or III) at a larger scale, because this opening mode is the most efficient to separate two surfaces. However the large scale configuration dictates the crack stability: we show that the crack is always stable under our (large scale) mode III loading whereas it can be unstable under mode I.

REFERENCES

1. Goldstein, R.V. and Sagalnik, R.L. (1974), *Int. J. Fract.* **10**, 507.
2. Mai, Y.W. and Cotterell, B. (1984), *Int. J. Fract.* **24**, 229-236.

3. Lazarus, V., Buchholz, F.-G. and Wiebesiek, J. (2006). In: *Institute of Fundamental Technological Research,* Polish Academy of Science.
4. Atkins, A.G. (1995), *Endeavour* **18**, 2-10.
5. Hamm, E., Reis, P., Leblanc, M., Roman, B. and Cerda, E. (2008) *Nature Materials*, vol.7, Issue 5, pp.386-390.