The Influence of Constraint Level on Crack Path

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ABSTRACT.
The main aim of the work was to estimate effect of the constraint on the predicted crack path and show the cases where the failure of the component could be described more precisely by constraint-based description. A special specimen geometry with a high level of constraint was used. Numerical predictions of the crack path in a modified CT specimen are compared with experimental results. The numerical model was built using commercial software ANSYS and a special procedure for the crack propagation was developed. The numerical prediction of the crack path based on a modified Maximum Tangential Stress (MTS) criterion (the criterion uses a two-parameter description of the stress field around the crack tip) is generally in agreement with the experimental data. The classical MTS criterion was found much more sensitive on the crack increment. It can therefore be concluded, that the modified criterion in the framework of two-parameter linear elastic fracture mechanics estimated the crack path more precisely only in the case of a high level of constraint.

INTRODUCTION

Crack propagation in a non-homogenous stress field generally displays a complicated trajectory. Accurate estimation of the crack path can aid the prediction of unexpected failures in engineering structures. The usual assessment of the crack trajectory and crack propagation rates is based on a phenomenological approach. According to classical linear elastic fracture mechanics two cracks display similar behaviour if the stress intensity factors are equivalent. Recently it has been shown that in some cases two-parameter fracture mechanics which take into account the constraint effect can describe the crack tip stress field more accurately. Consequently, the criteria used for estimation of the fatigue crack trajectory were also modified. There exist numerous different mix-mode criteria such as the maximum tangential stress criterion, the maximum energy the release rate criterion or the criterion based on the strain energy density factor [1]. All these criteria minimize mode II of the crack propagation, and predicted crack paths are more or less similar, especially for the small ratio K_{II}/K_I. For the work presented the commonly known (Maximum Tangential Stress) MTS criterion was used for estimation of the crack path. A special geometry with high level of constraint (modified CT specimen) was used to prove experimentally numerical predictions for the crack path calculated on the basis of classical and modified MTS criteria. The sensitivity of the numerical results with regard to finite element mesh, crack increment and constraint level is discussed.

CRACK DEFLECTION CRITERIA

In the framework of linear elastic fracture mechanics, the stress state near the crack tip is usually described by the stress intensity factor. In the case of a two-parameter description, T-stress is used to define the level of the constraint. Then the tangential stress near the crack tip can be determined by expression [2,3]:

$$\sigma_{\theta\theta} = \frac{1}{\sqrt{2\pi r}} \cos\frac{\theta}{2}\left[K_I \cos^2\frac{\theta}{2} - \frac{3}{2}K_{II}\sin\theta\right] + T\sin^2\theta, \tag{1}$$

where r, θ are the polar coordinates with their origin in the crack tip. K_I, K_{II} are the stress intensity factors for loading mode I and II and T is elastic T-stress.

For estimation of further crack propagation direction the knowledge of stress intensity factors K_I for loading mode I (opening mode) and K_{II} for loading mode II (in-plane shear) is necessary under conditions of one-parameter linear elastic fracture mechanics (LEFM). Using the MTS criterion it is possible to estimate the direction of the further crack propagation from following expression [4,5]:

$$K_I \sin\theta_P + K_{II}(3\cos\theta_P - 1) = 0 \ , \tag{2}$$

where θ_P is the crack propagation angle, see Fig. 1.

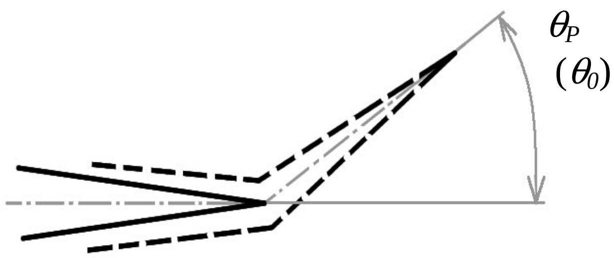

Figure 1. Estimated angle of crack propagation

In the case of two-parameter linear elastic fracture mechanics the MTS criterion must be modified in order to take in to account the influence of the constraint. The constraint is quantified by the T-stress and the modified MTS criterion can be expressed as [6,7]:

$$\left.\frac{\partial\sigma_{\theta\theta}}{\partial\theta}\right|_{\theta=\theta_o}=0,$$
$$\therefore\left[K_I\sin\theta_0+K_{II}(3\cos\theta_0-1)\right]-\frac{16T}{3}\sqrt{2\pi r_c}\cos\theta_0\sin\frac{\theta_0}{2}=0\ , \tag{3}$$

where T is T-stress, r_c is the size of the process zone, which must be determined and θ_0 is the further crack propagation direction.

There is a non-consistent view in the literature as to how to define parameter r_c. Some authors consider r_c to be a material constant [3, 6], some as a plastic or process zone [7]. In this work, the r_c is connected with the plastic zone size near the crack tip:

$$r_c=\frac{1}{\pi}\left(\frac{K_I}{\sigma_y}\right)^2\ , \tag{4}$$

where σ_y is the yield stress of the material studied.

EXPERIMENTAL MEASUREMENT

Experimental measurements on a modified CT specimen were performed to validate the numerical predictions. A standard CT specimen with two holes was chosen for experimental work, see Fig. 2. This geometrical configuration induces high level of the constraint in region where the crack is supposed to propagate.

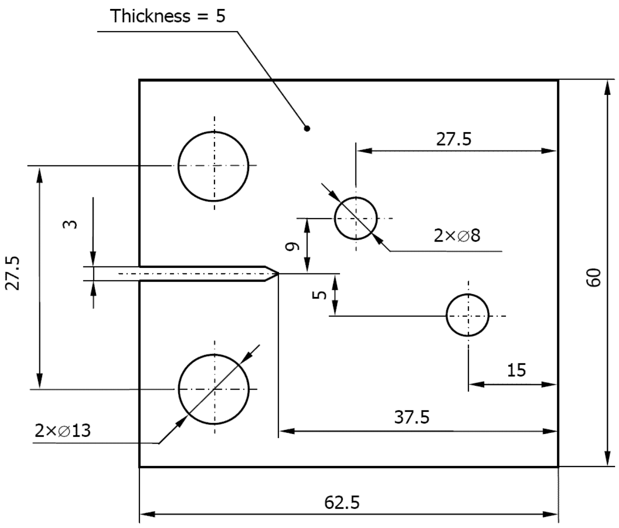

Figure 2. CT specimen used for experimental measurements

Three sets of experiments were performed in order to avoid errors such as internal defects or differences in the material behaviour or geometry of the specimens. The material used for the experiments was low carbon steel (labelled in Czech standard 15 313), see Table.1 for details of chemical composition. The experimentally determined material properties are Young's modulus E = 2.1×10^{5}MPa, Poisson's ratio ν = 0.3 and cyclic yield stress σ_o = 330MPa.

Table 1. Steel composition (wt %)

	C	**Mn**	**Si**	**P**	**S**	**Cr**	**Ni**	**Cu**	**Mo**
15 313 steel	0.10	0.60	0.40	0.035	0.035	2.3	0.60	0	1.05

All three experimentally determined paths had very similar trajectories. The average experimental data (see. Fig.3) were used for comparison with results obtained by numerical estimations.

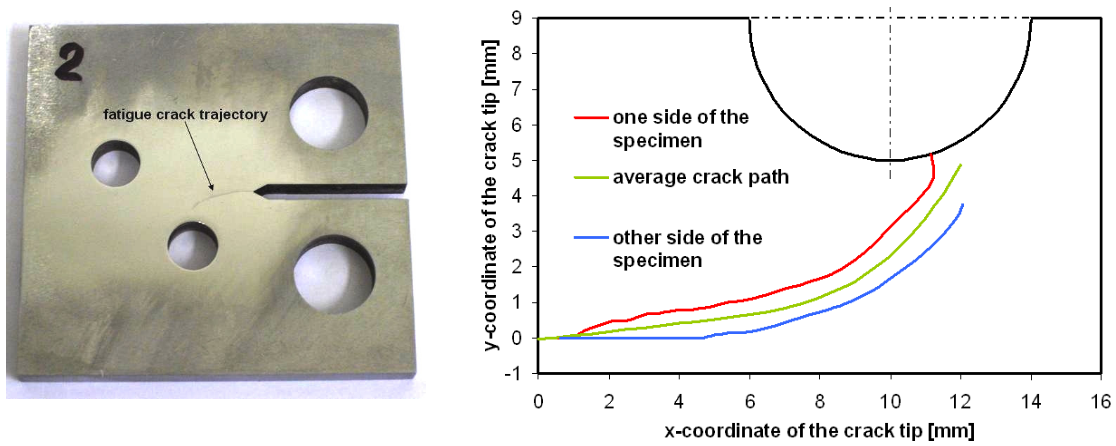

Figure 3. Experimentally obtained data of the crack paths

During all experiments the crack deflection leading to the first hole was observed, see Fig.3. Therefore, from the experimental point of view the crack behaviour was clearly determined.

NUMERICAL SIMULATION

The numerical model follows geometry of the CT specimen (see Figs. 2 and 4). The finite element mesh is non-homogenous with refinement in the vicinity of the crack tip in order to avoid numerical errors due to a strong stress gradient. Special crack elements simulating the stress singularity of $r^{-1/2}$ were used, see [2,9,10] for details. FE analysis was performed under plane strain conditions.

The crack paths were studied under the classical MTS criterion and modified MTS criterion in order to determine the influence of the constraint effect on the predicted path. The crack started in the V-notch of the CT specimen. The size of the initial crack was assessed as 1mm and the initial crack angle was adopted from the experimental observations. Crack propagation was simulated using incremental method, see [6,7,11]. Due to dependence of the predicted path on the increment size, the sensitivity analysis of the crack increment on the predicted crack path was done. The increment varied between 0.025 mm and 1 mm in order to find appropriate conditions for the numerical calculation, where the resulting path is independent of the crack increment. The initial crack angle was assumed as 0° for this analysis. The influence of the constraint effect was not taken into account. It was found that the increment sizes of 0.1mm and 0.025mm give similar but not identical results, see Fig. 5. The appropriate crack increment sizes of 0.25mm, 0.1mm and 0.025mm (for verification) were chosen and used for the following simulations.

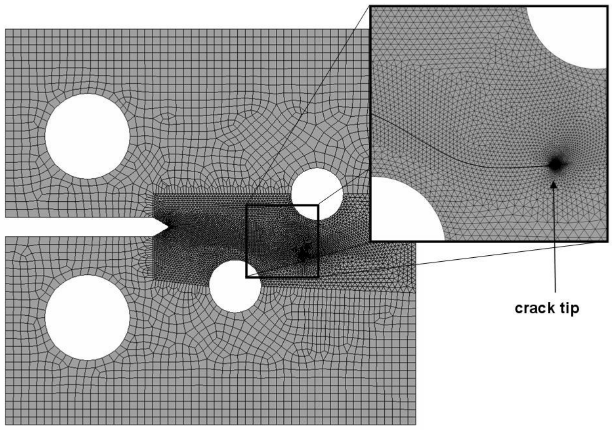

Figure 4. Typical mesh of finite elements used for numerical simulations.

A high level of constraint was expected in the presented modified CT specimen. Results of the numerical simulations using MTS (Eq.2) and modified MTS criteria (Eq.3) are shown in Fig.6.

A significant difference in the predicted paths was not observed until the crack reaches the length of 12mm. All criteria predicted the smooth trajectories that are close to the experimental path. Then the radical difference between the predicted paths occurred. After comparison between the path predicted using classical MTS (Eq.2) and modified criterion (Eq.3) for crack increment 0,25 mm, an important discrepancy was found. Eq.2 predicted crack propagation inside the specimen, which is in contradiction with experimental observations. The crack path estimation based on Eq.3 does not perfectly fit experimental data, but this estimation showed interaction between the crack and the nearer hole. The estimation of the crack path based on original form of MTS criterion is sensitive to crack increment size in comparison to its modified version, where the crack increment size plays an unimportant role (see Fig. 6), however with decrease of the increment size, the estimated path is closer to the experimentally observed one. To obtain approximately similar crack path trajectories the original MTS criterion needs a ten times smaller crack increment size than modified one. The considerable time-saving in the case of the modified MTS criterion is obvious. The computation time for the crack increment of 0.25mm was approximately 50× shorter than in case of 0.025mm.

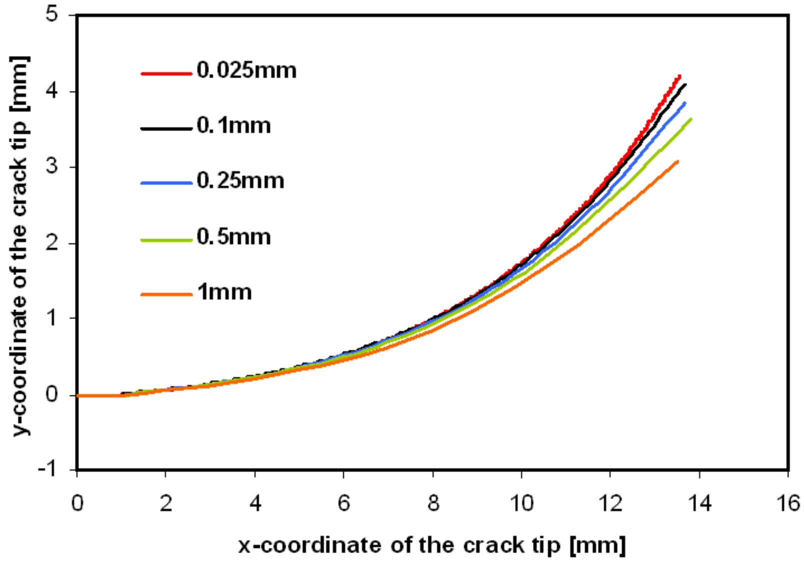

Figure 5. Predicted crack path calculated for different crack increment sizes in case of use of MTS criterion

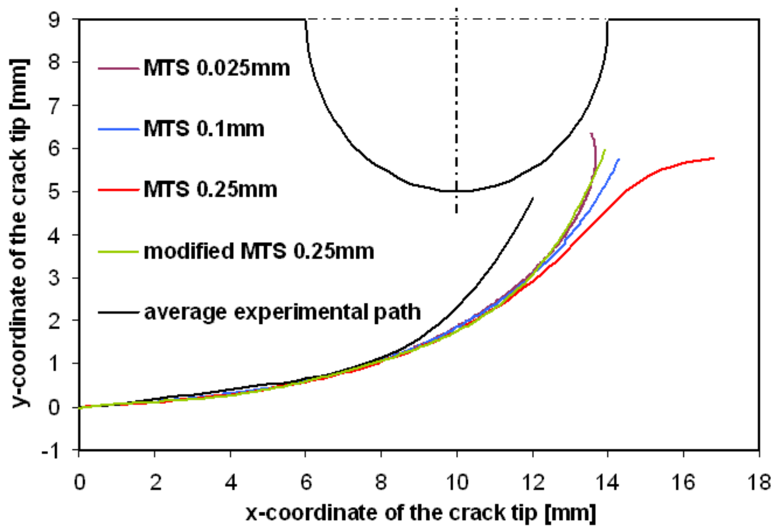

Figure 6. Predicted crack path calculated using MTS criterion and modified MTS criterion

The accuracy of the numerical estimations was more dependent on the crack increment size than on the criteria used. The smaller crack increment leads to better accuracy of prediction. Even for the modified MTS criterion, the difference between the experimental and numerically predicted path is still noticeable. Based on the literature data [12] and our own study it can be concluded that the accuracy of the MTS criteria depends on the level of the constraint. In the study presented the level of the constraint was expressed by the biaxiality parameter *B* defined as follows [2]:

$$B = \frac{T\sqrt{\pi a}}{K_I}. \tag{5}$$

The distribution of the biaxiality parameter along the crack path is shown in Fig. 7. If *B* is negative, the influence of the constraint on the crack propagation direction is negligible as well as for *B* = 0, see e.g. [12] for details.

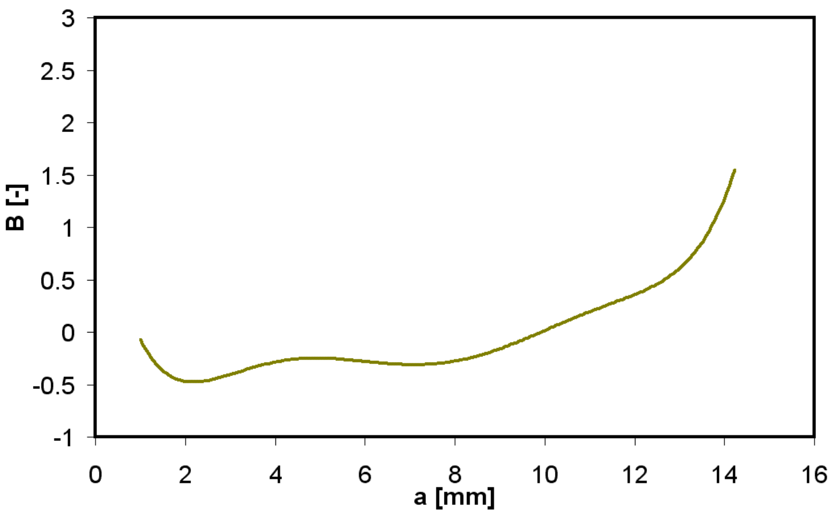

Figure 7. Dependence of biaxiality parameter *B* on the crack length *a*

For high values of *B* the influence of the constraint can be significant and can change the crack propagation direction, see Fig. 6 where the difference between the crack path predicted by the MTS criterion and the modified MTS criterion for a crack increment of 0.25mm is shown.

CONCLUSIONS

The effect of constraint on crack propagation direction was studied. The modified geometry of a CT specimen was chosen for testing. For this geometry, the mixed mode fracture behaviour and high level of constraint near the crack tip were expected. Two different forms of the MTS criterion (classical and considering T-stress) were used for numerical estimation of the crack propagation path in the geometry referred to. The results obtained were compared with experimental data to validate the numerical predictions and to find which form of the criteria considered is advantageous and more accurate under given conditions of high constraint level.

Results show that both the criteria are in agreement with experimental observations when a sufficiently small crack increment is used for simulation. The modified version of the MTS criterion converges more quickly to the real crack propagation path in comparison to the classical one. Therefore, in this case, the advantage is a significant reduction of computational time and smaller sensitivity to crack increment size.

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