Numerical simulation of crack paths in materials with a functional gradation perpendicular to the crack orientation

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ABSTRACT. *Within this paper several numerical simulations of cracks in a three-point-bending specimen with non-homogeneous material behaviour are under consideration. Thereby a functional gradation of the material as well as sharp interfaces are examined. In those simulations the initial crack is oriented perpendicular to the direction of the gradation. In the simulations it becomes apparent that in case of a sharp interface between two materials the mismatch ratio of the Young's moduli play an important role in the development of the crack paths. The crack is subjected to a distinct Mixed-Mode loading, that causes a (more or less) slight kinking towards the weaker material. The stress intensity factors calculated in those simulations show excellent agreement with already known results [1]. All numerical simulations are carried out with the three-dimensional crack simulation program ADAPCRACK3D, which has been developed at the Institute of Applied Mechanics at Universität Paderborn.*

INTRODUCTION

Functionally graded materials gain more and more attention in modern engineering, since they allow to meet the specialized demands to structural components in a simple and elegant manner. However, the material inhomogeneity arising from the functional gradation requires special treatment in the fracture mechanical assessment of such structures. Especially an automatic simulation of fatigue crack growth is much more complicated than in the homogeneous case.

Besides the influence on the absolute value of the stress intensity, that is caused by a material gradation or an interface, respectively, also a Mixed Mode loading might be induced. As a consequence the crack will change its direction and grow in a completely different manner.

Therefore this paper will present an examination of the stress intensity factor development and the crack paths in three-point-bending specimens, in which the initial crack is oriented perpendicular to a (nearly continuous) material gradation or within the interface in between two materials. The numerical evaluations will be carried out with the three-dimensional FE-based crack growth simulation program ADAPCRACK3D, that is ideally suited for such simulations also in non-homogeneous materials.

ADAPCRACK3D

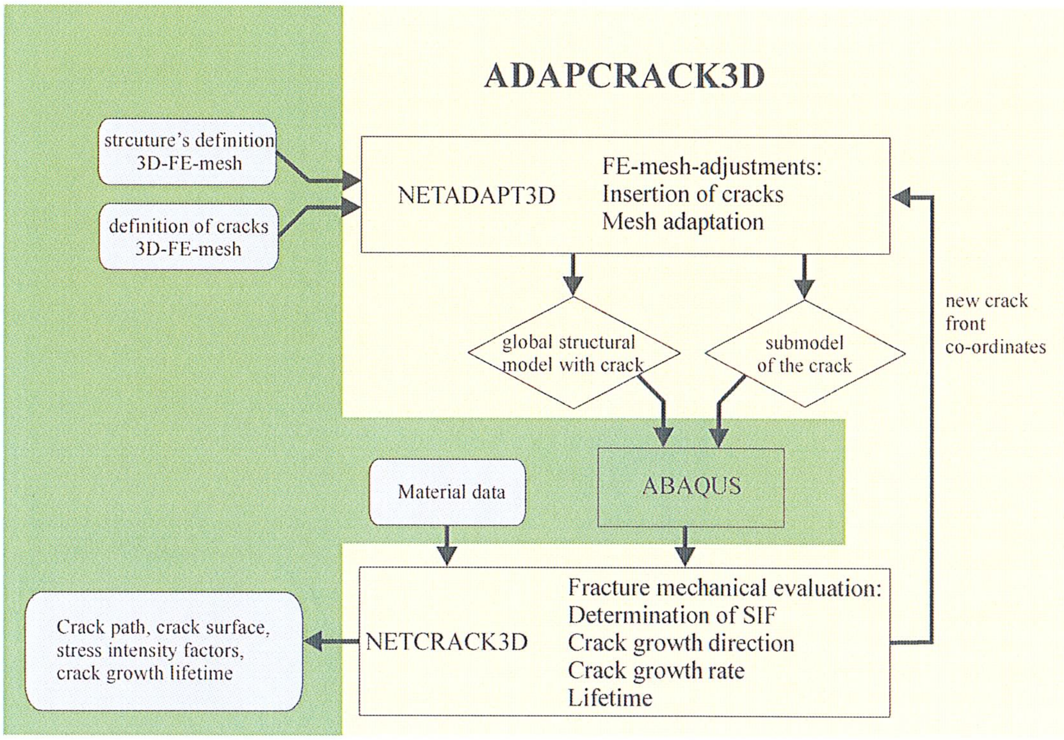


Figure 1. Simplified functionality scheme of the crack growth simulation program ADAPCRACK3D

ADAPCRACK3D consists of three independent modules, which in combination perform the fracture mechanical simulation (Figure 1). The necessary mesh adaptation that arises during a simulation is conducted by the module NETADAPT3D, while in the third module NETCRACK3D all fracture mechanical evaluations are carried out. The (linear-elastic) FE-computations are performed with the commercial code ABAQUS^{TM}. Dealing with inhomogeneous material behaviour in place of (classical) homogenous material requires especial mesh adaptation procedures, that are in detail explained in [2]. Further information concerning the functionality and capabilities of the program can be taken from [3-5].

THREE-POINT-BENDING-SPECIMEN

The three-point-bending specimen that are under consideration in this analysis are dimensioned according to experiments conducted by Marur and Tippur [6], see Figure 2. In those experiments material 1 was an Epoxy with E_1=3490MPa while material 2 on the glass-rich side had an E_2= 10790MPa. Between those "pure" materials, there is a gradient region with a width of 21mm. In [6] it was assumed, that the mechanical properties vary linearly over this gradient region from Material 1 to 2. The initial crack length of the three-point-bending specimen is set 6,6mm.

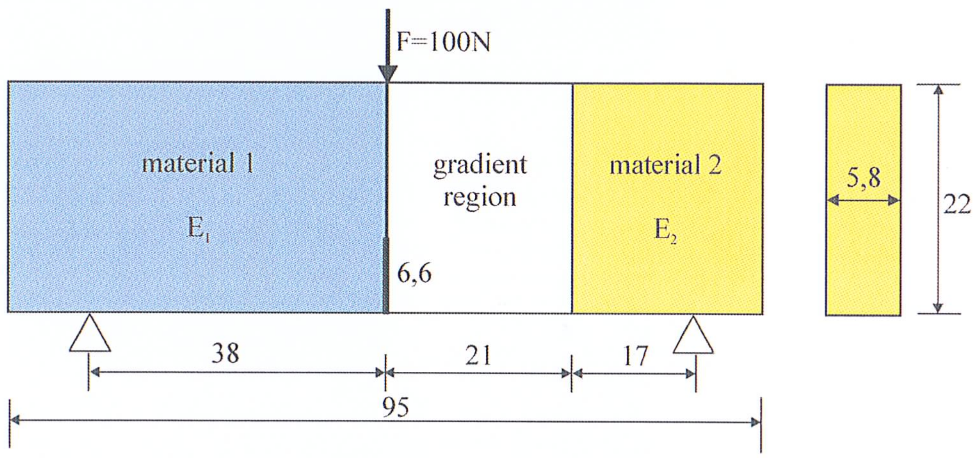


Figure 2. Three-point-bending-specimen with functionally graded material

In the subsequent evaluations two variations of this basic layout will be taken into consideration. As in the FE-calculation of ADAPCRACK3D standard elements with a distinct Young's modulus for each element are applied, the (continuous) gradient region is approximated by a number of interlayers (Fig. 3a). As a consequence the initial crack is located in an interface between a stiffer and a weaker material. The elasticity mismatch of the two adjacent materials of course then is a function of the number of interlayers chosen for the approximation of the gradient. In order to thoroughly investigate the behaviour of a crack located in an interface, in a second approach the gradient region will completely be defined as material 2, which allows to study the evaluating crack paths in dependence of the elasticity mismatch of the two materials (Fig. 3b)

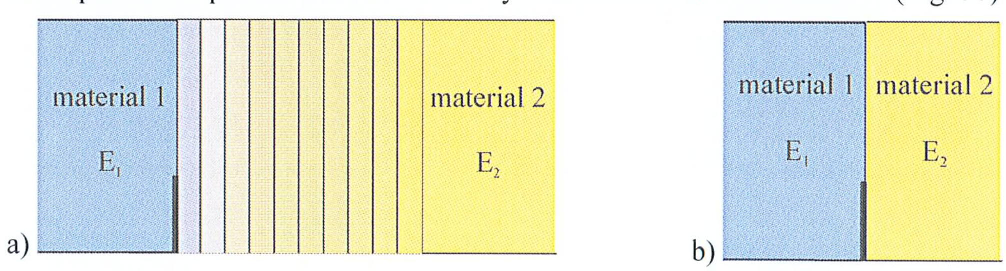


Figure 3. Material configuration in simulations: a) interlayers between material 1 and 2
b) sharp interface

NUMERICAL SIMULATION OF CRACK GROWTH

Preliminary investigation of stress intensity factors in a functionally graded material

Since in standard FE-analyses only elements with each a constant Young's modulus can applied, it is an important question, in how far a continuous variation of the elasticity can be approximated by a number of interlayers as depicted in Fig. 3a. If so, it needs to be evaluated, which number of interlayers is necessary in order to obtain reasonable results not only for the stress field, but for the stress intensity factors in cases of a crack occurrence. Therefore a number of simulations with different approximations of the gradation region are carried out. The results are shown in Table 1 in comparison with results by Marur/Tippur[6] and Kim/Paulino[1] .

Table 1. Simulation results for 3pbs with functional gradation

	# interlayers	K_I [MPam$^{1/2}$]	K_{II} [MPam$^{1/2}$]
ADAPCRACK3D	2	0,521	-0,026
	5	0,536	-0,020
	10	0,542	-0,017
	15	0,541	-0,008
Marur/Tippur[6]		0,589	-0,033
Kim/Paulino[1]		0,557	-0,028

It can be seen, that the discrete approach with interlayers with piecewise constant elasticity is justified by very good results for the stress intensities. As expected the obtainable results generally do improve with increasing number of interlayers. However, obviously even a relatively small number of interlayers already provide a reasonably good approximation. When having a closer look at the Mode II stress intensity factors, it becomes apparent, that those increase with a smaller number of interfaces (which equals a bigger elasticity mismatch at the interface of the crack plane). So it can be concluded, that a sharp interface will induce a notable Mixed Mode loading of the crack front, that causes a crack to kink out of its original direction.

Simulation of crack paths at the interface

For those investigations a layout according to Figure 3b is chosen. In order to simplify those studies, a constant E_2=1000MPa is set, while E_1 is variable. The influence of different stiffness ratios on the developing crack paths can be gathered from Figure 4. It becomes apparent, that the crack kinks towards the weaker material. Thereby the initial kinking is the more pronounced the bigger the mismatch ratio between the two materials is. After the first kinking the cracks for all stiffness ratios smoothly turn back towards the original orientation perpendicular to the global stress field in a three-point-bending-specimen.

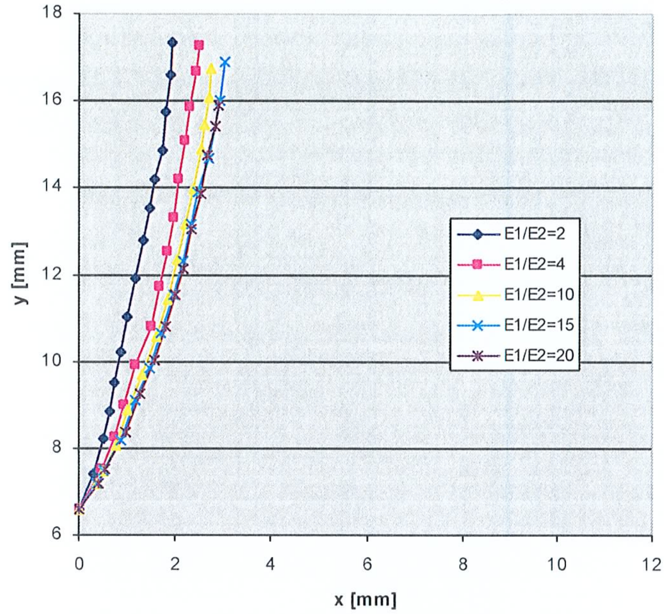


Figure 4. Development of the crack paths for different stiffness ratios

A detailed analysis of the kinking angle in the first simulation step (Figure 5) shows a monotonic increase with an increase of the stiffness mismatch at the interface. Even for relatively small E_1/E_2-ratios of 2 a pronounced Mixed-Mode loading situation and thus a kinking of $\varphi=20°$ can be detected. However, obviously the slope of the curve tends to zero, so that beyond stiffness ratios of $E_1/E_2=10$ no significant growth of the kinking angle can be found any more.

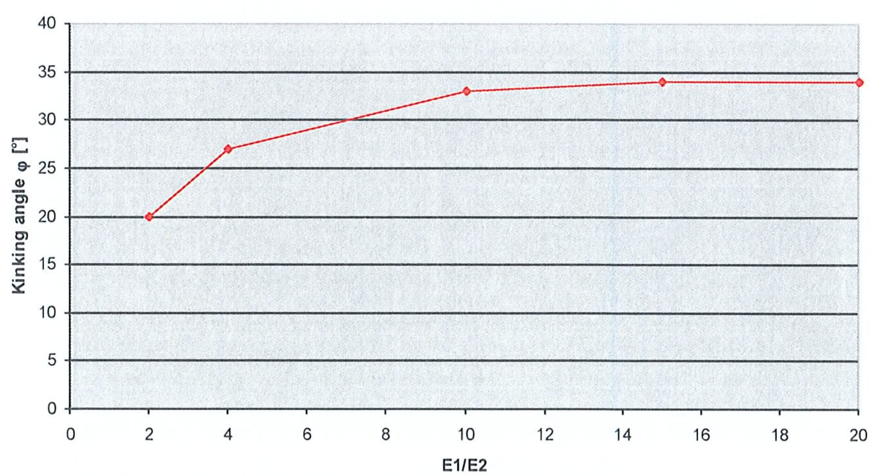
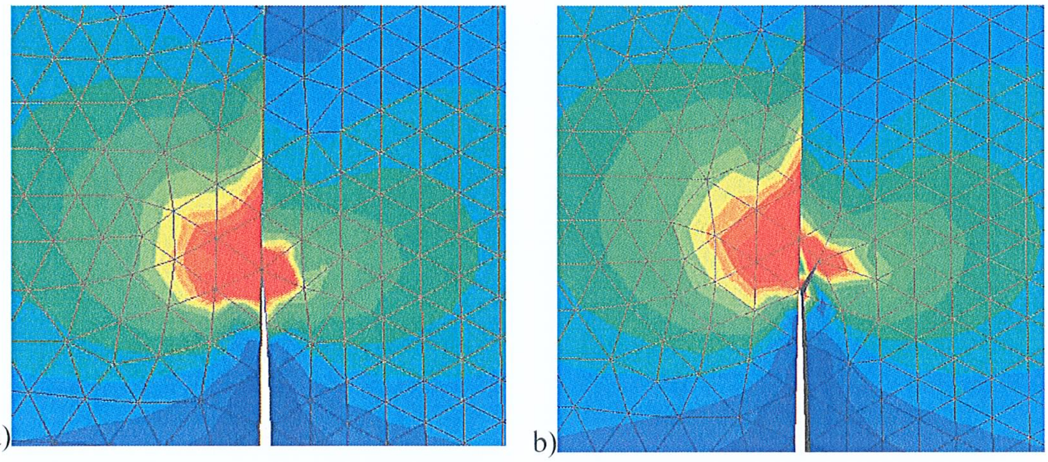


Figure 5. Kinking angles of the crack for different stiffness ratios

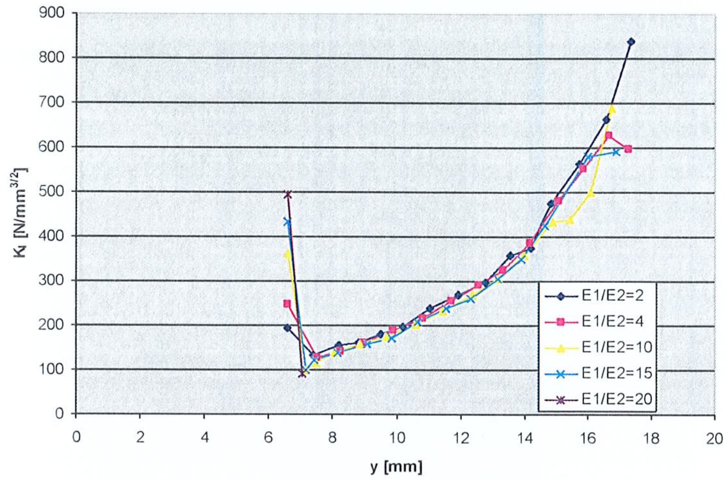


a) b)

Figure 6. Stress distribution in the vicinity of the crack front a) before and b) immediately after the kinking of the crack for a stiffness ratio 20:1

Figure 6 presents the stress fields, the crack is subjected to in the initial situation and directly after the kinking. In Fig. 6a it can be seen, that due to the sharp interface the crack front is located within a region of a non-symmetric stress field. This un-symmetry induces the Mixed-Mode loading causing the initial kinking towards the weaker material side (Fig. 6b). After the kinking two more or less „independent" stress fields can be distinguished: One still highly un-symmetric stress field caused by the interface and one relatively symmetric stress field around the new crack tip. So it can be concluded, that the interface-crack-field does not affect the crack notably any longer almost immediately after the initial kinking. So the crack subsequently grows – predominantly under the influence of the global stress field –as it would do in a homogeneous three-point bending-specimen. The related development of the stress intensity factors for Mode I and Mode II are given in Figure 7. It can be seen, that the different stiffness ratios basically only affect the first two simulation steps. The absolute values for both Mode I and II for the initial crack increase with an increasing mismatch at the interface. As soon as the crack has kinked out of the interface, not only Mode II drops down to near zero, but also the Mode I stress intensity notably decreases, which corresponds to the observations in the stress fields, that the crack is no longer affected by the stress concentration of the interface. The simulations indicate, that this effect of "drop-down" of the stress intensities increase with higher stiffness ratios.

Further simulations as well as experimental verifications need to be carried out in order to verify if this shielding effect of the stiffer material might also cause a crack to even drop below the Threshold-value for fatigue crack growth and thus to cause a crack arrest.



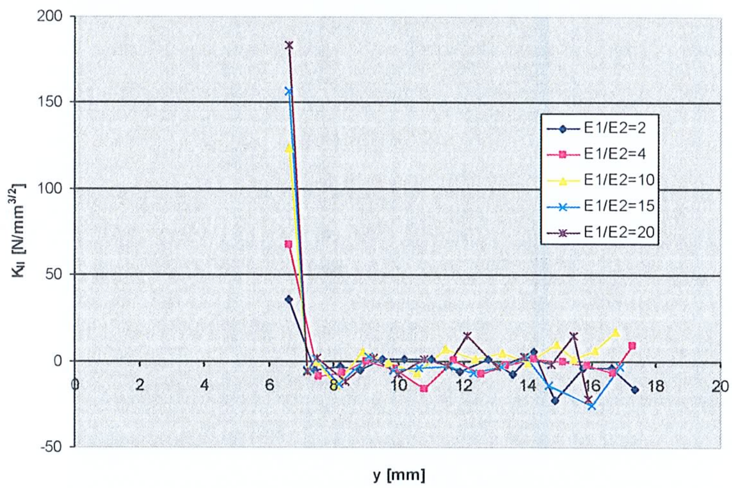


Figure 7. Development of Mode I and Mode II stress intensities with growing crack

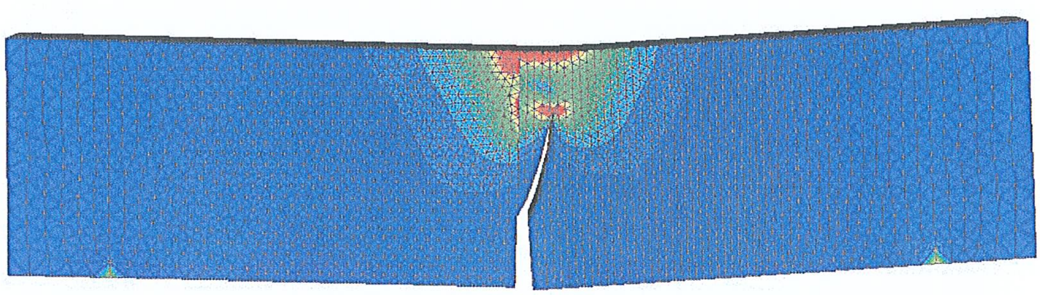


Figure 8. Crack path in a three-point-bending specimen with a sharp interface with stiffness ratio E_1/E_2=20

The final crack path in the specimen with a stiffness ratio $E_1/E_2=20$ is depicted in Figure 8. It becomes apparent, that the zone with the maximum stress in the interface advances with the advancing crack front, but does not directly affect the near-field of the crack .

CONCLUSIONS

Within this paper a numerical study of developing crack paths in a three-point-bending specimen with non-homogeneous material behaviour has been conducted. Thereby both sharp interfaces as well as a continuous gradation (by means of an approximation by stepwise constant parts) have been under consideration. It is shown, that the approximation by interlayers with piecewise constant material behaviour is a reasonable one, since the results show (even for a small number of interlayers) a good match with the continuous results. In cases, the crack is set up in a sharp interface in between two materials, a distinct Mixed-Mode-loading is induced, that causes the crack to kink out of the interface towards the weaker material. The kinking angle and the crack path thereby is highly influenced by elasticity mismatch of the two involved materials. As soon as the crack has left the interface, a notable drop of the stress intensity can be detected. Further simulations experiments need to be carried out in order to decide, under which circumstances this decrease of the stress intensity might even cause a crack arrest as a consequence of a drop below the Threshold-value for fatigue crack growth.

ACKNOWLEDGEMENT

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REFERENCES

1. Kim, J.H., Paulino, G. (2002) *J. Num. Meth. Engng.* **53**, 1903-1935.
2. Fulland, M., Steigemann, M., Richard, H.A., Specovius-Neugebauer M. (2008) *Proceedings of 17th European Conference on Fracture*, Brno, Czech Republik.
3. Fulland, M., Richard, H.A. (2003), *Steel research* **74,** No. 9, pp. 584-590.
4. Schöllmann, M., Fulland, M., Richard, H. A. (2003) *Engineering Fracture Mechanics* **70,** 221-230.
5. Fulland, M. Richard, H. A. (2003) In: *Advances in Fracture Mechanics*, pp. 79-84, Buchholz, F.G., Richard, H.A., Aliabadi, M.H. (Eds.) Trans Tech Publications, Zuerich.
6. Marur, P.R., Tippur, H.V. (2000) *Int. J. of Solids and Structures* **37**, 5353-5370.