Theoretical Prediction of the Overload Cycle Effect on Fatigue Crack Growth in Plates of Finite Thickness

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ABSTRACT. *This paper describes a theoretical approach for modelling fatigue crack growth after the application of an overload cycle in plates of finite thickness. Plate thickness effects are directly taken into account through the use of first-order plate theory, which eliminates the need for any empirical correction factors or extensive numerical simulations. Results are presented for the post-overload fatigue crack growth and are found to be in very good agreement with previous experimental data These results demonstrate the effectiveness of the new approach as well as the significance of accounting for plate thickness effects in fatigue crack growth phenomena.*

INTRODUCTION

Variable or random loading events such as overloads and underloads are frequently encountered throughout the service life of most engineering structures. It is therefore imperative that these events are accurately accounted for when making lifetime assessments using fatigue crack growth predictions. Numerous past experiments [1] have shown that the application of a tensile overload cycle can provide substantial crack growth retardation and even complete crack arrest. Furthermore, it has been found that the extent of overload retardation is not only dependent on the overload and baseline loading, but also on the thickness of the component [1-3]. In general, an increase in the plate thickness means a reduction in the extent of overload retardation [3].

The plate thickness effect, as well as the whole overload mechanism, can be readily explained by the concept of plasticity-induced crack closure (PICC). The underlying principle of PICC is that as a fatigue crack propagates the plastically deformed material ahead of the crack tip is left along the crack faces as a plastic wake. This leads to premature closure during the unloading portion of the load cycle. After the application of an overload cycle the amount of crack closure experienced is temporarily increased and thus the crack growth is slowed. If the plate thickness is increased, the crack tip stress state transitions from being overall plane stress to plane strain dominant. This results in an increase in the out-of-plane constraint and a smaller overload tensile plastic zone. There will therefore be a smaller increase in the crack closure levels post-overload, compared to a thinner plate, and less retardation in the crack growth.

Techniques for modelling crack closure include finite element methods [4] and a variety of strip-yield models [5-7]. Finite elements methods offer the advantage of being able to model complex 3D geometry; however, they usually require very extensive computational effort. On the other hand, simplified strip-yield models have proven to be very popular and form the basis of several commercial life prediction codes [5]. Plate thickness effects are often incorporated through the use of a plastic constraint factor [5], but this approach is limited as there is much ambiguity in choosing suitable values for the factor. In addition, interaction between the applied load and plate thickness is not properly accounted for when using an averaged value for the constraint factor.

This paper presents a new theoretical approach for predicting fatigue crack growth after the application of an overload cycle in plates of finite thickness. The developed approach is based on the plasticity-induced crack closure concept and a modified strip-yield model. Plate thickness effects are directly accounted for through the use of first-order plate theory [6-8]. The special case of small-scale yielding is considered in order to generalise the obtained results. These results are compared with previously published experimental data for a range of applied loadings and plate thicknesses.

THEORETICAL APPROACH

Description of the Model

The theoretical approach presented in this paper deals with a straight, through-the-thickness fatigue crack growing under mode I loading in a plate of finite thickness 2h (Fig. 1a). If the size of the tensile plastic zone is far less than that of any in-plane characteristic lengths, such as the crack length, and the ratio of the applied stress to the yield stress is less than 0.3, then the small-scale yielding (SSY) assumption can be employed. This generalisation therefore allows the developed models to be utilised for the analysis of a wide range of plate geometries and loading conditions.

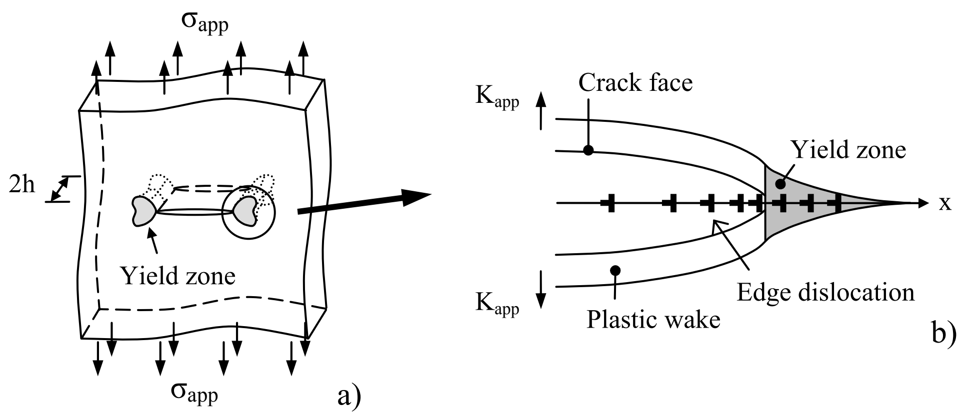

Figure 1. Through-the-thickness crack in a plate of finite thickness: a) crack in an arbitrary plate, and b) small-scale yielding representation.

The SSY assumption makes it only necessary to consider a small region surrounding the crack tip and thus a semi-infinite crack geometry is adopted (Fig. 1). It is further assumed that the plastic wake is fully developed and that prior to the application of the overload cycle, the crack has been growing for a significant period under constant amplitude loading. This allows for the plastic wake that is already developed along the crack length to be treated as a layer of constant thickness. A rigid-perfect-plastic strip-yield model is utilised for the regions of plastic deformation. The crack and plastic zones are then mathematically modelled by way of a distribution of edge dislocations (see Fig.1b). It is understood that the strip-yield hypothesis is most applicable to a plane stress analysis; however, it does provide a suitable approximation and modelling simplification for the three-dimensional case [6-8].

Plate thickness effects are incorporated into the analysis through the use of first order plate theory, which assumes that the out-of-plane strain is uniform across the plate thickness. It is also assumed that the stress components, crack opening displacement and plastic stretch are uniform across the thickness of the plate. Additional simplifications are made in the current analysis whereby strain hardening and the Bauschinger effect are not included. The developed approach is therefore limited to situations where these factors do not significantly influence the fatigue crack growth. Minor correction for these effects can be made, however, through the chosen value of the flow stress (e.g. use an average of the yield and ultimate strengths). Lastly, as the developed approach is based on the concept of plasticity-induced crack closure it is only applicable to materials and load conditions where this mechanism is dominant.

Mathematical Procedure

In this section we outline the mathematical procedure for the analysis of a fatigue crack subjected to a variable amplitude load sequence after an initial period of constant amplitude loading. A full description of the technique has already been presented in earlier work by the authors [6,7] and only a brief overview is provided here. Consider the through-the-thickness fatigue crack described in previous section and displayed in Fig. 1. The cracked region and zones of plastic deformation can be modelled by a continuous distribution of edge dislocations such that the resultant stress field due to the dislocations is the same as the original crack problem. The dislocation density function, $B_y(\xi)$, is related to the crack opening displacement and plastic stretch curves, $g(\xi)$, through:

$$B_y(\xi) = -\frac{dg(\xi)}{d\xi}. \tag{1}$$

When a mode I stress intensity factor, K, is remotely applied the resultant stress field is:

$$\sigma(x,y) = \frac{1}{\pi}\int_{-\infty}^{r_p} B_y(\xi)G(x,y;\xi)d\xi, \tag{2}$$

where r_p is the size of the tensile plastic zone, and $G(x,y;\xi)$ is the dislocation influence function at the point (x,y) for a dislocation located at ξ along the x-axis. The influence functions utilised in this study are those developed by Kotousov and Wang [8] for a through-the-thickness edge dislocation in a plate of finite thickness. For brevity, the influence functions are not included here.

To investigate the effects of a variable amplitude load sequence it is required to incrementally grow the crack. For the sake of computational efficiency, the crack closure calculations are not carried out for every single load cycle. Instead, the crack is extended a given increment, Δa, over which the crack opening load is held constant. The number of load cycles needed to extend the crack is found by stepping through the load sequence cycle-by-cycle and summing up the growth for each individual cycle. Any typical crack growth law may be utilised, such as the well known modified Paris equation, written in terms of the effective stress intensity factor range:

$$\Delta K_{eff} = K_{max} - K_{op}, \tag{3}$$

where K_{max} and K_{op} are the maximum and opening load stress intensity factors.

The crack opening load stress intensity factor for each block of load cycles is determined by implementing the distributed dislocation model. During the crack growth calculations the load sequence is monitored to find the highest maximum load, $K_{max,H}$, and the lowest minimum loads applied before, $K_{min,B}$, and after, $K_{min,A}$, the highest load. These values are then used to determine the crack opening displacement and plastic stretch curves, and hence the crack opening load. The basic algorithm involves:

1. Apply $K_{min,B}$ at current crack length (provided $K_{min,B}$ < previous $K_{min,A}$),
2. Apply $K_{max,H}$ at current crack length,
3. Extend crack by an increment of Δa,
4. Apply $K_{min,A}$ at new crack length,
5. Apply and determine crack opening load, K_{op}, at new crack length,
6. Calculate the block of load cycles for the next crack increment.

The solution procedure follows by enforcing the known boundary conditions over the length of the crack and plastic zones. Along the region of the crack that was formed prior to the variable loading, i.e. the 'pre-overload' region where the wake thickness is uniform, there are two possibilities. The crack may be open and therefore the crack faces traction free, or the crack closed and in compression. In this section of the crack it is assumed that no yielding will occur. This provides the boundary conditions:

$$g = \delta_w, \quad \sigma_{yy} < 0, \quad \text{crack closed}, \tag{4a}$$
$$\sigma_{yy} = 0, \quad g > \delta_w, \quad \text{crack open}, \tag{4b}$$

where σ_{yy} is the normal stress component in the y-direction, δ_w is the wake thickness, and g is the displacement between the crack faces plus the wake thickness.

Ahead of the crack tip and within the plastic yield zone there are three possibilities for the boundary conditions. The material may yield in either compression or tension, or the residual stretch may remain unchanged. If a Tresca yield criterion is used then:

$$g = g', \qquad -\sigma_f < \sigma_{yy},\ \sigma_{yy} - \sigma_{zz} < \sigma_f, \qquad \text{no yield}, \tag{5a}$$
$$\sigma_{yy} - \sigma_{zz} = \sigma_f, \qquad g > g', \qquad \text{tensile yield}, \tag{5b}$$
$$\sigma_{yy} = -\sigma_f, \qquad g < g', \qquad \text{compressive yield}, \tag{5c}$$

where g' refers to the displacement from the previous load configuration, σ_f is the uniaxial flow stress, and σ_{zz} is the out-of-plane stress component. The assumption has been made that compressive yielding occurs with negligible out-of-plane constraint and thus when $|\sigma_{yy}| = \sigma_f$. This approximation has been made in numerous past efforts to model plasticity-induced crack closure using the strip-yield hypothesis [5-7].

The final section of interest is the incremental growth region which has formed due to the application of a variable load sequence. In this region the plastic wake is non-uniform and its distribution must be determined. Compressive yielding of the wake is allowed and therefore the three boundary conditions are:

$$g = g', \qquad \sigma_{yy} > -\sigma_f, \qquad \text{crack closed}, \tag{6a}$$
$$\sigma_{yy} = 0, \qquad g > g', \qquad \text{crack open}, \tag{6b}$$
$$\sigma_{yy} = -\sigma_f, \qquad g < g', \qquad \text{compressive yield}. \tag{6c}$$

Numerical results are obtained for each applied load, namely $K_{min,B}$, $K_{max,H}$, $K_{min,A}$, and K_{op}, within each block of load cycles. Along the pre-overload crack length Gauss-Chebyshev quadrature is utilised, while direct placement of the edge dislocations is favoured for the incremental growth region and crack tip plastic zone [6,7]. For each load case, an initial guess is made for the boundary conditions in the various regions along the crack length. The solution is then checked to ensure that the obtained stress and displacement fields meet all necessary requirements given in Eqs 4 to 6. Iteration is used until the final solution is reached and convergence criteria are met.

RESULTS

Crack Opening Load

Consider the case of a single tensile overload in otherwise constant ΔK loading. This idealisation is of great practical importance as overloads are common place in many real structures, e.g. in an aircraft during take-off and landing. Figure 2 displays the results for the crack opening load ratio determined using a plane stress dislocation influence function. Here the overload stress intensity factor is K_{ov}, the overload ratio is K_{ov}/K_{max}, and the load ratio $R = K_{min}/K_{max}$, where K_{max} and K_{min} are the maximum and minimum values, respectively, of the constant ΔK loading. The crack extension in Fig. 2a is normalised by the plane stress tensile overload plastic zone size, $r_{p,ov}$. With an increase in the overload ratio, there is an accompanying increase in the peak opening load ratio and the distance to recover the steady state value. This translates to an increase in the amount of crack growth retardation (see next section). Figure 2b shows the peak opening load ratio as a function of the overload ratio and the R ratio. Lines of constant $\Delta K_{ov}/\Delta K$ are also indicated as this is an alternative definition for the overload ratio.

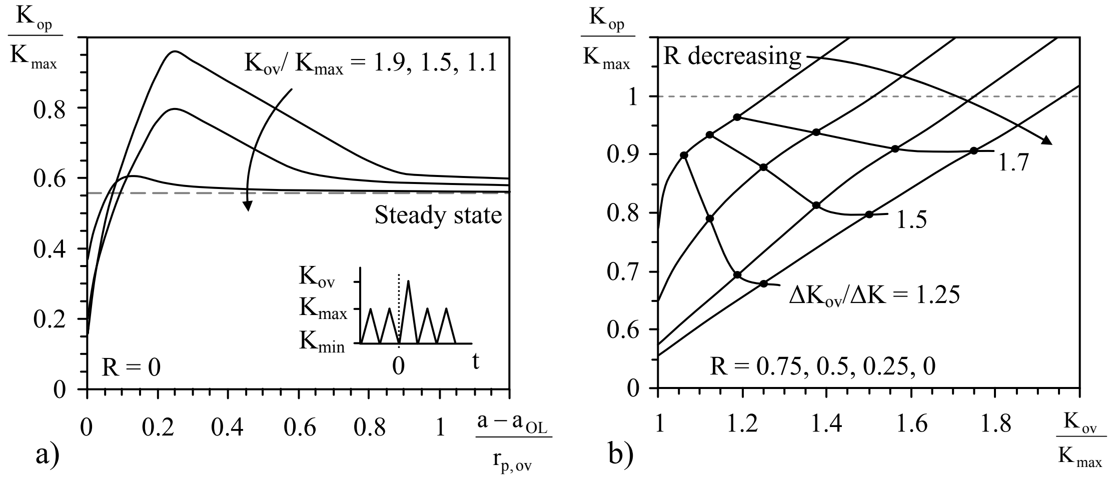

Figure 2. Plane stress crack opening load ratio following the application of an overload cycle: a) as a function of crack extension [6], and b) peak value [6].

Figure 3 displays the results for the crack opening load ratio as determined using the finite thickness model. In Fig. 3a, the crack extension is thus normalised by the finite thickness overload plastic zone size. Curves for several values of the non-dimensional parameter $\eta = K_{max}/(\sigma_f\sqrt{h})$ are provided. The peak opening load ratio is shown in Fig. 3b as a function of K_{ov}/K_{max} and η. It can be seen in both figures that a decrease in the parameter η leads to a reduction in the change in opening load ratio. Therefore, for a fixed K_{max}, R and σ_f an increased plate thickness will result in less crack growth retardation. Figures 2b and 3b also demonstrate the potential for complete crack arrest by choosing an appropriate overload ratio for the given material and load conditions.

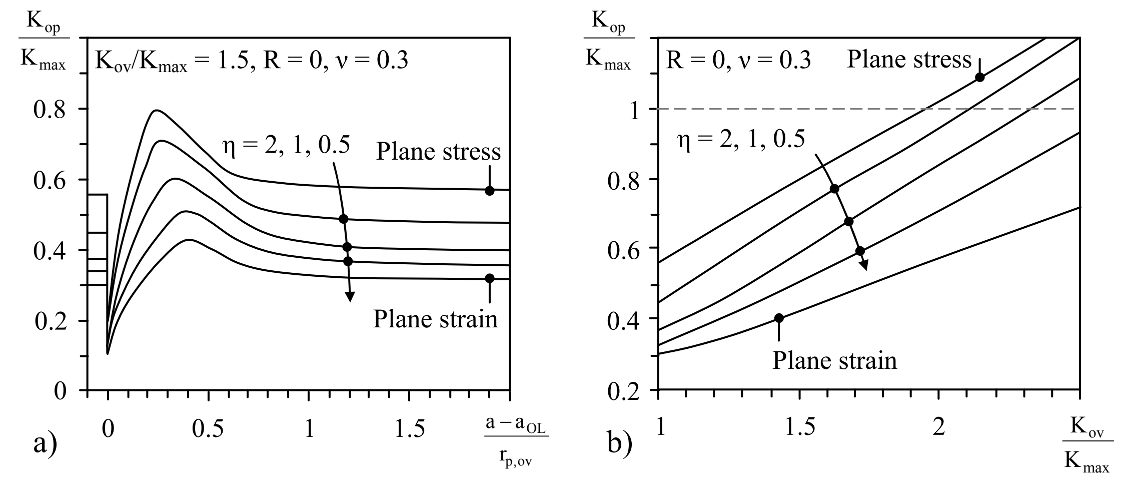

Figure 3. Thickness effect on the crack opening load ratio following the application of an overload cycle: a) as a function of crack extension [7], and b) peak value [7].

Fatigue Crack Growth

To demonstrate the adequacy of the developed approach, we will now compare the predictions with previously published experimental data. Again, we consider the case of a single tensile overload in constant ΔK loading. The first data to be investigated is that of Borrego et al. [2] who tested 6082-T6 aluminium alloy centre cracked tension specimens. Figure 4 shows a comparison between predictions made with plane stress and finite thickness dislocation influence functions. In these tests ΔK = 8 MPa√m, R = 0.05, K_{ov}/K_{max} = 1.95, and in the finite thickness case ν = 0.3 and 2h = 3 mm. The plane stress prediction severely overestimates the retardation effect, while the finite thickness prediction provides a very good estimate. In both cases, however, the predicted growth rate returns towards the pre-overload rate within the respective overload plastic zones. The experimental data shows continued retardation well outside of the plastic zone. This can be partly attributed to strain hardening, which is neglected in the present analysis.

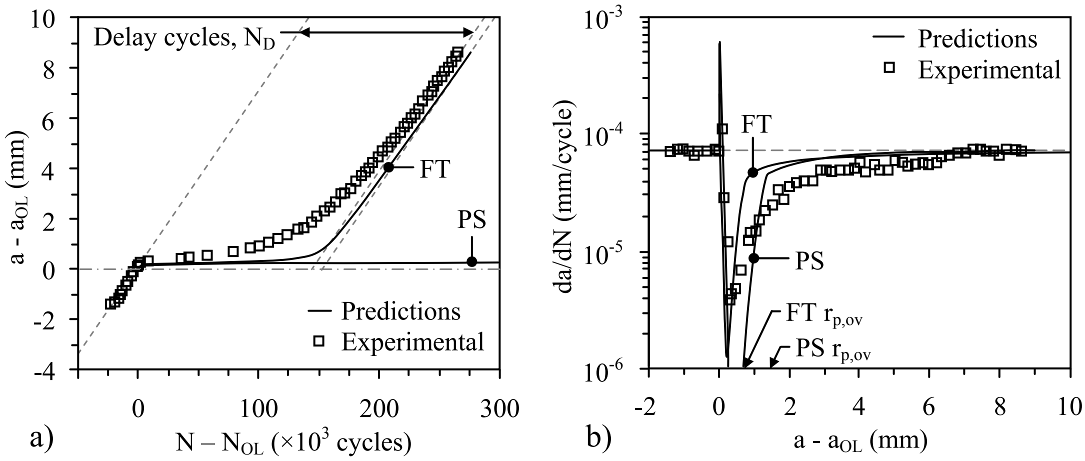

Figure 4. Comparison of the plane stress (PS) and finite thickness (FT) predictions for a) the crack growth, and b) the crack growth rate.

One measure of the extent of the retardation effect is the number of delay cycles produced by the overload cycle. The number of delay cycles refers to the number of load cycles required to restore the pre-overload crack growth rate minus the number of cycles needed to reach the same crack length without an overload applied (see Fig. 4a). Figure 5a shows the number of delay cycles predicted by the finite thickness model compared to the experimentally measured values for the data of Borrego et al. [2]. A range of baseline loading and overload ratios is considered. Further results for the number of delay cycles are presented in Fig. 5b demonstrating the effect of plate thickness on overload retardation. These particular results (Fig. 5b) are for BS 4360 Grade 50D structural steel compact tension specimens from a study by Shuter and Geary [3]. The tests were conducted at a baseline ΔK = 25 MPa√m with R = 0.1 and K_{ov}/K_{max} = 2. The predicted and experimental results show the same trend of a reduction in the amount of retardation with an increase in the plate thickness.

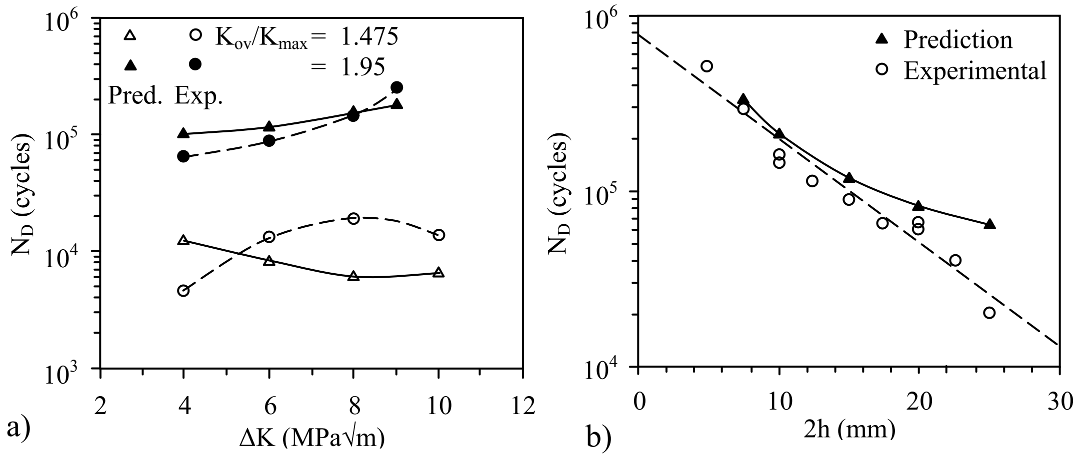

Figure 5. Number of delay cycles as a function of a) baseline loading and overload ratio [7], and b) the plate thickness [7].

SUMMARY

A new theoretical approach was presented for predicting fatigue crack growth after the application of an overload cycle. This approach directly takes into account the plate thickness through the use of first-order plate theory and eliminates the need for any empirical fitting parameters. Predictions for the fatigue crack growth following an overload cycle were compared with previous experimental data and found to be in very good agreement. This demonstrates the significance of correctly accounting for plate thickness effects when modelling fatigue crack growth phenomena.

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