Detection of Damage in Beams Utilising the Principle of Strain Compatibility

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ABSTRACT. *This paper discusses a new method for damage detection based on the most fundamental concept in continuum mechanics: strain compatibility. Compliance with this principle implies a deformed material is free from discontinuities, which are indicative of many types of structural damage. Therefore the principle of strain compatibility, in its ability to identify discontinuities, is very promising as a new foundation for future research into non-destructive evaluation and structural health monitoring technologies. The proposed method has many advantages compared to existing damage detection techniques, such as its invariance to material properties and the geometry of the structure. To illustrate the application of this technique, the detection of damage in beam structures is investigated. The formulation of the strain compatibility equation for beam structures is introduced and numerical simulations carried out to detect crack and delamination damage in a cantilever beam. The simulations demonstrate that the strain compatibility technique shows high potential for locating and quantifying the severity of damage in beam structures.*

INTRODUCTION

Non-destructive evaluation (NDE) and structural health monitoring (SHM) techniques are normally based on general physical principles or phenomena accompanying the presence or development of structural damage. However, previous research into damage detection has overlooked one of the fundamental principles of continuum mechanics – the principle of strain compatibility. Compliance with this principle implies a deformed material is free from discontinuities, indicative of many types of structural damage such as cracks and delamination. Therefore the principle of strain compatibility is very promising as a new technique for future NDE and SHM technologies [1].

Strain compatibility equations were first derived by Saint-Venant in 1860 and are used to solve various problems, particularly in the theory of elasticity where the concept of compatibility has mathematical and physical significance [2]. From a mathematical point of view, this theory asserts the components of displacement match the geometrical boundary conditions and are single-valued, continuous functions of position, with which the strains are associated. In general, the kinematical relationships, between strains and displacement components, connect six independent components of the strain matrix to only three components of the displacement vector; therefore the strain components are not independent of one another. This idea, represented by the equations of compatibility, establishes the geometrically possible forms of strain variations from point to point within a body.

Physically, the principle of strain compatibility implies a deformed body must be pieced together with no gaps, overlaps or other discontinuities, as shown in Fig.1. It is evident from the above interpretation that the nature of the principle of compatibility is closely related to typical structural damage and, therefore, can serve as a basis for the development of NDE techniques.

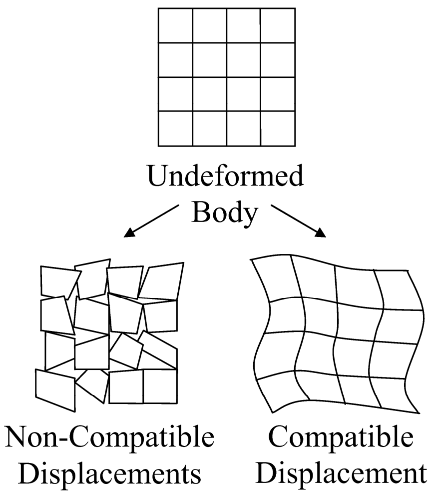


Figure 1. Illustration of strain compatibility principle

Conceptually a damage detection system can consist of built-in (or surface-mounted) passive elements (eg. strain gauges or piezoelectric patches) or use non-contact strain measurement techniques (eg. laser Doppler vibrometry [3]). The loading can be due to normal operation or induced intentionally (by active sources) to detect damage. The sensor elements form clusters of various shapes and sizes, which can be used to measure the non-compliance of the measured strain field with the theoretical strain compatibility conditions.

The proposed method has many advantages in comparison with existing NDE technologies. The most important of these is that the method is applicable to isotropic and anisotropic materials experiencing elastic and non-elastic deformations, in curved or flat surfaces. In addition, the method is robust as the output signals from the clusters are invariant to loading conditions, for example, accidental loading or changes in the boundary conditions will not lead to false alarms

Interestingly, methods based on modal curvature measurements are closely related to strain compatibility. These methods have been employed and investigated in the past fifteen years for structural damage identification, however, the links between modal curvature methods and the principle of strain compatibility has never been recognized. These curvature methods include the absolute difference method, damage factor, damage index, shape method, gapped smooth method [4, 5], a trous Laplace operator [6], smoothed Teager energy operator [6], frequency response function curvature method, damage localization vector method [7-9] to name a few. Assuming that the original healthy structure produces smooth curvature without irregularity, these methods normally utilise a curve-fitting technique to find the local variability in structural stiffness associated with delamination damage. Curvature measurement based methods have proven to be very effective in detecting, locating, and quantifying local damage. However, the success of damage identification depends strongly on the quality and selection of the parameters involved when curve fitting, which could be different for varying geometries and loading conditions [1].

Therefore, the strain compatibility principle provides a rigorous theoretical foundation to the existing curvature based NDE techniques and will allow the development of damage detection procedures that are invariant to material properties, type and intensity of loading, and the geometry of the structure.

STRAIN COMPATIBILITY ALGORITHM

Thin Plate and Shell Components

The most general form of three-dimensional strain compatibility conditions are represented by a system of six homogeneous partial differential equations. It is well known that in the case of a very thin isotropic plate (utilising Kirchhoff hypotheses, small-deflection theory or classical theory) these six equations reduce to a single homogeneous Laplace equation with respect to the sum of the principle strain components [2] as

$$\partial^2(\varepsilon_x + \varepsilon_y)/\partial x^2 + \partial^2(\varepsilon_x + \varepsilon_y)/\partial y^2 = 0 , \qquad (1)$$

where ε is the normal strain and x and y are the Cartesian coordinates.

For the practical implementation of Eq. 1, a finite difference representation can be utilised. In recent work conducted by Wildy et al. [2], a central difference scheme was used to model a damage detection system to monitor cracks in wide plates. However, this work will focus on detection of crack damage and delamination damage in cantilever beams.

Beam Components

For beam components (one-dimensional system) in bending, the strain compatibility equation (Eq. 1) takes the simple form of

$$\partial^2\varepsilon_x/\partial x^2 = 0 , \qquad (2)$$

where ε_x is the extensional deformation on the surface of a slender beam and can be calculated through the deflection of the beam as

$$\varepsilon_x = h\, \partial^2 w/\partial x^2 , \qquad (3)$$

where h is the height of the beam and $\partial^2 w/\partial x^2$ (= κ) is the local curvature of the beam.

Finally, the compatibility equation can be reduced to the following equation, which will hold for the undamaged beam:

$$\partial^4 w/\partial x^4 = 0 . \qquad (4)$$

The use of a 4th order least squares fit of the out-of-plane displacements can be utilised to determine strain compatibility (Eq. 4) and alleviate the adverse effect of noise. At each measurement point on the beam the fitted out-of-plane displacement is

$$P(x_i) = a_i\, x_i^4 + b_i\, x_i^3 + c_i\, x_i^2 + d_i\, x_i + e_i , \qquad i = 1,2,\dots, N \qquad (5)$$

where a_i, b_i, c_i, d_i and e_i are real constants, $P(x_i)$ is the 4th order least square fit of the out-of-plane displacement at measurement point x_i, i is the measurement point of interest and N is the total amount of measurement points. The constants can be solved by evaluating,

$$\mathbf{W}\begin{bmatrix} 1 & x_{i-n} & x_{i-n}^2 & x_{i-n}^3 & x_{i-n}^4 \\ : & : & : & : & : \\ 1 & x_i & x_i^2 & x_i^3 & x_i^4 \\ : & : & : & : & : \\ 1 & x_{i+n} & x_{i+n}^2 & x_{i+n}^3 & x_{i+n}^4 \end{bmatrix}\begin{bmatrix} e_i \\ d_i \\ c_i \\ b_i \\ a_i \end{bmatrix} = \mathbf{W}\begin{bmatrix} w_{i-n} \\ : \\ w_i \\ : \\ w_{i+n} \end{bmatrix}, \qquad n = 2, 3, 4, \dots \qquad (6)$$

where $(2n + 1)$ is the number of neighbouring measurement points used for the fit around each point and **W** is a diagonal weighting matrix.

In the subsequent simulations the following weighting function was used to determine the diagonal weighting coefficients, which biases the fit around points in the vicinity of the point i.

$$W_j = \cos\left[\,\pi\,(x_{i+j} - x_i) \,/\, (x_{i+n} - x_{i-n})\,\right], \qquad j = -n, -n+1, \dots, n\,. \qquad (7)$$

The above weighting function can only be applies to regular spaced grids and, thus, the measurement grid chosen in the subsequent simulations is equally spaced.

Consequently, if Eq. 4 is applied to Eq. 5, the residual strain compatibility is left

$$\Delta_i = a_i \qquad (8)$$

where Δ_i is the residual strain compatibility.

NUMERICAL SIMULATIONS

To investigate the potential of the strain compatibility technique for damage identification, two damage scenarios have been considered for a cantilever beam with a force applied at its end; a single transverse edge crack and a delaminated section. The two models were developed and non-dimensionalised with out-of-plane displacement and beam position parameters represented, respectively, as

$$w^* = (w\,E\,I\,) \,/\, (P\,L^3) \qquad \& \qquad x^* = x \,/\, L\,. \qquad (9)$$

To simulate real measurement conditions, a moderate level of noise was added (SNR = 65 dB) to the numerically generated displacements and a realistic number of measurement points was selected (N = 1001, eg. for a one meter beam, the measurement point would be spaced 1mm apart, which is feasible for a scanning laser vibrometer).

Crack Damage

To model the single transverse edge crack, using Euler-Bernoulli beam theory, a linear rotational spring (K_T) was used to approximate the crack (as seen in Fig. 3) and is a function of beam height (h) and crack length (a). Details of this approximation have been omitted in this paper and can be found in [10]. For the following simulation a beam height (h) of $0.006L$ (eg. for a one meter beam, the beam height would be 6mm) and a crack position (L_c) of $0.4L$ was used.

Figures 4 and 5 illustrate the effect of varying parameters on the acquired strain compatibility (Eq. 8) along the cracked cantilever beam. Firstly, the effect of incremental crack growth (Fig. 4) and, secondly, the effect of the number of measurement points used to determine the least squares fit ($2n + 1$) (Fig. 5). Table 1 details the specific parameters use in each figure.

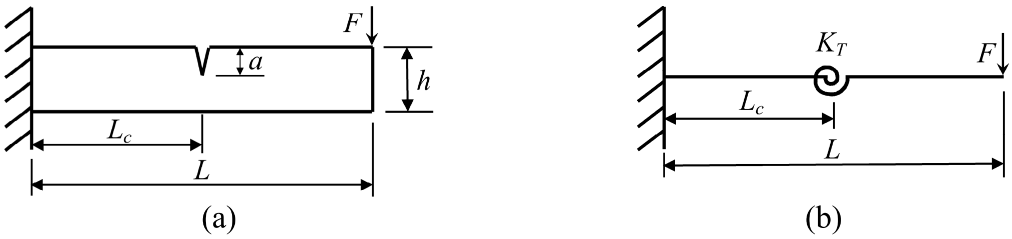


Figure 3. (a) Physical and (b) theoretical model of the cracked cantilever beam.

Table 1. Parameters used in Figs 4 and 5

Figure	a/h	n
4	0, 0.3 & 0.5	100
5	0.3	50, 100 & 150

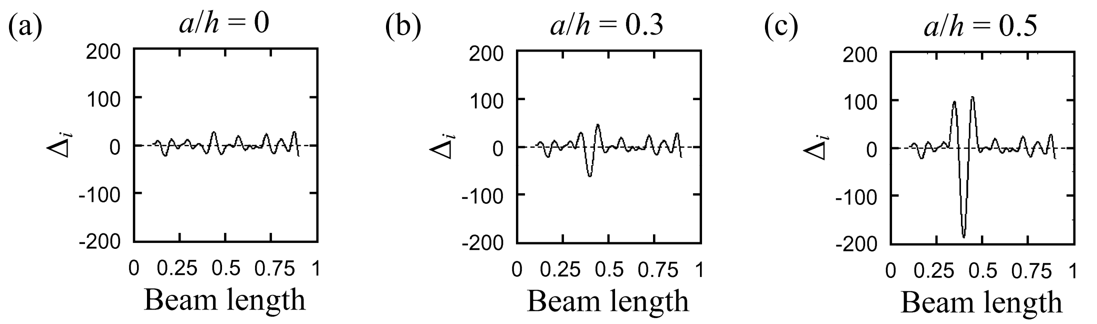
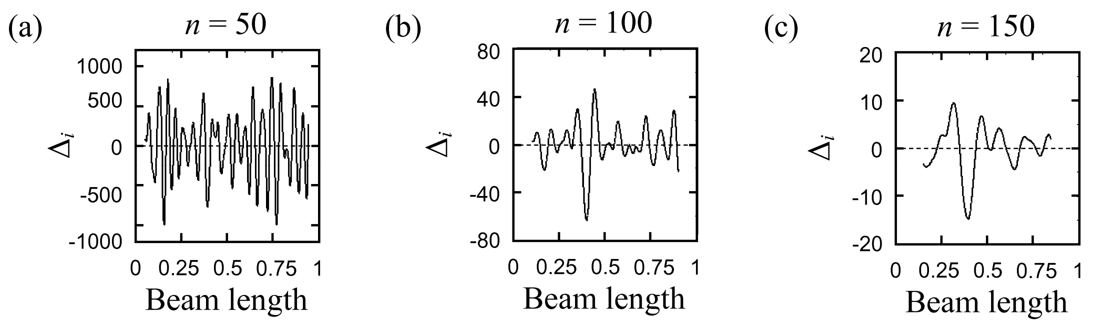
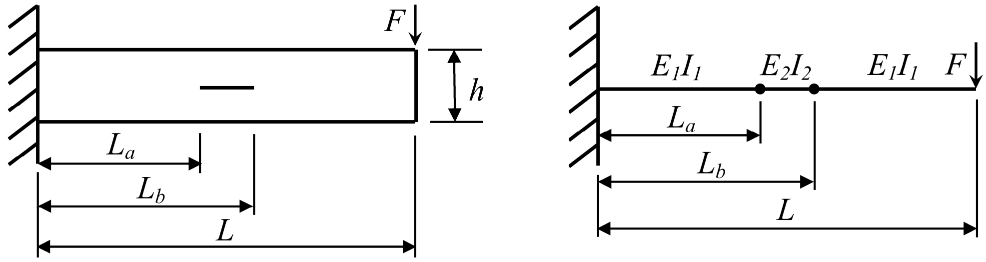


Figure 4. Acquired strain compatibility (Δ_i) at a range of locations along the cracked cantilever beam for three different crack length to beam width ratios (a/h).



Figures 5. Acquired strain compatibility (Δ_i) at a range of locations along the cracked cantilever beam for three different amounts of measurement points ($2n + 1$) used to evaluate the least squares fit of the out-of-plane displacement.

Delamination Damage
To model the delaminated cantilever beam, a section of the beam is given a reduced flexural rigidity, as seen in Fig 6. Figures 7 to 9 illustrate the effect of varying parameters on the acquired strain compatibility along the delaminated cantilever beam. Firstly, the effect of incremental damage severity in the delaminated zone (Fig. 7), secondly, the effect of the number of measurement points used to determine the least squares fit of the out-of-plane displacement ($2n + 1$) (Fig. 8), and finally, the effect of incremental growth of the delaminated section (Fig. 9). Table 2 details the specific parameters use in each figure.



(a) (b)

Figure 6. (a) Physical and (b) approximated model of the delaminated cantilever beam.

Table 2. Parameter used in Figs 7 to 9.

Figure	E_2I_2/E_1I_1	n	L_a/L	L_b/L
7	0.8, 0.65 & 0.5	100	0.4	0.5
8	0.65	50, 100 &150	0.4	0.5
9	0.65	100	0.4	0.41, 0.45 & 0.5

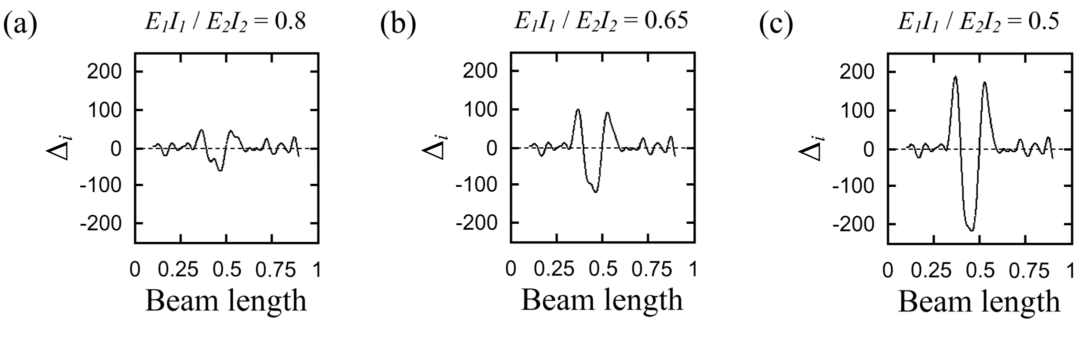


Figure 7. Acquired strain compatibility (Δ_i) at a range of locations along the delaminated cantilever beam for three different reductions in modulus of elasticity and moment of inertia in the delaminated section (E_2I_2/E_1I_1).

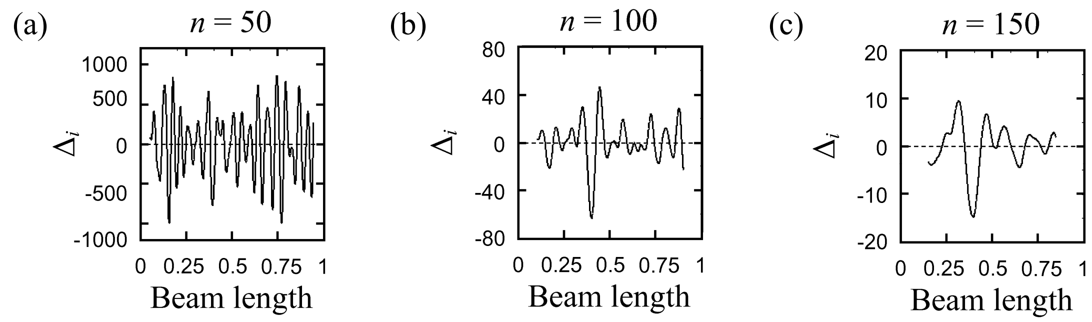


Figure 8. Acquired strain compatibility (Δ_i) at a range of locations along the delaminated cantilever beam for three different amounts of measurement points used to evaluate the least squares fit of the out-of-plane displacement ($2n + 1$).

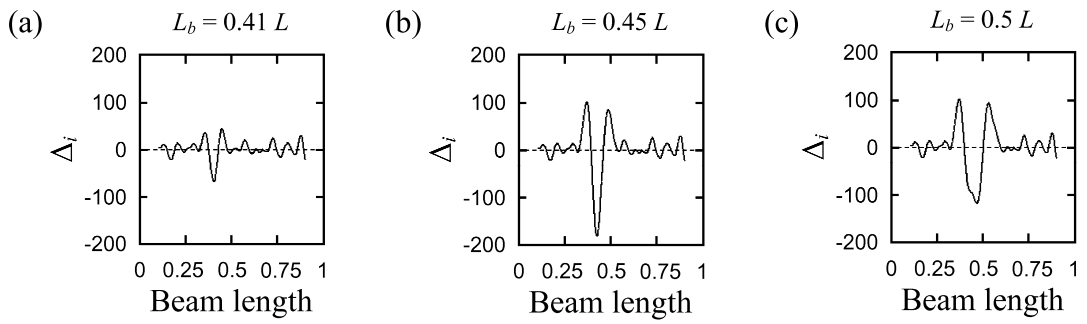


Figure 9. Acquired strain compatibility (Δ_i) at a range of locations along the delaminated cantilever beam for three different delaminated section sizes.

SUMMARY OF RESULTS

Figures 4 and 7 show the residual strain compatibility (Δ_i) for crack and delamination damage, respectively, in a cantilever beam versus increases with damage intensity. As the intensity of the damage increases, a spike in the residual strain compatibility is observed at the location of the damage, which increases with the intensity.

Figures 5 and 8 show the residual strain compatibility (Δ_i) for crack and delamination damage, respectively, in a cantilever beam versus three different amounts of measurement point used to evaluate the least squares fit of the out-of-plane displacement ($2n + 1$). These figures illustrate that, with an increase in the amount of measurement points used to evaluate the least squares fit, there is an increased ability to detect the damage. However, with an increase in the amount of measurement points used there is also a reduction of the same amount in the range in which the residual strain can be calculated for. Therefore, an optimum number of measurement points is required to evaluated the least squares fit, in order to detect damage effectively.

Figure 9 shows the residual strain compatibility (Δ_i) for delamination damage in a cantilever beam versus increases in the delaminated section size. As seen in Figs 4 and

7, an increase in the size of the delaminated section produces a spike in residual strain compatibility at the damage location. However, an increase in the size of the delaminated section causes the spike to increase in magnitude, but also in width, matching the size of the delaminated section.

CONCLUSION

A new technique for the detection of damage in beam structures loaded in bending, which is based on the concept of strain compatibility, has been presented. This technique utilizes strain compatibility conditions to determine violations of these conditions for localized areas and, therefore, indicates the presence of cracks, delamination and other types of damage. The major features of this technique are that the strain compatibility equations hold for all material properties and the technique can be applied to structures with elastic and non-elastic deformations.

Simulations were conducted, utilising the newly proposed damage algorithm, for two damage scenarios: delamination damage and crack damage in a cantilever beam. The results from these simulations demonstrate that this algorithm has high potential for locating and quantifying the severity of damage in beam structures.

Future work will focus on validating the technique for identifying delamination damage in laminated composite beams, involving the use of a Polytec 3D laser scanning vibrometer to measure the out-of-plane displacements and, in addition, investigate the potential of various algorithms for determining strain compatibility.

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