Limitations of K_{II}-criterion for crack-path computation in ceramics

T. Fett[1], S. Fünfschilling[1], R. Oberacker[1], M.J. Hoffmann[1], M. Lengauer[2], R. Bermejo[3], R. Danzer[3]

[1]Universität Karlsruhe, Institut für Keramik im Maschinenbau, Karlsruhe, Germany
[2]FH Joanneum GmbH, Graz, Austria
[3]Montanuniversität Leoben, Institut für Struktur- und Funktionskeramik, Leoben Austria

ABSTRACT. *Ceramic rolls for wire hot rolling at multi-line rolling mills may fail by contact overloading. The present paper deals with a refinement of first publications on this topic. In the first part the relations for stress intensity factor computation via the weight function method are compiled. Then it is shown whether the mixed-mode stress intensity factors of the curved cracks can be applied for the prediction of crack path via the condition of local symmetry.*

INTRODUCTION

Due to the excellent mechanical high temperature properties and wear resistance of silicon nitride, this class of ceramics is foreseen for rolls for wire hot rolling at multi-line rolling mills. In first applications, and under extreme conditions, delayed failure of rolls occurred by spontaneous crack extension with cracks in the order of 0.4–1.2 mm. The problem of path development and path stability has been investigated very extensively in the fracture mechanics literature. All these investigations using different fracture criteria have in common the feature that a crack can extend along the continuously turning tangent direction only if K_{II}=0. This condition, called the *criterion of local symmetry*, in principle allows the prediction of the crack path. First analyses of the failure behaviour of silicon nitride rolls based on such criterion have already been given in literature by Lengauer et al. [1]. In our contribution their analysis will be refined by taking into consideration mode-II stress intensity factors. The shielding term acting also for mode-II loading will be discussed with respect to K_{II}-R-curve behaviour.

STRESS INTENSITY FACTOR COMPUTATION

A crack extending in a milling roller is shown in Fig. 1a. In order to understand extension of this crack, computations of the related mode-I and mode-II stress intensity factors are necessary. Figure 1b shows a first-order approximation of the real crack (solid line) by an auxiliary slant crack (dash-dotted line) with a kink situation near the tip. The auxiliary crack is chosen from two conditions: First requirement is that its tip coincides with the tip of the real crack. Second the slope as the second free parameter results from the condition of minimizing the sum of y^2.

Results of mode-I stress intensity factors for *arbitrary* crack shapes deviating from straight cracks are seldom reported. An analysis of a straight crack in an infinite body assuming a small perturbation ahead of the tip of the initial crack was early presented by Cotterell and Rice [2]. They computed the mixed-mode stress intensity factors and crack paths under restrictions of a first-order analysis, i.e. for small deviations from the prospective extension of initial crack plane. Their computation procedure originally derived for cracks in an *infinite* body was extended in [3] to semi-infinite bodies by means of the weight function method.

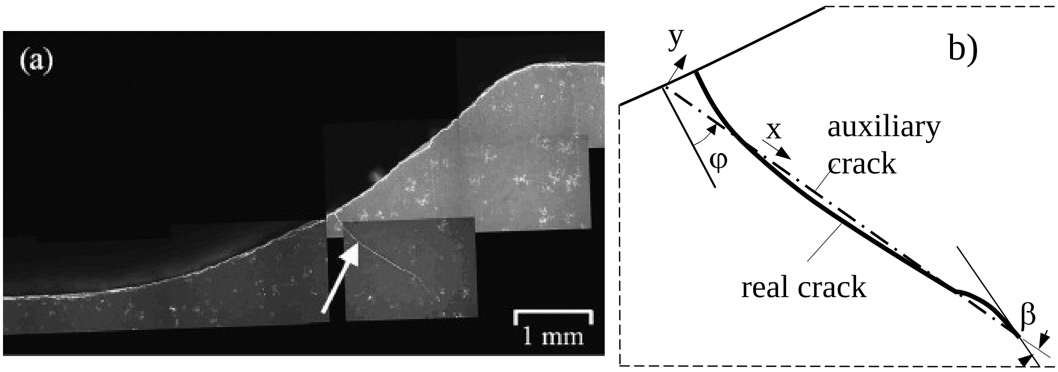

Fig. 1 a) Crack in a milling roller, b) curved crack (solid line) replaced by an averaged slant crack (dash-dotted line) with a kink of angle β at the tip.

In order to minimize the deviations between the real curved crack and the straight-line approximation by a slant crack, let us construct the auxiliary crack so that its tip coincides with the tip of the real crack but exhibiting an infinitesimally small kink under the angle β into the direction of the real crack (Fig. 1b). The angle between the auxiliary crack (length a) and the free surface is denoted as φ, the coordinate system with respect to the auxiliary crack is (x, y). For the example shown in Fig. 1 it holds φ=27°. In this coordinate system, the conditions for an application of a first-order stress intensity factor analysis read

$$y \ll a\,, \quad dy/dx \ll 1 \tag{1}$$

As an important result it was found in [2] and [3] that a simple first-order evaluation of mode-I stress intensity factors is even possible for arbitrarily shaped cracks with an acceptable error margin if $dy/dx \leq 0.2$ and $y/a < 0.1$ are fulfilled.

The mixed-mode stress intensity factors $K_{I,II}$ for non-linear cracks can be computed by the weight function method as

$$K_{\mathrm{I}} = \int_0^a \sigma_n h_{11} \, dx + \int_0^a \tau_{xy} h_{12} \, dx \tag{2}$$

$$K_{\mathrm{II}} = \int_0^a \sigma_n h_{21} \, dx + \int_0^a \tau_{xy} h_{22} \, dx \tag{3}$$

with the stresses normal on the crack, σ_n, and the shear stress parallel to the crack, τ_{xy}. The weight function terms h_{11}, h_{12}, h_{21}, and h_{22} necessary in Eqs 2 and 3 are of course hardly available for a crack of arbitrary shape. In general these functions have to be determined for any shape and any crack length. Such an approach of course would make the weight function method highly inefficient.

Following the suggestion in [3] the weight functions for the rather complicated crack can be approximately derived in a first-order analysis by superposition of known stress intensity factor and weight function solutions (for details see [3]). In the following considerations, the stress intensity factor for the real crack is composed by a K of the slant auxiliary crack (dash-dotted line in Fig 1b) and that of a kink under angle β resulting in

$$K_{\mathrm{I}} \cong K_{\mathrm{I,slant}} + K_{\mathrm{I,kink}} + O(y'^2) \tag{4}$$

$$K_{\mathrm{II}} \cong K_{\mathrm{II,slant}} + K_{\mathrm{II,kink}} + O(y') \tag{5}$$

Stress intensity factors for the slant crack

For the special case of a straight slant crack, the four weight functions were determined in [4].

$$K_{\mathrm{I,slant}} = \int_0^a (h_{11,slant} \sigma_n + h_{12,slant} \tau_{xy}) dx \tag{6}$$

$$K_{\mathrm{II,slant}} = \int_0^a (h_{21,slant} \sigma_n + h_{22,slant} \tau_{xy}) dx \tag{7}$$

The mode-I and mode-II *weight functions* can be described by

$$h_{11,slant} \cong \sqrt{\frac{2}{\pi a}}\left(\frac{1}{\sqrt{1-x/a}} + D_1^{(11)}(1-x/a)^{1/2} + D_2^{(11)}(1-x/a)^{3/2}\right) \tag{8}$$

$$h_{22,slant} \cong \sqrt{\frac{2}{\pi a}}\left(\frac{1}{\sqrt{1-x/a}} + D_1^{(22)}(1-x/a)^{1/2} + D_2^{(22)}(1-x/a)^{3/2}\right) \tag{9}$$

with the coefficients D which can be represented for $\varphi < \pi/4$ by

$$D_1^{(11)} = 0.568 + 1.48\varphi^2 + 1.05\varphi^4, \qquad D_2^{(11)} = 0.284 + 0.327\varphi^2 + 0.508\varphi^4 \tag{10}$$

$$D_1^{(22)} = 0.568 + 0.587\varphi^2 + 0.344\varphi^4, \qquad D_2^{(22)} = 0.284 + 0.0965\varphi^2 - 0.861\varphi^4 \tag{11}$$

with φ in radian. The *mixed* weight functions for $\varphi < \pi/6$ are given in [4]. They read for small angles of $\varphi \le 30°$

$$h_{12,slant} \cong -\sqrt{\frac{2}{\pi a}}\left(0.942\sqrt{1-x/a} - 0.411(1-x/a)^{3/2}\right)\varphi \tag{12}$$

$$h_{21,slant} \cong -\sqrt{\frac{2}{\pi a}}\left(0.689\sqrt{1-x/a} + 1.2501(1-x/a)^{3/2}\right)\varphi \tag{13}$$

Stress intensity factors for the kink crack

Since the tangent to the real crack and the auxiliary straight crack include a finite angle β, a kink stress intensity factor has to be added, i.e.

$$K_{\mathrm{I,kink}} = K_{\mathrm{I,slant}}\, g_{11} + K_{\mathrm{II,slant}}\, g_{12} \tag{14}$$

$$K_{\mathrm{II,kink}} = K_{\mathrm{I,slant}}\, g_{21} + K_{\mathrm{II,slant}}\, g_{22} \tag{15}$$

using the functions g_{ij} according to [2]

$$g_{11} = \cos^3(\beta/2) \tag{16a}$$

$$g_{12} = -3\sin(\beta/2)\cos^2(\beta/2) \tag{16b}$$

$$g_{21} = \sin(\beta/2)\cos^2(\beta/2) \tag{16c}$$

$$g_{22} = \cos(\beta/2)(1-3\sin^2(\beta/2)) \tag{16d}$$

In a first-order analysis it trivially holds: g_{11}=g_{22}→1.

MODE-II STRESS INTENSITY FACTOR AND CRACK PATH

The solution of Eqs. 4-16 at different times t and crack lengths a results in the individual thin curves of Fig. 2a for mode-I and the curves in Fig. 2b for mode-II. The dashed curve in Fig. 2a represents the envelope of the individual K_{appl}-curves. A maximum stress intensity factor of $K_{I,max}$ = 6 MPa√m is reached at about t = 1.6 ms for a crack length of a = 150 µm. This value is about 1.5 MPa√m smaller than obtained by the assumption of a straight crack normal to the free surface [1].

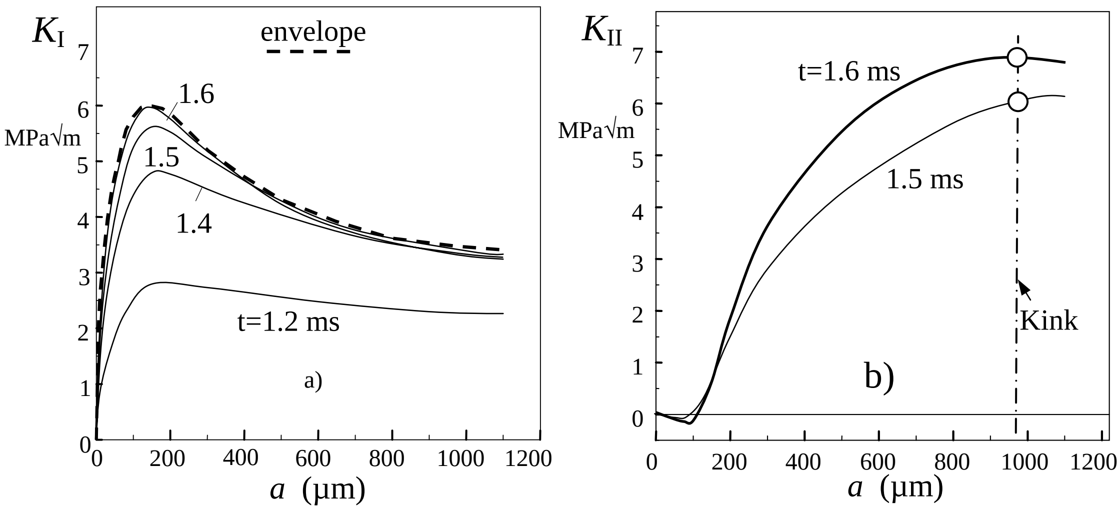

Fig. 2 Applied stress intensity factors as a function of crack depth a at different times t, a) mode-I, b) mode-II stress intensity factor, dash-dotted line indicates the crack length at which the crack rather abruptly kinked (Fig. 1).

Crack path: The problem of path development and path stability was investigated very extensively in fracture mechanics literature (for references see for instance the introduction chapter of Cotterell and Rice [2]). All these investigations using different fracture criteria have in common the feature, that a crack can extend with continuously turning tangent direction only if (in cases of traction free crack faces) the applied mode-II stress intensity factor $K_{II,appl}$ disappears

$$K_{II,appl} = 0 \tag{17}$$

This condition is called the *criterion of local symmetry*. If for a given crack the condition Eq. 17 is not fulfilled, the crack must spontaneously kink by an angle of Θ out of the initial crack plane and will propagate then under K_{II}=0. For small values of K_{II} /K_I, the crack kink angle Θ can be expressed by

$$\Theta = -2\frac{K_{\mathrm{II,appl}}}{K_{\mathrm{I0}}} \tag{18}$$

A complication of this simple behaviour occurs for ceramic materials. In addition to the well known mode-I effect of bridging interactions between the crack surfaces, it has to be expected that crack-face interactions may also affect crack extension under pure or superimposed mode-II loading as for instance outlined for frictional bridging in [5,6]. It was outlined that the shear tractions generated under small mode-II load contributions may cause a shielding stress intensity factor $K_{II,sh}$ which reduces the applied stress intensity factor $K_{II,appl}$. The total mode-II stress intensity factor, also called the crack-tip stress intensity factor $K_{II,tip}$, reads

$$K_{\mathrm{II,total}} = K_{\mathrm{II,tip}} = K_{\mathrm{II,appl}} + K_{\mathrm{II,sh}}\ , \quad K_{\mathrm{II,sh}} < 0 \tag{19}$$

In the case of crack shielding (i.e. in the presence of a mode-II R-curve), $K_{II,tip}$ must disappear during crack propagation

$$K_{\mathrm{II,total}} = K_{\mathrm{II,tip}} = 0 \tag{20}$$

Equation 19 enables to compute the mode-II shielding stress intensity factor $K_{II,sh}$. From Eqs 19 and 20 it simply results

$$K_{\mathrm{II,sh}} = -K_{\mathrm{II,appl}} \tag{21}$$

The actual mode-II crack tip stress intensity factor $K_{II,tip}$ present at the crack tip results from

$$K_{\mathrm{II,tip}} = \begin{cases} 0 & \text{for} \quad K_{\mathrm{II,appl}} + K_{\mathrm{II,sh}} \le 0 \\ K_{\mathrm{II,appl}} + K_{\mathrm{II,sh}} & \text{else} \end{cases} \tag{22}$$

This stress intensity factor governs the local stability of crack paths. If the value $K_{II,tip}$ does not disappear, the crack must kink by an angle of Θ out of the initial crack plane and will propagate then under $K_{II,tip}$=0. For small values of $K_{II,tip}/K_{I0}$, the crack kink angle Θ can be expressed similar to Eq. 18 by

$$\Theta = -2\frac{K_{\mathrm{II,tip}}}{K_{\mathrm{I0}}} \tag{23}$$

The mode-II stress intensity factor curves in Fig. 2b show that K_{II} is clearly not disappearing as might be expected from Eq. 17. This is a clear indication for the occurrence of *mode-II-shielding* as had to be expected even from the occurrence of *mode-I-shielding*.

At the moment of crack kinking (in Fig. 1 at about 960 µm), it must be fulfilled $K_{II,appl} = -K_{II,sh}$. From this condition we find a shielding (bridging) stress intensity factor between about $K_{II,sh} = K_{IIR} = -6$ MPa√m and −7 MPa√m, i.e. an effect in the same order of magnitude as observed for the mode-I R-curve. This of course is a material specific and not necessarily a general result.

So far we exclusively discussed the stress intensity factors for the existing crack as specified in Fig. 1a. For a real prediction of a crack path one needs knowledge of the mode-II R-curve as a predisposed material property. For this material no information of mode-I and mode-II shielding terms were available.

CONCLUSIONS

The criterion of local symmetry demands that cracks follow a path where the crack tip mode II applied stress intensity factor, $K_{II,appl}$, disappears. The analysed crack in the ceramic roll shows a kink what implies that for the kinked crack this condition is fulfilled. Since the contributions of the applied stress intensity factor $K_{II,appl}$ are considerable, this condition can only be fulfilled if considerable (negative) contributions of the crack shielding does occur. But we also have to conclude that exact crack-path predictions on disappearing $K_{II,appl}$ are not possible so far the “mode-II R-curve” of the material is not available as an independent material property (geometry, loading,…).

REFERENCES

1. Lengauer, M. Danzer, R. (2008) *J. Eur. Ceram. Soc.* **28**, 2289-2298.
2. Cotterell, B. Rice, J.R. (1980) *Int. J. Fract.* **16**, 155-169..
3. Fett, T., Rizzi, G., Bahr, H.A., Bahr, U., Pham, V.B., Balke, H. (2008) *Engng. Fract. Mech.* **75**, 2246-225.
4. Fett, T., Rizzi, G. (2005) *Int. J. Fract.* **132**, L9-L16.
5. Fett, T., Rizzi, G., Hoffmann, M., Oberacker, R., Wagner, S. (2008) *J. Mater. Sci.* **43**, 2077-2081.
6. Fett, T., Rizzi, G., Munz, D., Hoffmann, M., Oberacker, R., Wagner, S. (2006) *J. Ceram. Soc. Japan* **114**, 1038-1043.