# Design of Layered Ceramics with Crack Bifurcation/Deflection Mechanisms

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***ABSTRACT.*** *Layered ceramics have been proposed as an alternative for the design of structural ceramics with an improved fracture toughness and reliability. The use of energy release mechanisms such as crack branching, crack deflection and/or crack bifurcation can improve the crack growth resistance of the material. The tendency of a crack to be deflected along the interface or to penetrate through it is associated with the elastic and mechanical properties of the layers and the architectural design. Additionally, the residual stresses generated in these laminates during cooling down from sintering may also influence the crack path. In this work the conditions for crack propagation are investigated on an alumina-zirconia layered ceramic based on experimental observations under distinct loading scenarios. A crack deflection/penetration criterion for bimaterials has been used as theoretical framework. An optimal design for maximum energy consumption is proposed based on such theoretical analysis and experimental observations, which can be extended to other layered architectures aiming to improve the crack growth resistance of the material.*

## INTRODUCTION

Crack propagation in structural ceramics has been investigated by many authors in the last decades in order to understand the fracture process in ceramic materials. The brittle fracture of monolithic ceramics has been overcome by introducing layered architectures of a different kind, i.e. geometry, composition of layers, residual stresses, interface toughness, etc. In this regard, layered structures designed with weak interfaces [1], crack growth resistance (R-curve) behaviour through microstructure design [2] and/or residual stresses [3-9] among others, have shown an outstanding potential for structural applications showing enhanced fracture toughness by means of energy dissipating mechanisms such as interface delamination and/or crack deflection/bifurcation phenomena.

It is known that the flaw distribution (size, location, etc.) and size effect in ceramic materials yield a statistical strength distribution (described by the Weibull theory [10-12]), which conditions the mechanical reliability of ceramic components. Despite the outstanding features of colloidal processing in terms of flaw size reduction (i.e. increase of strength) [13], the presence of processing and/or machining defects in the ceramic material is in most cases unavoidable. In this regard, trends to design "flaw tolerant" materials rather than reducing the size of such defects have been the focus of many researchers in the last decades [4, 5, 7, 9, 14-16]. The main goal of multilayered ceramic designs has been to enhance the fracture energy of the system on the one hand and to increase the strength reliability of the end component on the other hand. The utilisation of tailored residual stresses in layered ceramics, generated during cooling down from sintering, to act as physical barriers to crack propagation under different loading conditions has succeeded in many ceramic systems [4, 5, 7, 9, 14, 17-19]. In addition to such "flaw tolerance" capability, an increase in the fracture energy of the material associated with the shielding effect of such compressive stresses in the layers has been achieved [7, 20-23]. The presence of energy release mechanisms such as crack branching, crack deflection and/or crack bifurcation during crack propagation can significantly improve the crack growth resistance of the material.

The efforts for designing layered materials with enhanced mechanical properties have focussed either on individual properties or on particular loading scenarios where such properties are evaluated. For instance, bending loading can yield a different response depending on the disposition of the layers, either parallel or normal to the loading axis [16, 24, 25]. The combination of flaw tolerant designs with enhanced toughness, being maintained regardless of the mode of loading is a difficult task that requires, in general, taking into account several parameters (often coupled), such as layer composition and thickness, elastic properties, residual stresses, interface toughness, loading mode, etc.

The motivation of this work is to investigate the conditions which may favour the presence of different energy release mechanisms in a unique layered ceramic architecture during crack propagation, considering the architectural design and material properties. Among the different mechanisms available, crack bifurcation and crack deflection (interface delamination) are studied in detail based on a crack deflection/penetration criterion for bimaterials as theoretical framework [26] and on experimental results of a reference layered structural ceramic (alumina-zirconia) previously investigated [8, 27].

## THEORETICAL APPROACH

A fracture mechanics analysis is here recalled based on a crack deflection/penetration criteria proposed by He and Hutchinson in 1989 [26]. In such work the conditions for a crack to penetrate into or deflect along the interface of two dissimilar materials with different elastic and/or mechanical properties were investigated. The tendency of a crack meeting at 90º the interface between dissimilar materials **B** and **A** to either penetrate through the next layer or deflect along the interface depends on whether the ratio $G_i/G_{layer}$ (i.e. fracture energy of the **A/B** interface/fracture energy of the adjoining layer per unit area) is either greater or lower than the ratio $G_d/G_p$ (i.e. energy release rate of the deflecting/penetrating crack given by the loading conditions and geometry configuration). The variables of interest depend only on two non-dimensional combinations of the material parameters associated with its elastic properties, the so called Dundurs' parameters, $\alpha$ and $\beta$ [28]:

$$\alpha = [\mu_A(1-\upsilon_B) - \mu_B(1-\upsilon_A)]/[\mu_A(1-\upsilon_B) + \mu_B(1-\upsilon_A)] \tag{1a}$$

$$\beta = [\mu_A(1-2\upsilon_B) - \mu_B(1-2\upsilon_A)]/[\mu_A(1-\upsilon_B) + \mu_B(1-\upsilon_A)] \tag{1b}$$

Where $\mu_i$ and $\upsilon_i$ are the corresponding shear modulus and Poisson's ratio respectively. The first and more important parameter can be easily interpreted when expressed as:

$$\alpha = \frac{E'_A - E'_B}{E'_A + E'_B} \tag{2}$$

where $E_i' = E_i/(1-\upsilon_i^2)$ is the plain strain elastic modulus, $E_i$ the Young's modulus and $\upsilon_i$ the Poisson's ratio of the corresponding layers **A** and **B**. Assuming a bi-material with a reference small crack length $a$ with the tip at the interface, when a symmetrical load is applied with respect to the crack plane (Fig. 1), the traction ahead of the crack in material **A** is given by the following equation:

$$\sigma_{xx}(0, y) = k_I (2\pi y)^{-\lambda} \tag{3}$$

where $k_I$ is proportional to the applied load and $\lambda$ is a real number that depends on $\alpha$ and $\beta$. More details can be found in [29].

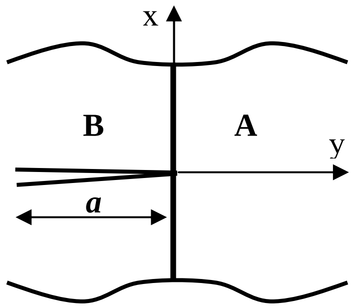


**Figure 1**. Scheme of a crack approaching an interface.

The crack may advance mainly in two ways: a) **straight**, penetrating into layer **A**, or b) **deflecting** along the interface of layers **A** and **B**.

<u>*In case of penetration*</u>, the stress state at the crack tip is pure mode I. The stress intensity factor depends on $k_I$ and $a$ according to:

$$K_I = c(\alpha, \beta) \cdot k_I a^{1/2-\lambda} \tag{4}$$

where $c$ is a dimensionless parameter as a function of $\alpha$ and $\beta$ that normally ranges between 0.8 and 1.2 [30]. The associated energy release rate can be expressed as:

$$G_p = \frac{1-\upsilon_A}{2\mu_A} K_I^2 = \frac{1-\upsilon_A}{2\mu_A} c^2 k_I^2 a^{1-2\lambda} \tag{5}$$

<u>*In case of crack deflection*</u>, the traction on the interface directly ahead of the deflected crack tip is characterised using a complex notation by [31]:

$$\sigma_{xx}(x,0)+i\sigma_{xy}(x,0)=(K_1+iK_2)\cdot(2\pi r)^{-1/2}r^{i\varepsilon} \tag{6}$$

where $K_1$ and $K_2$ can be considered as the conventional mode I and mode II stress intensity factors, $r$=$x$-$a$, and $\varepsilon$=(1/2π)ln((1-$\beta$)/(1+$\beta$)). Dimensional considerations require that:

$$K_1+iK_2=k_\mathrm{I}a^{1/2-\lambda}[d(\alpha,\beta)a^{i\varepsilon}+e(\alpha,\beta)a^{-i\varepsilon}] \tag{7}$$

where $d$ and $e$ are dimensionless complex functions of $\alpha$ and $\beta$, which have been evaluated through integral equation methods by HH in [26].

Thus, the energy release rate of the deflected crack results in:

$$G_\mathrm{d}=\left[\left(\frac{1-\upsilon_\mathrm{A}}{\mu_\mathrm{A}}\right)+\left(\frac{1-\upsilon_\mathrm{B}}{\mu_\mathrm{B}}\right)\right](K_1^2+K_2^2)/(4\cosh^2\pi\varepsilon) \tag{8}$$

Where

$$K_1^2+K_2^2=k_\mathrm{I}^2a^{1-2\lambda}[|d|^2+|e|^2+2\,\mathrm{Re}(d\cdot e)] \tag{9}$$

In order to establish a criterion for crack deflection/penetration the ratio $G_\mathrm{d}/G_\mathrm{p}$ must be evaluated. It can be observed that this ratio is independent of $a$ (and $k_\mathrm{I}$) and is given by:

$$G_\mathrm{d}/G_\mathrm{p}=[(1-\beta^2)/(1-\alpha)]\cdot[|d|^2+|e|^2+2\,\mathrm{Re}(d\cdot e)]/c^2 \tag{10}$$

The ratio $G_\mathrm{d}/G_\mathrm{p}$ is plotted in Fig. 2 as a function of $\alpha$.

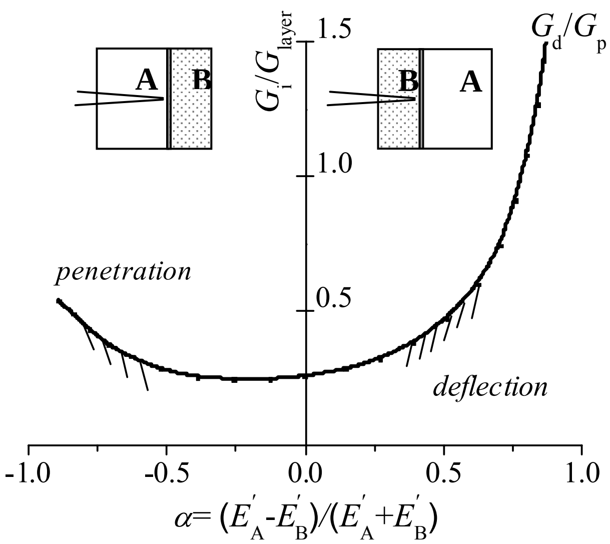


**Figure 2.** Crack deflection/penetration criterion for a crack propagating normal to the interface of two dissimilar materials **B** and **A**.

The influence of the parameter $\beta$ is not significant, as it can be inferred from Eq. (10), where $\beta$ appears explicitly only to order $\beta^2$, thus $\beta$ has been assumed equal 0 for every case. Considering the fracture energy of the interface and the fracture energy of the neighbouring layer, a crack propagating from layer **A** to layer **B** or vice versa would be likely to deflect along the interface if: $G_i/G_B < G_d/G_p$ or $G_i/G_A < G_d/G_p$ respectively. Likewise the crack will tend to penetrate when the inequalities are reversed.

## EXPERIMENTAL FEATURES ON MULTILAYERED CERAMICS

### Material of study

A multilayered ceramic consisting of alternated layers of $Al_2O_3$-5vol.%t$ZrO_2$, named **A**, and $Al_2O_3$-30vol.%m$ZrO_2$, referred to as **B**, was fabricated by slip casting following the procedure described elsewhere [32]. Samples were sintered at 1550 °C for 2 hours using heating and cooling rates of 5 °C/min. As a result, a symmetrical multilayered system with 4 thin **B** layers sandwiched between 5 thick **A** layers was obtained (Fig. 3). Due to the differential thermal strain between adjacent layers during cooling from sintering, biaxial residual stresses (parallel to the layer plane) appear within the layers, being tensile in the **A** layers and compressive in the **B** ones [8]. In table 1 the material properties measured in layers **A** and **B** are presented.

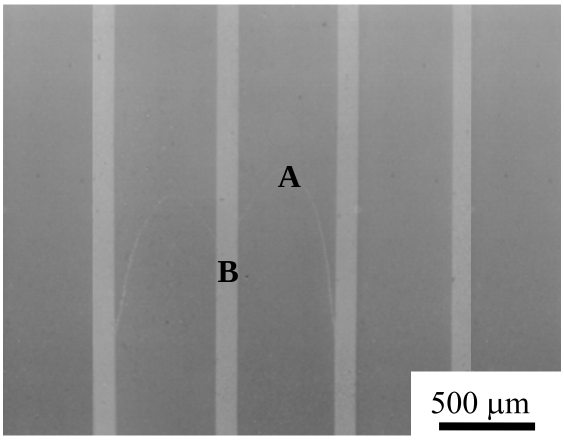


**Figure 3.** SEM micrograph of an alumina-zirconia layered architecture designed with residual stresses.

Table 1. Material properties

| Layer | Thickness [µm] | $E$ [GPa] | $\upsilon$ [–] | $\alpha$ (x10$^{-6}$) [°C$^{-1}$] (20-1200°C) | Res. Stress [MPa] | $\sigma_f$ [MPa] | $K_{Ic}$ [MPam$^{1/2}$] | $G_c$ [J/m] |
|---|---|---|---|---|---|---|---|---|
| **A** | 540 ± 10 | 390 ± 10 | 0.22 | 9.82 | +100 | 482 ± 65 | 3.2 ± 0.1 | 26 ± 1 |
| **B** | 95 ± 5 | 280 ± 15 | 0.22 | 8.02 | -690 | 90 ± 20 | 2.6 ± 0.1 | 23 ± 1 |

## Mechanical behaviour

The mechanical response of this multilayered ceramic under different loading scenarios has been investigated elsewhere [8, 18, 19, 27]. The high compressive biaxial stress in the thin **B** layers yields a so called "threshold strength", i.e. a minimum stress level below which the material does not fail [9]. As a consequence, the presence of relative large cracks in the outer layer (layer **A**) would not lead to catastrophic failure of the layered structure, thus increasing the reliability of the system.

The further propagation of the arrested cracks into the next layer under applied stress may occur by either penetration into the **B** layer or deflection along the **A**/**B** interface, according to a crack deflection/penetration criterion discussed in the previous section. In this regard, experimental observations of fracture surfaces have shown that the propagation of a crack from layer **A** to layer **B** always took place under penetration conditions. The explanation for that can be analysed according to Fig. 2 by energetic considerations and will be assessed in the next section. Once the applied stress intensity factor is increased (applied load increases) the crack enters the **B** compressive layer propagating in a stable manner. After a certain distance, a bifurcation mechanism takes place owed to the combination of high compressive stresses and relative thickness of the **B** layers [33]; the crack propagates along the centre of the compressive layer, as shown in Fig. 4.

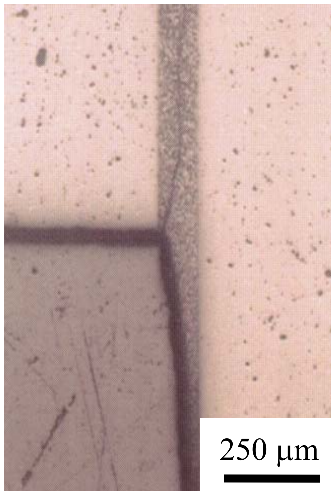


**Figure 4.** Crack bifurcation mechanism along the centre of the thin **B** compressive layer.

## Crack deflection/penetration criterion applied to multilayers

### *Effect of residual stresses*

As mentioned above in the previous section, the loading conditions and geometry configuration of the system may influence the energy release rate for crack deflection/penetration in a bimaterial. In this regard, the inherent architecture and composition of such systems may be associated with the presence of residual stresses as a result of the different thermo-elastic properties of each material. He, Evans and Hutchinson extended the above criterion for crack penetration/deflection for architectures with residual stresses [30]. Hence, the presence of residual stresses in the layers may affect the conditions for crack deflection/penetration, and thus the ratio $G_d/G_p$ represented in Fig. 2 may be modified affecting the crack propagating mode. For the case of layered structures, which may develop residual stresses during sintering, this feature should be taken into account for a correct plot of the crack propagation mode. In such case, in the presence of normal ($\sigma_{nor}$) and/or tangential ($\sigma_{tag}$) residual stresses, two additional non-dimensional length parameters, i.e. $\eta_{nor}$ and $\eta_{tag}$, become important, and are defined as [30]:

$$\eta_{nor} = \frac{\sigma_{nor} \cdot a_d^{\lambda}}{K_I}; \qquad \eta_{tag} = \frac{\sigma_{tag} \cdot a_p^{\lambda}}{K_I} \tag{11}$$

where $a$ is the length of the crack branch either at the interface ($a_d$) or in the next layer ($a_p$), $\lambda$ is a stress singularity exponent for the main crack, and $K_I$ is a factor proportional to the applied stress field (as defined above). In layered ceramics, the $\eta_{nor}$ parameter (related to the normal stresses at the interface) is usually zero, and the occurrence of interface delamination is dominated by $\eta_{tag}$, which accounts for the tensile or compressive in-plane residual stresses in the layers and represents the boundary region between crack deflection (delamination) and crack penetration. For the case of thin layers with relative high elastic modulus (large $E$) and a relative low thermal expansion coefficient (low $\alpha$) that results in a negative $\eta_{tag}$, interface delamination effects are favoured. On the other hand, when the elastic mismatch is not so significant crack penetration is enhanced. In Fig. 5 the $\eta_{tag}$ curves corresponding to a layered ceramic with residual stresses previously studied by the authors [27] are represented on a HH plot [30]. Such multilayered architecture consists of thick **A** layers alternated with thin **B** layers (see Fig. 3), which has ≈100MPa and ≈–690MPa residual stresses respectively [8]. The case for null residual stresses, $\eta_{tag} = 0$, is also presented with a point-line for comparative purposes.

It can be observed that in case the crack propagates normal to the interface from layer **A** to layer **B,** the compressive residual stresses in layer **B** yields an upwards shift of the $G_d/G_p$ curve, thus enhancing crack deflection. On the other hand, for crack propagating from the compressive to the tensile layer there is not significant effect. Therefore, the presence of high compressive stresses in layer **B** might favour the crack deflection at the interface when the crack propagates from layer **A** to layer **B**. However, by representing the corresponding $G_i/G_B$ and/or $G_i/G_A$ values in Fig. 5 (see full symbols)[ξ], it can be inferred that this effect is not significant for multilayer ceramics with strong interfaces, i.e. $G_i \approx G_{layer}$, even in presence of relative high residual stresses. It can be observed that, in any case, the crack propagating normal to the interface from layer **A** to layer **B** or vice-versa lies in the region of penetration. The effect of the residual stresses does not play any significant role for the crack deflection/penetration conditions, when the crack approaches the interface with an angle of ca. 90°.

[ξ] The interface fracture toughness was assumed as the toughness of layer **B**, i.e. 2.6 $MPam^{1/2}$, based on indentation fracture (IF) experiments.

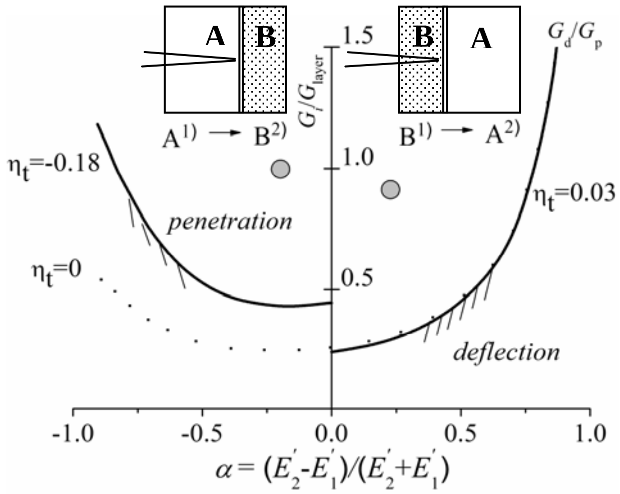


**Figure 5.** Crack deflection criteria for a crack propagating normal to the interface where the layers have residual stresses. $G_i/G_B$ and $G_i/G_A$ are represented as full symbols, which remain in the region of crack penetration.

*Effect of crack propagation angle*

As demonstrated by He et al. [26], the tendency for a crack to delaminate increases for small impinging angles. Under the same conditions and geometry, it is more likely for an inclined propagating crack to delaminate along an interface than for a crack which faces the interface with 90° degrees (i.e. normal to the interface). He, Evans and Hutchinson analysed the influence of the angle of the impinging crack on deflection/penetration mechanisms [30].

In this regard, experimental observations of the crack path in the multilayered ceramics here studied showed bifurcation effects in the compressive layers, which set a new angle of propagation for the crack. Therefore, in Fig. 2 an “upwards” correction of the curves should be recalled. Hence, a tendency for crack deflection might be now feasible. Figure 6a sketches the new curves for crack deflection of a crack propagating with different angles towards the interface of the multilayer of study. In such materials, and under certain conditions (e.g. loading mode, geometry, residual stresses), a propagating crack may deviate from the straight crack path when entering the layer with compressive residual stresses. An special case is that of crack bifurcation, which has been reported for these layered ceramics [8], as shown above in Fig. 4. In such cases, the crack branches (as it enters the compressive **B** layer) and thus faces the **B**/**A** interface with a certain angle. Under these conditions, and considering the correct angle of crack propagation (in the example ≈25°), the inequality $G_i/G_A < G_d/G_p$ may now be fulfilled (empty symbol in Fig. 6a), and thus crack deflection along the interface is likely to occur, thus enhancing the fracture energy of the system and maintaining intact the structure underneath (see Fig. 6b). This tendency of a bifurcating crack to deflect along the interface (in this case along the **B**/**A** interface) has been only evidenced by the authors in some particular layered ceramics under certain loading conditions (e.g. flexural loading at relative high temperatures (e.g. 800° C) [27] and also under cyclic loading at room temperature [19]).

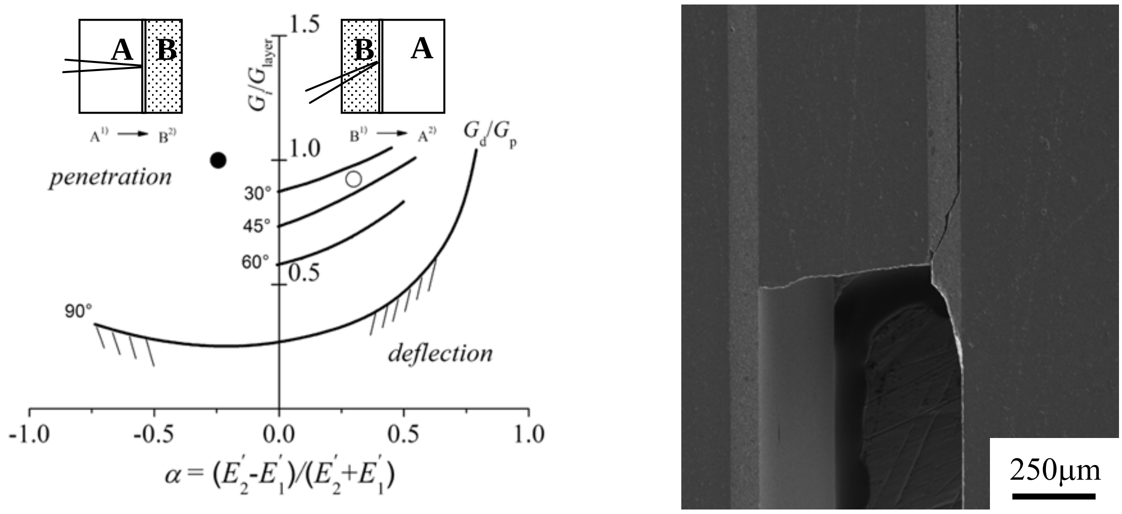


**Figure 6.** a) Crack deflection criteria for a crack propagating with different angles towards the interface; b) Example of a bifurcating crack approaching the **B**/**A** interface and delaminating.

These experimental observations supported by the theoretical approach using the HH diagram raise the query whether an optimal design for multilayered architectures which combines crack bifurcation mechanisms favouring interface delamination should be pursued. The effect of both energy release mechanisms should enhance significantly the total fracture energy of the system, in comparison with monolithic ceramics, as shown in different multilayered systems [8, 34-36]. Moreover, the fact that the crack propagates along the interface prevents the material from catastrophic failure, as for the case of layered ceramics with weak interfaces [1, 37], thus increasing the mechanical reliability of the component.

## GUIDELINES FOR MULTILAYER CERAMIC DESIGNS

The main mechanisms to increase the fracture energy of many layered ceramics are crack bifurcation, crack deflection and interface delamination, as commented in the introduction. Based on a fracture energy criterion and experimental observations an optimal design in terms of laminate geometry and internal residual stresses may be found. Energy release mechanisms such as crack bifurcation and interface delamination should be combined to maximise the crack growth resistance of layered architectures.

Concerning the multilayer ceramics here studied, an important mechanism for toughness enhancement is the crack bifurcation taking place in the thin compressive layer, i.e. the propagating crack deviates from the straight path as it enters the compressive layer (see Fig. 4). The conditions for the appearance of crack bifurcation have been addressed by many authors [8, 33, 38-44]. In a previous work, the authors showed that an optimal design should consist of a layer architecture where the compressive layers are thin enough to ensure a relative high threshold strength (i.e. the thinner the compressive layer, the higher are the compressive residual stresses, and thus higher strength), but thick enough to induce crack bifurcation as the crack enters the compressive layer, thus yielding higher fracture energy [8]. For a layered ceramic subjected to flexural loading, where the load is applied normal to the layer plane, a crack propagating perpendicular to the layers is prone to bifurcate in the compressive layer if the product $t \cdot \sigma^2_c$ (where $t$ is the thickness of the layer and $\sigma_c$ the compressive residual stresses) is larger than a critical value [45]. This statement has been mainly based on experimental observations [45] and has been the topic of many attempts using finite element analyses [15, 42-44]. Although several explanations have been given for the onset of crack bifurcation, a 3-dimensional model might be still required to account for the triaxial stress near free surface and other effects such us edge cracking, which is claimed to be close related to bifurcation mechanisms. Beside the appearance of crack bifurcation, another important parameter is the angle with which the bifurcating cracks approach the next interface, which may lead to additional energy consumption through interface delamination. It can be inferred from Fig. 6a that the smaller the angle the higher is the ratio $G_d/G_p$, i.e. the condition $G_i/G_A < G_d/G_p$ for crack deflection can be fulfilled. It has been shown that the bifurcation angle is associated with 1) the level of compressive stresses [46] and 2) the thickness of the compressive layer [8]. An optimal design that favours small crack bifurcation angles should contain high compressive stresses, which can be obtained with thin compressive layers, bearing I mind that the thickness should always remains above the critical thickness for crack bifurcation.

Another important parameter which may favour crack delamination is the Young's modulus of the layers. The coefficient $\alpha$ (given by Eq. 2) should be then as large as possible, so that the deflection region in Fig. 6a can be favoured. In a previous work [23] the authors showed that, for layered ceramics with compressive residual stresses in the internal layers, the effect of variation of Young's modulus between layers will not lead to important changes in terms of optimal strength and toughness for the multilayer. However, it may condition the level of residual stresses (responsible for crack bifurcation). Based on the material properties reported in Table 1, i.e. $E_A$=390 MPa and $E_B$=290 MPa, the coefficient $\alpha$ results in ≈±0.15. By increasing the stiffness of layer **A** in a 20%, the coefficient would result in ≈±0.20. On the other hand, reducing the stiffness of layer **B** by 20%, the coefficient would result in ≈±0.25. The latter (more effective) may be achieved, for instance, by increasing the porosity of the layer in approx. a 10% [47]. Assuming the new value for $E_B$, i.e. ≈230 MPa, the corresponding compressive stresses in the thin layers would vary from –690 MPa to –580 MPa. This is still a relative high level of compressive stresses, which would maintain the crack bifurcation features, occurring at a relative small bifurcation angle.

Summarising, an optimal design that favours crack bifurcation mechanisms and delamination at the interface is strongly dependent on the level of compressive stresses which is associated with the multilayer architecture and elastic properties of the layers. These parameters are intrinsically related and should be taken into account when modelling such layered structures. This analysis based on experimental observations on alumina-zirconia multilayer ceramics and analytical models may be extended for other multilayer systems where such energy release mechanisms have been reported.

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