Crack growth rates for microstructurally short fatigue cracks

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ABSTRACT. *The influence on the crack growth rate on a microstrucurally short edge crack, subjected to fatigue loading, due to changes in crack length, distance to grain boundaries and applied load, is investigated. The crack is assumed to grow in a single shear mechanism due to nucleation, glide and annihilation of dislocations along preferred slip planes in the material. The geometry is modelled by distributed dislocation dipole elements in a boundary element approach under quasi-static and plane strain conditions. The evolving plasticity is described by discrete dislocations situated along one single slip plane in front of the crack, coinciding with the crack direction.*

INTRODUCTION

It is well known that the behaviour of microstructurally short cracks subjected to fatigue loading is influenced by features of the surrounding microstructure in the material, such as grain boundaries, slip plane orientation and local plasticity near the crack tip. Such cracks grow in a single shear mechanism, cf. Suresh [1], as a result from nucleation, glide and annihilation of dislocations. The crack grows along specific slip planes within the grains of the material, and not perpendicular to the loading axis as typically is observed for long fatigue cracks on a global scale. Due to the low growth rates observed for short cracks it is important to account for individual dislocations created during the fatigue process, building the plastic zone. Similar models taking individual dislocations into account have been developed by Riemelmoser et. al. [2] to study the cyclic crack tip plasticity for a long mode I crack, and by Bjerkén and Melin [3] to study the influence of grain boundaries on a short mode I fatigue crack, among others. A similar approach was also used by Krupp et. al. [4], who instead of discrete dislocations, used dislocation dipole elements to describe the plasticity in order to study the growth of a short crack in duplex steel.

In this study a discrete dislocation model, describing both the geometry and the plasticity by discrete dislocations, is used to study the fatigue growth of a microstructurally short edge crack. The aim of this paper is to evaluate how the crack growth rate changes in relation to crack length, distance to grain boundary and applied load.

PROBLEM UNDER CONSIDERATION

The fatigue growth of a microstructurally short edge crack located within one grain in a bcc structure, subjected to fatigue loading, have been investigated under plane strain and quasi-static conditions. The crack grows in a single shear mechanism due to nucleation, glide and annihilation of discrete dislocations along specific slip planes in the material. In this study, it is assumed that only one slip plane is active in order to ensure that the crack will remain straight and not grow in a zigzag pattern on alternating slip planes as obtained in general cases, cf. Hansson and Melin [5, 6]. From [5,6] it was found that changes in growth direction strongly influenced the growth rate which would shadow the influence from other parameters such as grain size, external load, crack length and the presence of a grain boundary, investigated in this study.

The initial crack, of length a, inclined an angle θ to the normal of the free edge, is located within a semi-infinite body, cf. Figure 1, with the only active slip plane having the same direction as the initial crack. In front of the crack one or two grain boundaries is introduced, hindering the spread of the plasticity. The external fatigue load, σ_{yy}^{∞}, is applied at infinity, parallel to the free edge and is varied between a maximum value, σ_{yymax}^{∞}, and a minimum value, σ_{yymin}^{∞}.

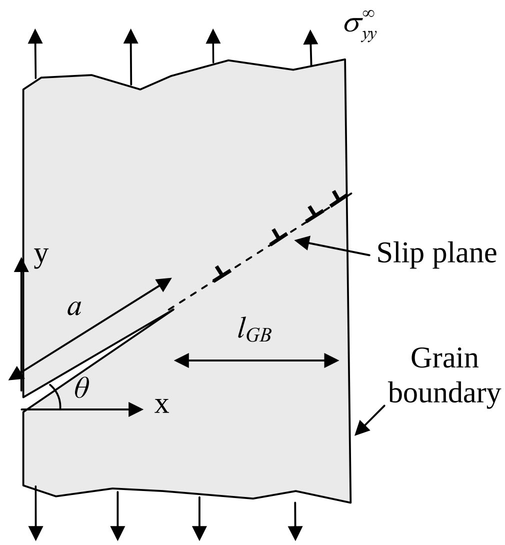

Figure 1. Geometry of the short edge crack with $\perp$ denoting the position of one positive edge dislocation in the plastic zone.

BOUNDARY ELEMENT APPROACH

The modelling in this study rests solely on a dislocation formulation, were both the geometry of the short edge crack and the plasticity are described by dislocations in a boundary element approach. Only plane problems are addressed, and, therefore, only edge dislocations are needed in the formulation.

External boundary modelling

The external boundary, here defined as the free edge together with the crack, cf. Figure 1, is modelled by dislocation dipole elements. Each dipole element consists of two climb and two glide dislocations, situated at the end points of the dipole element, according to Figure 2. Using both climb and glide dislocations make it possible to determine the opening between as well as the shearing between the crack surfaces. The stresses in an element are calculated at the collocation point, CP, at the centre of each element, cf. Figure 2. For the elements along the crack, the magnitudes of the climb dislocations directly correspond to the crack opening, whereas the magnitudes of the glide dislocations correspond to the shearing between the crack surfaces.

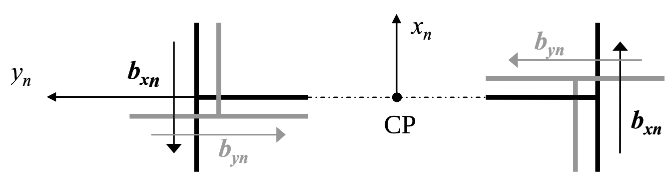

Figure 2. Dislocation dipole element consisting of four edge dislocations, with (x_n, y_n) denoting the local coordinate system and CP the collocation point at which the stresses are calculated.

The stresses at an arbitrary point in the body are calculated by adding the stress contributions from all dislocations in the dipole elements at this specific point, calculated according to Eq. 1, cf. Hills et al. [7], and the externally applied load. Eq. 1 describes the stress field from one edge dislocation situated at the origin of the local coordinate system (x, y), where b_k is the Burgers vector, μ the shear modulus, κ the Kolosov constant, and $G_{ij}^k(x,y)$ the influence functions given in [7].

$$\sigma_{ij}(x,y) = \frac{2\mu}{\pi(\kappa+1)} \sum_{k=x,y} G_{ij}^k(x,y) b_k, \; i,j = x,y \quad (1)$$

The sizes of the dislocations in the dipole elements are calculated from an equilibrium equation, describing the normal and shear stresses along the external boundary. Knowing that the normal and shear stress along the external boundary must equal zero, if the crack is assumed to be fully open, the magnitudes of the dislocations in the dipole elements can be determined.

Plasticity

The developing plastic zone is, in this approach, described by discrete dislocations, each of magnitude b, the Burgers vector of the material, spread out along the slip plane in front of the crack. The dislocations contribute to the stress field and must therefore be included in the equilibrium equation, needed to calculate the magnitudes of the dipole dislocations. The equilibrium equation, taking this into account, is given by Eq. 2:

$$\mathbf{G}\mathbf{b}_{\mathbf{boundary}} + b\mathbf{G}_{\mathbf{internal}} + \boldsymbol{\sigma}^{\infty} = \mathbf{0} \tag{2}$$

Where $\mathbf{G}$ is a matrix holding the influence functions for the dislocations in the dipole elements, $\mathbf{b}_{\mathbf{boundary}}$ is a vector containing the magnitudes of the dipole dislocations, $\mathbf{G}_{\mathbf{internal}}$ is a vector holding the influence functions for the dislocations constituting the plastic zone and $\boldsymbol{\sigma}^{\infty}$ is a vector containing the shear and normal stress contributions, due to the external load.

In the simulations it is assumed that no exist within the material prior to the first loading cycle. As the external load is raised the stresses in front of the crack tip increases, and it is assumed that when the resolved shear stress, τ_{slip}, calculated according to Eq. 3, reaches the nucleation stress, τ_{nuc}, a new dislocation pair is nucleated at the crack tip. In Eq. 3 θ is the angle between the global x-axis and the slip plane. In this study the crack tip is assumed to be the only source for dislocation nucleation.

$$\tau_{slip}(\theta) = \frac{\sigma_{yy} - \sigma_{xx}}{2} \sin 2\theta + \sigma_{xy} \cos 2\theta \tag{4}$$

The dislocation pair consists of two dislocations of equal size but opposite sign, separated a small distance. The dislocation having a Burgers vector pointing inwards in the material is called positive dislocation and the one pointing towards the free edge is called a negative dislocation. The nucleation stress is defined as the lowest stress at the nucleation point needed to ensure that the positive dislocation in the newly nucleated pair travels inwards in the material directly after nucleation. This value is found to be geometry dependent and most therefore be calculated for each new crack geometry.

Crack growth

As the applied load gets sufficiently high dislocation pairs will nucleate at the crack tip, creating a plastic zone in front of the crack. The positive dislocation will glide inwards into the material directly after nucleation along the slip plane as long as the resolved shear stress at its momentary position exceeds the lattice resistance of the material, τ_{crit}, whereas the negative dislocation will remain at the crack tip. The dislocations creating the plastic zone shield the crack tip and, therefore, the load must be further increased before more dislocation pairs can nucleate. This process of nucleation and glide of dislocations continues until the maximum load is reached and the load starts to decrease. Load reversal will, eventually, result in that the dislocations start to glide in the opposite direction, back towards the crack. When a positive dislocation gets close to its negative counterpart at the crack tip the two annihilate, resulting in crack growth in the corresponding direction by one b, under the assumption that no healing of the crack surfaces is allowed. This growth process due to annihilation of dislocations continues until minimum load is reached and a new load cycle starts. More details about the developed model used in this study are found in [5, 6].

RESULTS AND DISCUSSION

Initial conditions

The material used in this study is pure iron, with a bcc crystal structure and which is assumed to be linear elastic. The material parameters are shown in Table 1, cf. Askeland [8], together with the geometrical data for the initial short edge crack seen in Figure 1 and the calculated value of the nucleation stress.

Table 1. Material properties and geometrical parameters.

Shear modulus, μ	80GPa	Initial crack length, a	10000-80000b
Poisson's ratio, ν	0.3	Crack angle, θ	45°
Burgers vector, b	0.25nm	Distance to grain boundary, l_{GB}	2000-15000b
Lattice resistance, τ_{crit}	40MPa	Applied load, $\sigma^{\infty}_{yy\max}, \sigma^{\infty}_{yy\min}$	220MPa, 20-80MPa
Nucleation stress, τ_{nuc}	1.59GPa		

Crack growth rate as function of distance to the grain boundary

The distance of the grain boundary from the crack tip, l_{GB}, influences the crack growth rate da/dN since it controls the plasticity spread and the static plastic zone size. It is also a measure of the grain size, keeping the crack length constant. As an example, with a=20000b, and $\sigma^{\infty}_{yy\,min}$=40MPa $\sigma^{\infty}_{yy\,max}$=200MPa, the number of dislocations along the slip plane at maximum and minimum load as functions of l_{GB} is shown in Figure 3.1. It is found that both the maximum and minimum number of dislocations is increased as l_{GB} is increased. It is also seen that the difference between maximum and minimum number of dislocations, corresponding to the number of annihilated dislocations, i.e. the crack growth rate per cycle, increases somewhat with increasing l_{GB}. When studying a number of different $\sigma^{\infty}_{yy\,min}$, holding $\sigma^{\infty}_{yy\,max}$ constant it was, however, found that this is not always the case, as seen in Figure 3.2, where the growth rate for four different $\sigma^{\infty}_{yy\,min}$ as function of l_{GB} is seen. As can be seen, for low values of $\sigma^{\infty}_{yy\,min}$ the growth rate increases with crack length. For higher values of $\sigma^{\infty}_{yy\,min}$ it was, however, found that the growth rate first increases with l_{GB} and then decreased for larger values of l_{GB}. This is because, for large values of l_{GB}, a large number of dislocations can remain along the slip plane also at low applied loads. This results in that the applied load when the first annihilation occur, decreases as l_{GB} increases, resulting in that the lowest $\sigma^{\infty}_{yy\,min}$ needed to obtain crack growth is decreased when l_{GB} is increased.

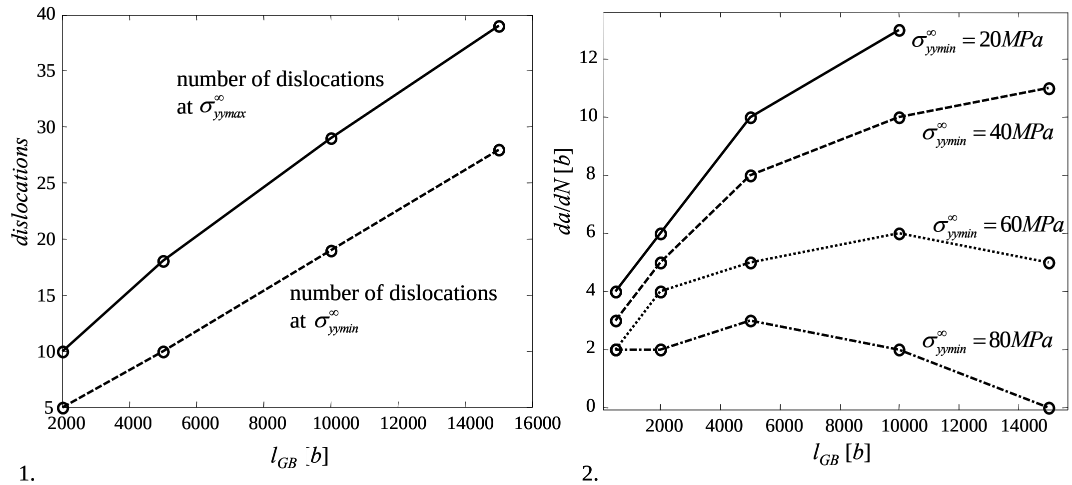

Figure 3. Number of dislocations along the slip plane at $\sigma^{\infty}_{yy\,min}$=40 MPa and $\sigma^{\infty}_{yy\,max}$=200MPa as functions of l_{GB}. 2. Crack growth rate for different $\sigma^{\infty}_{yy\,min}$ as a functions of l_{GB}.

Crack growth rate as function of ΔK

To compare the results from the simulations to typical behaviour for long cracks the crack growth rate is calculated as a function of the stress intensity factor range, *ΔK*, [1]. *ΔK* is calculated according to Eq. 5, where no consideration to the mixed mode loading, or influence of the free edge is taken into consideration. Thus *ΔK* is merely treated as a way of measuring how crack length and external load influences the stresses in front of the crack.

$$\Delta K = \left(\sigma^{\infty}_{yymax} - \sigma^{\infty}_{yymin}\right)\sqrt{\pi a} \tag{5}$$

First da/dN as function of *ΔK*, with *ΔK* increasing due to increasing *a*, was calculated with l_{GB1}=5000*b* and $\sigma^{\infty}_{yy\,max}$=200 MPa for different $\sigma^{\infty}_{yy\,min}$ with four different crack lengths; *a*=10000*b*, 20000*b*, 40000*b* and 80000*b*, cf. Figure 4.1. It is found that, for all but the lowest growth rates, the curves for the different load amplitudes show very good agreement. The discrepancies for the lowest loads are due to that at already $\sigma^{\infty}_{yy\,min}$=40 MPa, the crack growth rate equals zero. Therefore, the first point, with $\sigma^{\infty}_{yy\,min}$=60 MPa and $\sigma^{\infty}_{yy\,min}$=80 MPa, is not the limit for crack growth.

In Figure 4.2, showing da/dN as function of ΔK, ΔK is instead increasing due to increasing $\sigma^{\infty}_{yy\,max}$, in the interval $\sigma^{\infty}_{yy\,max}$=140-360 MPa, with a=20000b and l_{GB1}=5000b. The curves in Figure 4.2 show good agreement, especially for the highest growth rates. The differences in the threshold values of ΔK, below which no crack growth will occur, is also clearly seen in the figure and it was found that the highest growth rates, given a constant ΔK, is obtained with $\sigma^{\infty}_{yy\,min}$=80 MPa and the lowest with $\sigma^{\infty}_{yy\,min}$=20 MPa. This is because it requires a smaller increase in $\sigma^{\infty}_{yy\,max}$ to nucleate further dislocations than the required increase in $\sigma^{\infty}_{yy\,min}$ to annihilate one less dislocation, which leads to an increase in threshold value with decreasing $\sigma^{\infty}_{yy\,min}$.

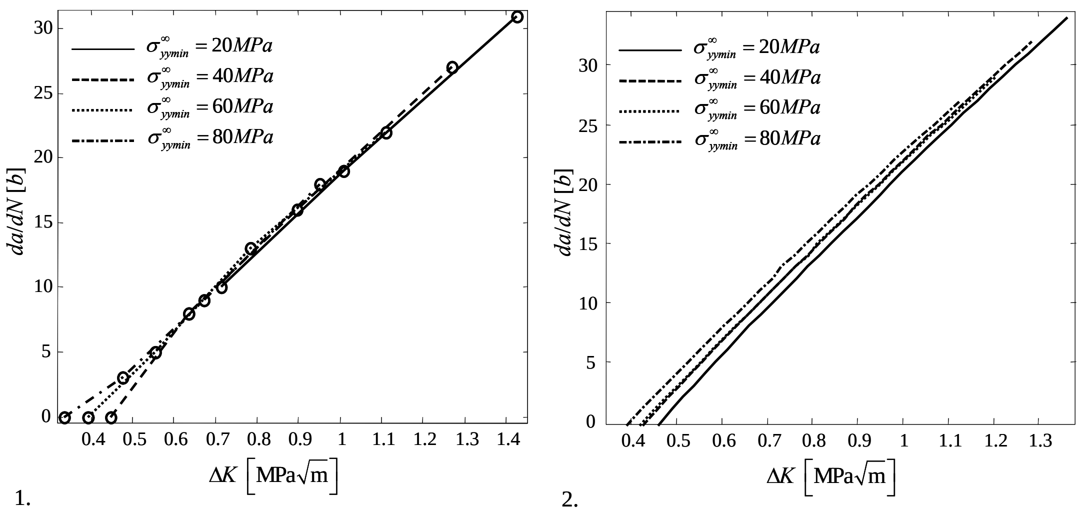

Figure 4.Crack growth rate, da/dN, as function of stress intensity factor, ΔK. 1. Constant $\sigma^{\infty}_{yy\,max}$ increasing a. 2. Constant a increasing $\sigma^{\infty}_{yy\,max}$.

Also a comparison between the two different ways of increasing ΔK, by either increasing a or increasing $\sigma^{\infty}_{yy\,max}$, has been performed, using $\sigma^{\infty}_{yy\,min}$=40 MPa and l_{GB1}=5000b. The result is seen in Figure 5 and it is seen that, for low growth rates, the two ways of increasing ΔK give similar results. For higher growth rates, however, it is found that the curves shows different slopes and, by increasing a, results in lower growth rates as compared to increasing $\sigma^{\infty}_{yy\,max}$. When using the method of increasing $\sigma^{\infty}_{yy\,max}$, an almost linear relation between ΔK and da/dN was observed as compared to what is obtained for long cracks were ΔK, typically, is of the power of two and four [1] in relation to da/dN.

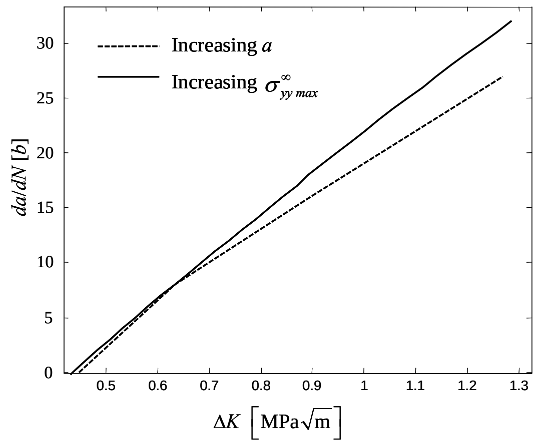

Figure 5. Comparison of crack growth rate, *da/dN*, as function of *ΔK*. Increase in *ΔK* is either due to increasing crack length (dashed line) or increase in $\sigma^{\infty}_{yy\,max}$ (solid line).

CONCLUSIONS

The crack growth rate might both increase and decrease for increasing distance between the crack tip and a grain boundary, depending on the chosen load range. It was also found that the crack growth rate increases, approximately, linearly with stress intensity factor range, contrary to long cracks which follows Paris' law with an exponent of two to four. Also good agreement was found when comparing the growth rates for different load ranges, changing either the crack length or the maximum load. However, some small differences were found when comparing the increase in crack growth rate due to an increase in crack length or due to increasing in maximum load.

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