# An experimental investigation of the crack growth under proportional and non proportional fatigue loading

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***ABSTRACT.*** *In the present investigations a series of fatigue crack growth experiments has been conducted using thin-walled, hollow cylinders with a notch made of fine-grained steel S460N. Pure tension-compression, pure torsion, proportional loading resulting from these two load components and out-of-phase loading with phase angles of 45° and 90° have been analyzed. Only constant amplitude loading with R=-1 has been examined. Within the experiment the crack-initiation, the failure and the crack growth life as well as the crack growth curves and the crack paths have been identified. Depending on the loading type, two to four cracks initiate at different positions of the notch. Each crack initiates as a corner crack on the inner side of the slot. Under pure tension-compression and pure torsion the expected crack growth behavior can be observed. Furthermore the crack growth behavior of specimens under proportional and out-of-phase loading with a phase angle of 45° are similar. Under out-of-phase loading with a phase angle of 90° various different types of crack initiation and growth have been identified. The crack-initiation correlates well with the local stresses and strains. Crack propagation lives vary with the type of loading.*

## INTRODUCTION

Fatigue cracks commonly initiate at stress concentrations, such as notches. In general these cracks are exposed to multiaxial and non proportional loading. The crack initiation life, the crack growth behavior and therefore the life to total failure of the component are strongly dependend on the applied loading.

To investigate the crack initiation and crack growth behavior under proportional and non proportional loading, Brüning [1] tested thin-walled, hollow cylinders with a slot.

## EXPERIMENTAL INVESTIGATIONS

23 thin-walled, hollow cylinders according to Figure 1 have been tested. Chemical composition and basic mechanical properties of the material S460N are given in Tables 1 and 2. To achieve large crack growth phases a slot was inserted into the specimens. The dimensions of the slot are given in Figure 2.

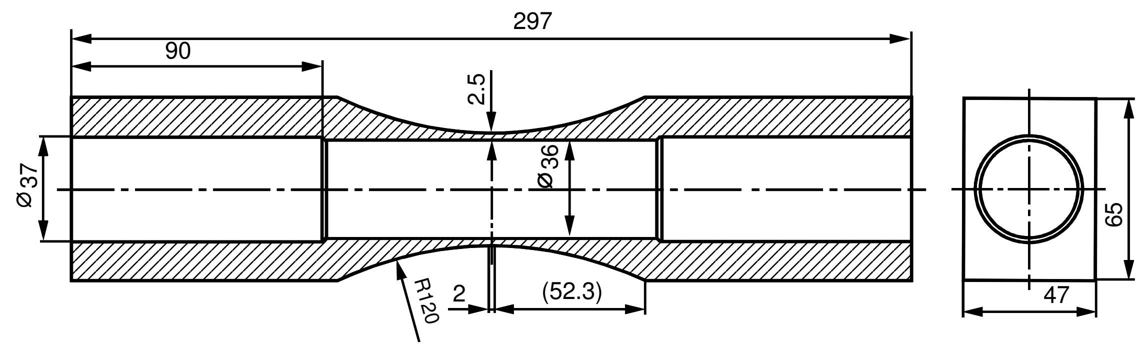


Figure 1. Geometry of the specimens

Table 1. Chemical composition in [wt.%] from [2]

| C | Si | Mn | P | S | Al | Cr | Mo | N | Ni | V |
|---|---|---|---|---|---|---|---|---|---|---|
| 0.18 | 0.44 | 1.54 | 0.016 | 0.001 | 0.013 | 0.022 | 0.004 | 0.019 | 0.27 | 0.17 |

Table 2. Mechanical properties from [3]

| Yield stress $R_{p0.2}$ [MPa] | Ultimate stress $R_m$ [MPa] | Elongation after fracture [%] | Cyclic hardening coefficient K' [MPa] | Cyclic hardening exponent n' [-] |
|---|---|---|---|---|
| 500 | 643 | 26.2 | 1115 | 0.161 |

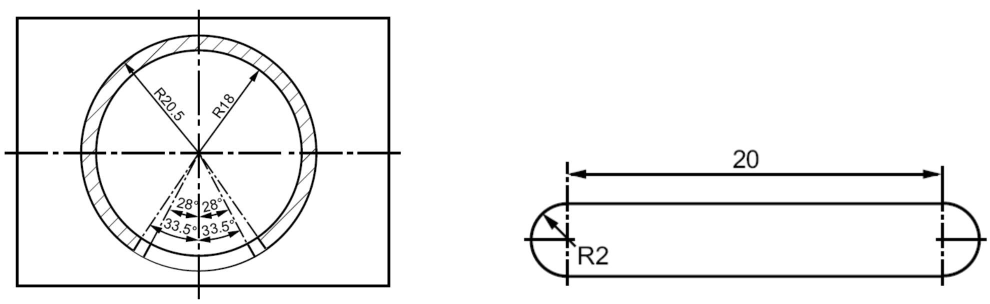


Figure 2. Geometry of the slot: top view of a cut through the specimen on the left side and front view of the uncoiled geometry on the right side

Five different loading types have been considered (refer to Figure 3): pure tension-compression loading, pure torsional loading, proportional loading resulting from these two load components and out-of-phase loading with phase angles of 45° and 90°. So far only constant amplitude loading with an R-ratio of -1 has been examined. The experiments have been conducted using a servo-hydraulic four-pillar testing machine.

## RESULTS

For all loading types the cracks initiate as corner cracks on the inner side of the slot and grow up to a through-thickness crack. The crack length *a* is defined as the direct connection between the starting point of a through crack at the outer side of the slot and the crack tip on the surface of the specimen. Crack initiation has been defined for a crack length of c=0.5 mm in thickness direction. Depending on the loading type, two to four cracks initiate.

Due to the limited space, only the crack-initiation, the failure and crack growth lives for pure tension-compression loading are depicted in Figure 14 und Figure 15. As expected, the crack paths for this loading are under pure mode I.

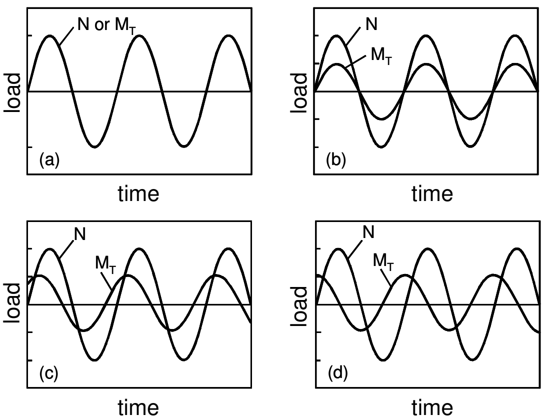


Figure 3. Loading types: (a) pure tension-compression or pure torsion, (b) proportional loading, (c) out-of-phase loading with a phase angle of 45° and (d) out-of-phase loading with a phase angle of 90°

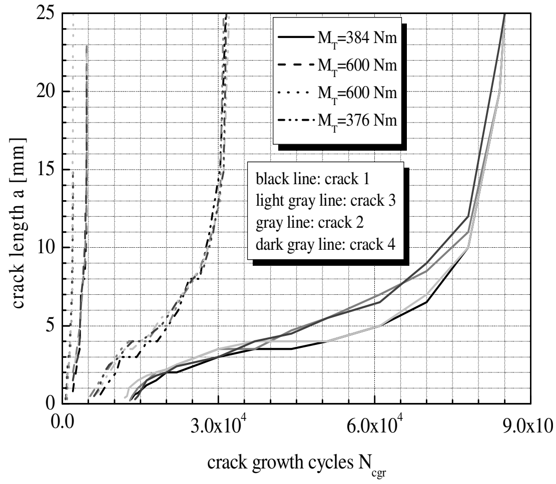


Figure 4. Crack growth curves under pure torsional loading

### *Pure torsional loading*

For the experiments with pure torsional loading, four specimens have been used. The torsional moments have been set to $M_T$=376 Nm , $M_T$=384 Nm and two times $M_T$=600 Nm. Again as expected, four almost symmetric cracks initiate approximately at the same time with an angle of about 45° to the longitudinal axis. As the cracks become larger, they grow in a plane, which is almost perpendicular to the longitudinal axis. The reasons are not only due to the loading, but also due to the geometry of the specimen, because wall thickness increases to the ends (refer to Figure 1). This crack growth behavior is exemplarily shown for a torsional loading of $M_T$=600 Nm in Figure 5.

In Figure 4 the crack growth curves for the specimens under torsional loading are depicted. It is obvious, that again the symmetric cracks of one specimen grow almost with the same rate.

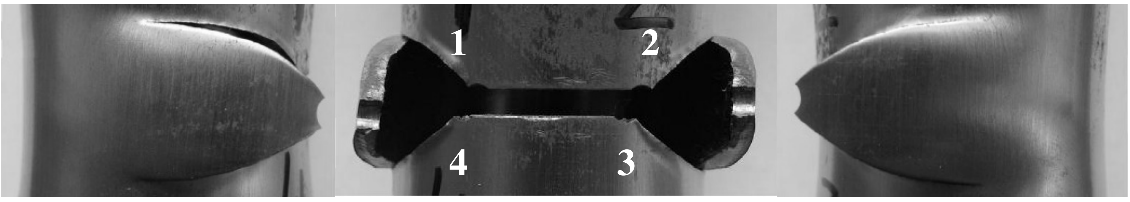


Figure 5. Crack path under pure torsional loading with $M_T$=600 Nm

***Proportional loading***

Five specimens have been tested under proportional loading with the following (N;$M_T$)-values: (25.0 kN; 384 Nm), (16.5 kN; 247 Nm), (19 kN; 224 Nm), (29.5 kN; 344 Nm) and (29 kN; 275 Nm).

Under proportional loading two cracks initiate at the positions 1 and 3 at first. Only for the specimen with (25.0 kN; 384 Nm) further cracks initiate later at the positions 2 and 4, but do not become failure critical. This phenomenon leads to the conclusion that the local stresses at the positions 1 and 3 are much higher than at 2 and 4.

The crack growth behavior of the cracks 1 and 3 is almost the same. First the the cracks grow perpendicularly to the slot surface and then they turn into a plane, which is perpendicular to the longitudinal axis of the specimen, as shown exemplarily for (19 kN; 224 Nm) in Figure 6. The crack path for the specimen with (25.0 kN; 384 Nm) is shown in Figure 7. The almost identical crack growth rate can also be seen from the crack growth curves in Figure 8. Only for the specimen with (25.0 kN; 384 Nm) the crack growth for crack 1 and 3 differs from each other. This is because crack 2 is much larger than crack 4 (refer to Figure 7). Thus, the stress at the crack tip is smaller for crack 3 than for crack 1 and therefore the crack growth rate is smaller.

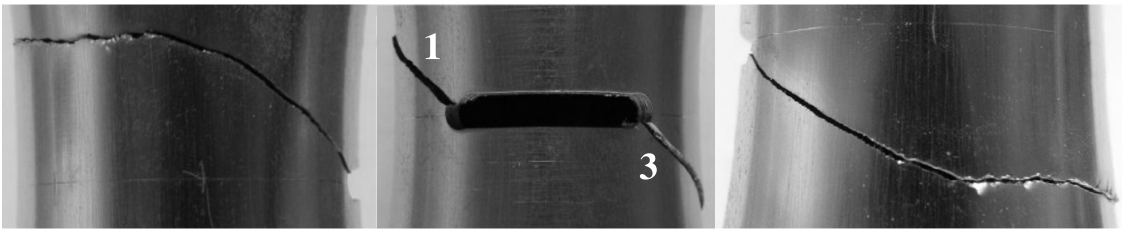


Figure 6. Crack path under proportional loading with (N;$M_T$)=(19 kN; 224 Nm)

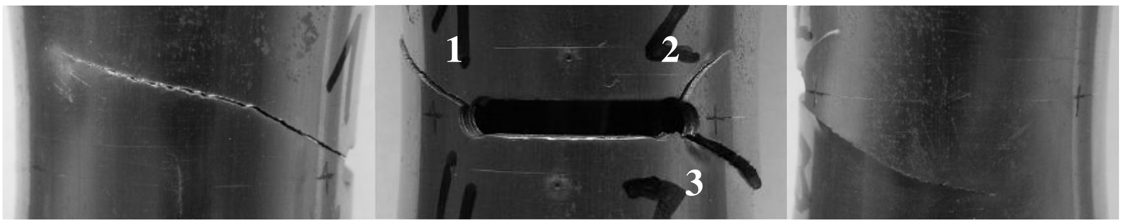


Figure 7. Crack path under proportional loading with (N;$M_T$)=(25.0 kN; 384 Nm)

***Non proportional loading with a phase angle of 45°***

To analyze the crack growth behavior under non proportional loading and a phase angle of 45° five specimens with (N; $M_T$)-values of two times (22.5 kN; 272 Nm), (32 kN; 300 Nm), (35 kN; 330 Nm) and (39 kN; 370 Nm) have been tested.

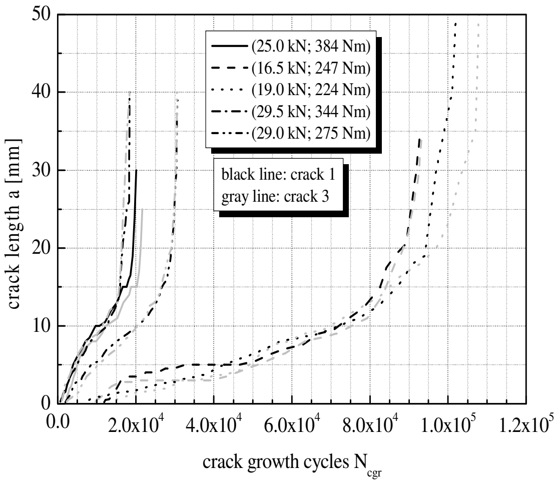


Figure 8. Crack growth curves under proportional loading

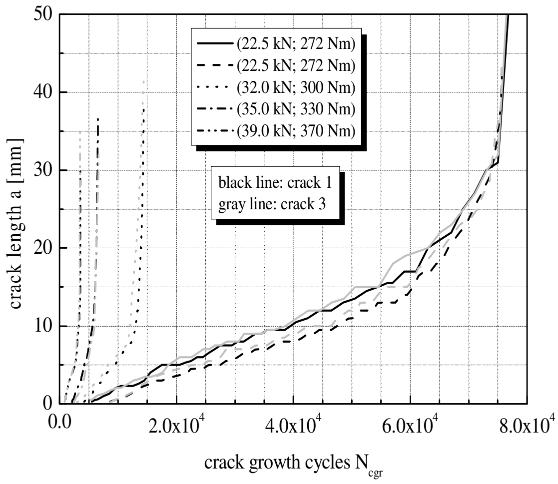


Figure 9. Crack growth curves under out-of-phase loading with a phase angle of 45°

The results obtained from these experiments are pretty equal to the ones obtained for proportional loading. First two cracks initiate at position 1 and 3. Crack 2 and 4 initiate either later or not at all. The crack paths are initially almost straight-line. Under lower load levels, i.e. (22.5 kN; 272 Nm) and (32 kN; 300 Nm), again the cracks turn into a plane, which is perpendicular to the longitudinal axis (refer to Figure 10), as for the proportional loading. Under higher load levels, i.e. (35 kN; 330 Nm) and (39 kN; 370 Nm), this tendency is not monitored and the cracks stay straight-line (refer to Figure 11). As for proportional loading, the crack growth rates for crack 1 and 3 are almost the same, which can be seen from Figure 9.

***Non proportional loading with a phase angle of 90°***

The greatest amount of experiments has been conducted under non proportional loading with a phase angle of 90°. Seven specimens with (N; $M_T$)-values of two times (27 kN; 408 Nm), two times (18 kN; 272 Nm), (33.5 kN; 392 Nm), (35 kN; 330 Nm) and (46 kN; 430 Nm) have been investigated.

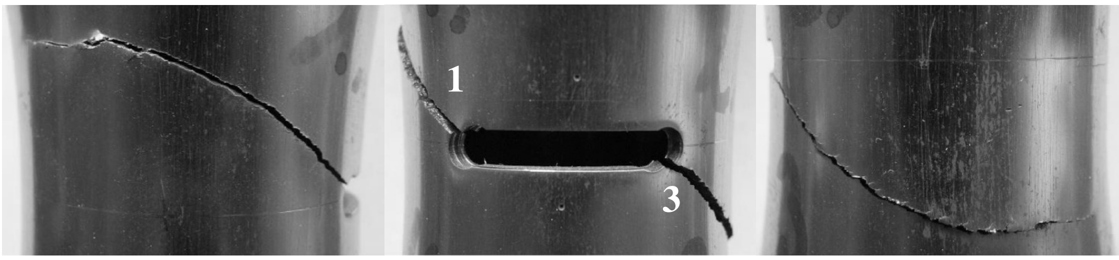


Figure 10. Crack path under out-of-phase loading with a phase angle of 45° with (N; $M_T$)=(22.5 kN; 272 Nm)

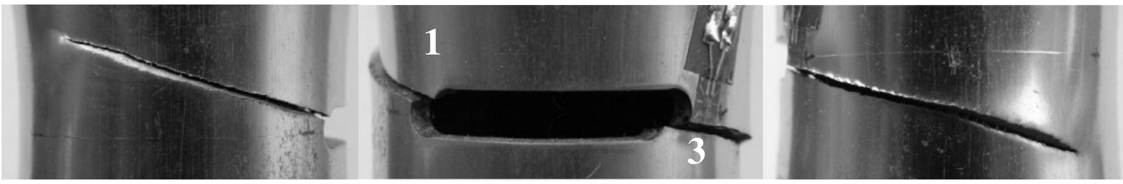

Figure 11. Crack path under out-of-phase loading with a phase angle of 45° with (N; $M_T$)=(35 kN; 330 Nm)

From these experiments three different kinds of crack growth behavior have been observed. For (27 kN; 408 Nm) and (33.5 kN; 392 Nm) four cracks initiate nearly at the same time and grow almost with the same rate. The resulting crack paths are similar to the ones for pure torsional loading shown in Figure 5. For (18 kN; 272 Nm) the cracks 1 and 3 are the dominating cracks. The cracks 2 and 4 initiate, but arrest at a very small crack length. The cracks 1 and 3 grow with almost the same velocity and very curviliniearly. After a certain crack length crack branching occurs, whereby the failure critical branch growths initially in a direction, which is perpendicular to the direction before branching. The crack path is exemplarily shown for one specimen in Figure 12. For (35 kN; 330 Nm) and (46 kN; 430 Nm) all four cracks initiate, but crack 2 and 4 arrest again at a very small crack length. The cracks 1 and 3 grow almost with the same rate in an almost straight-line manner. The crack paths are very similar to the ones obtained for out-of-phase loading with a phase angle of 45° under higher load levels, as depicted in Figure 11.

The crack growth curves for these experiments are summarized in Figure 13.

***Summary***

In Figure 14 the crack-initiation and failure lives are depicted. For every crack of one specimen the crack-initiation life is noted, so that two to four markers are shown for one specimen. For the ordinate in Figure 14 the maximum tangential stress max $\sigma_e$ on the surface of the slot has been chosen. This fictitious stress has been calculated by finite element method selecting linear elastic material behavior. This value is proportional to the applied loads. It is clear that at the crack initiation site severe cyclic plastic conditions prevail. The advantage of this choice of $\sigma_e$, however, is that results of all types of loading can be summarized in one figure. Additionally the regression lines for crack initiation and failure lives obtained from all results are depicted in Figure 14 together with their scatterbands.

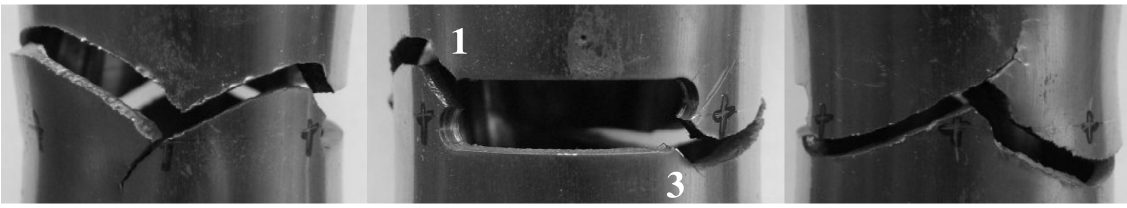

Figure 12. Crack path under out-of-phase loading with a phase angle of 90° with (N; $M_T$)=(18 kN; 272 Nm)

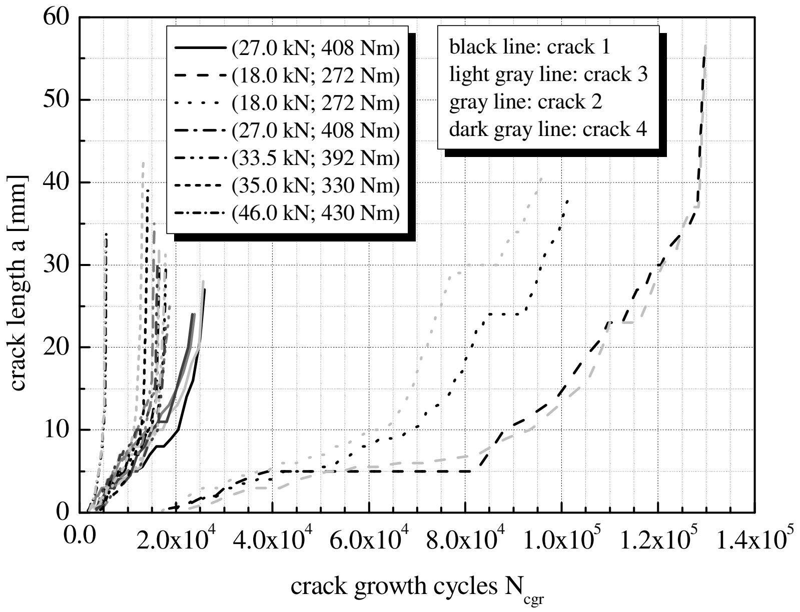


Figure 13. Crack growth curves under out-of-phase loading with a phase angle of 90°

For the crack initiation lives no significant differences between the five loading types can be found. A reason for this is that the stresses on the surface of the slot, which are controlling the crack initiation are almost uniaxial and therefore the loading type is not relevant. However, looking at the failure lives, considerable differences between the loading types occur. Under pure tension-compression and proportional loading the failure lives are by trend longer than the ones predicted with the regression line. For out-of-phase loading with a phase angle of 90° it is the vice versa. The lives predicted with the regression line and for out-of-phase loading with a phase angle of 45° are almost identical. A clear tendency for pure torsional loading is not identifiable.

To analyze the crack growth behavior more detailed, Figure 15 contains the crack growth lives for all specimens. Out of Figure 15 the same tendencies as for the failure lives can be found for the loadings types. The only difference is, that for out-of-phase loading with a phase angle of 45° the results for higher loading levels tend to crack growth lives, which are shorter than the ones predicted by the regression line, but for lower loading levels the lives are longer.

## CONCLUSIONS

In the present paper experimental investigations of slotted, thin-walled, hollow cylinders made of fine-grained steel S460N under proportional and non proportional loading have been presented. Five different loading types have been analyzed: pure tension-compression loading, pure torsional loading, proportional loading resulting from these two load components and out-of-phase loading with phase angles of 45° and 90°.

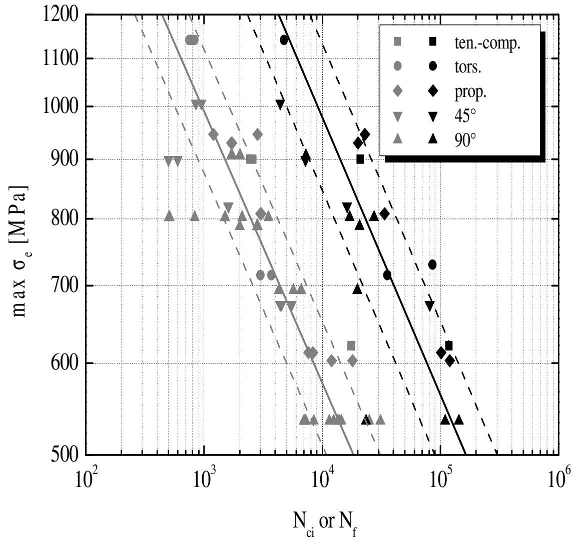


Figure 14. Crack-initiation and failure lives

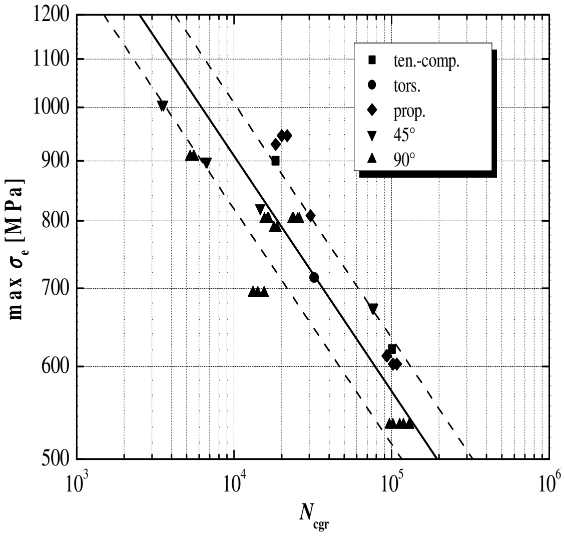


Figure 15. Crack growth lives

Out of these experiments the following conclusions can be drawn:

- Depending on the loading type, two to four cracks initiate as corner cracks on the inner side of the specimen on different positions within the slot.
- Under pure tension-compression loading pure mode I crack growth has been observed.
- Under pure torsional loading four cracks initiated and grow firstly under an angle of about 45° degree to the longitudinal axis. With increasing crack length the cracks grow in a plane which is perpendicular to the longitudinal axis.
- The crack growth behavior of specimens under proportional and out-of-phase loading with a phase angle of 45° are similar.
- Under out-of-phase loading with a phase angle of 90° various different types of crack initiation and growth have been identified.
- The crack initiation life is almost independent of the load type
- Considerable differences for the failure and crack growth life have been found for the five loading types

## REFERENCES


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