Load Separation Method in Plastic J-integral Estimation for Inclined Cracks

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ABSTRACT. *The load separation method was employed to determine the mixed mode plastic* η_{pl} *and* η_{pl}^{COD} *factors for the tension plate with an inclined centre crack in the case of power law hardening materials. The crack orientation angle was varied from pure mode I (*$\theta = 0$*) to mixed (modes I+II) mode. The separation parameters* S_{ij} *have been calculated from the FE computed load – plastic displacement (or COD) curves for each case. The slope of the* $\log(S_{ij}) - \log(b_i / W)$ *lines gives the* η*-factors. It was shown that the mixed mode plastic* η_{pl} *factor is a function of both the crack angle orientation* θ *and the strain hardening exponent. At the same time, the normalized mixed mode plastic* $\eta_{pl}(\theta)/\eta_{pl}(\theta = 0)$ *factor does not depend on specimen configuration.*

The mixed plastic η_{pl}^{COD} *factor exceeds the value of* η_{pl} *over the whole domain of the crack orientation angle.*

INTRODUCTION

Application of the J-integral concept to analyze the mixed elastic-plastic fracture in materials is able to develop the J-integral criteria of a mixed crack initiation and to measure the mixed fracture toughness. Moreover, the prediction of crack re-orientation angles during the crack propagation under mixed mode loading could be based on these criteria.

However, problems concerning the variations of the mixed J-integral, the J-integral of mode I component and the J-integral of mode II component versus mixed I+II ratios and loading level, the measurement of J-integral values for the initiation of a stable mixed crack growth and the establishment of criterion of mixed J-integral have not been solved systematically [1, 2]. It is clear that the calculation of the J-integral under mixed mode loading is more complex than pure mode I or mode II, so there is increased scope for uncertainty in the mixed fracture toughness.

In the present paper the finite element and the load separation procedures were employed to study the plastic component of the mixed J-integral in the case of the tension plate with an inclined centre crack for power law hardening material.

THEORETICAL BACKGROUND

Calculation of J_{pl} values

Calculation of the plastic component J_{pl} of the mixed J-integral can be based on the energy rate interpretation as a function of the plastic component U_{pl} of the total area under the load–displacement curves, the ligament length b/W and the plastic η_{pl} factor

$$J_{pl} = \eta_{pl} \frac{U_{pl}}{B(W-a)}. \tag{1}$$

Here, B is specimen thickness, W is width, a is crack length.

The other J_{pl} estimation approach can be based on load- plastic component of the crack opening displacement (COD) δ_{pl}

$$J_{pl} = \eta_{pl}^{COD} \frac{U_{pl}^{COD}}{B(W-a)}, \tag{2}$$

where U_{pl}^{COD} is the plastic work based on the load versus plastic COD curve. In this case, the plastic η_{pl}^{COD} factor is different from the η_{pl} solution [3].

Load separation method

To determine the η_{pl} factor under mixed loading, the load separation method has been employed in the present study. The method assumes that the load P can be represented as a product of two functions, namely, a crack geometry function and a material deformation function. The load separation concept introduces a separation parameter $S_{ij} = \frac{P(a_i, v_{pl})}{P(a_j, v_{pl})}$ as the ratio of loads $P(a, v_{pl})$ of same specimens but with two different crack lengths a_i and a_j over the whole domain of the plastic displacement v_{pl}. The η_{pl} factor should be estimated by testing or computing the load versus plastic displacement curves at least 3 specimens with different crack aspect ratio. The separation parameter S_{ij} for each specimen curve P-v_{pl} is obtained by dividing the specimen load record by the reference specimen load record for the same plastic displacement. The separation constants S_{ij} versus the uncracked ligament b_i/W are estimated from the approximately constant separation parameter region in the S_{ij} plot versus the plastic displacement. The straight line fits through these points and the slope of the $\log(S_{ij}) - \log(b_i / W)$ curve for the cracked specimen is the η_{pl} factor under mixed mode loading. The details of the load separation method are given in Ref. [3-5].

FINITE ELEMENT CALCULATIONS

The load separation method was used to determine η_{pl} and η_{pl}^{COD} plastic factors for the tension plate with an inclined centre crack for power law hardening material.

Two-dimensional, nonlinear finite element analyses of specimens with different crack lengths with the crack aspect ratio $a/W = 0.4$, 05, 0.6 and crack orientation angles were conducted using the ANSYS finite element software to construct the load–displacement curves under the plane strain condition. The crack orientation angle was varied from pure mode I ($\theta = 0$) to mixed (modes I+II) mode. The total length of the specimen was 250 mm, the width W was 50 mm, and the thickness B was 23 mm. The gage length of the specimen was 150 mm.

The finite element model of the tension plate with an inclined centre crack is represented in Fig. 1. The mesh was built up with eight node isoparameter elements. The blunting model with radius of 0.5 mm was used as the original crack tip in all meshes. Calculations were performed under displacement control.

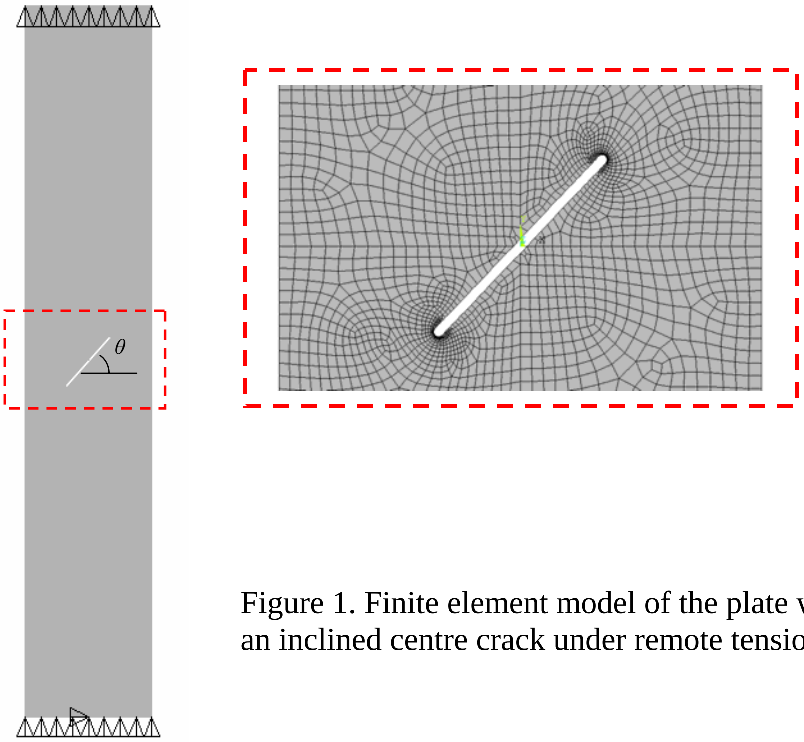

Figure 1. Finite element model of the plate with an inclined centre crack under remote tension

The elastic-plastic finite element analysis was performed by using the incrementaltheory of plasticity with a von Mises flow rule and multilineal isotropic hardening. The following Ramberg-Osgood material constants were introduced in a material model: σ_0=385 MPa, E=72 GPa, ν=0.33 and α=0.86. To analyze the effect of hardening, two strain hardening exponents n=10 and 3 were considered.

As a result of calculations, the load-total displacement and load-crack opening displacement curves were created. To determine the load versus plastic displacement v_{pl} record (Fig. 2), the elastic displacement was subtracted from the total displacement v, i.e. $v - P \cdot C$. The compliance C was calculated from the elastic part of the load versus total displacement curves.

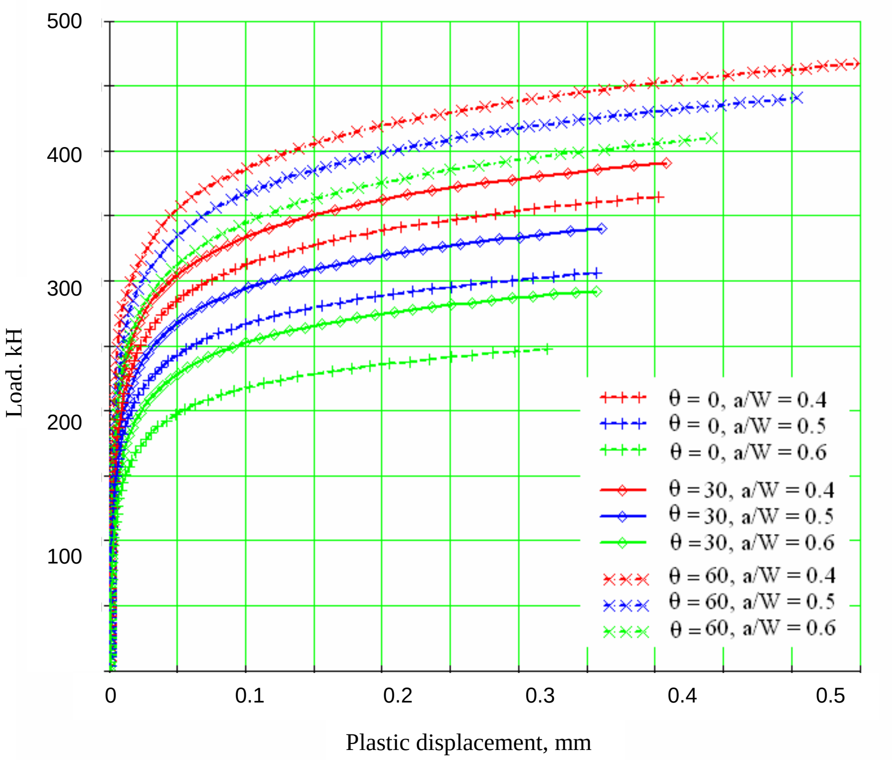

Figure 2. Load versus plastic displacement curves of the tension plate with an inclined centre crack for n=10

Moreover, the load–plastic COD curves (not shown) have been also obtained to analyze the plastic η_{pl}^{CTOD} factors for inclined cracks. In the original blunting crack, the points of intersections 1 and 2 of two 45^{o} lines, drawn back from the crack tip with the deformed profile, are defined as the original measured points of the COD, as shown in Fig. 3a. After mixed loading, the blunting profile is changed to a blunting-sharpening model [6] and the COD is the line segment shown in Fig. 3b.

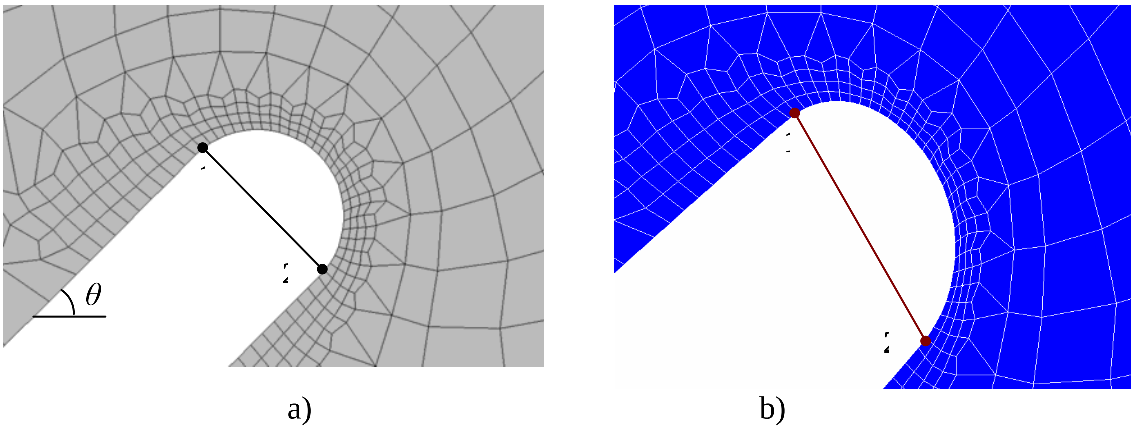

Figure 3. Definition of the crack opening displacement (COD) for an inclined centre crack (n=3 and θ=45^o): (a) before deformation and (b) after deformation

RESULTS AND DISCUSSION

The separation parameter S_{ij} for each cracked specimen record was obtained by dividing the specimen load record by the reference specimen load record for the same plastic displacement. The specimen with the crack aspect ratio a/W =0.5 was used as the reference specimen. Figure 4 shows the separation parameters of the tension plate with an inclined centre crack with respect to plastic displacement.

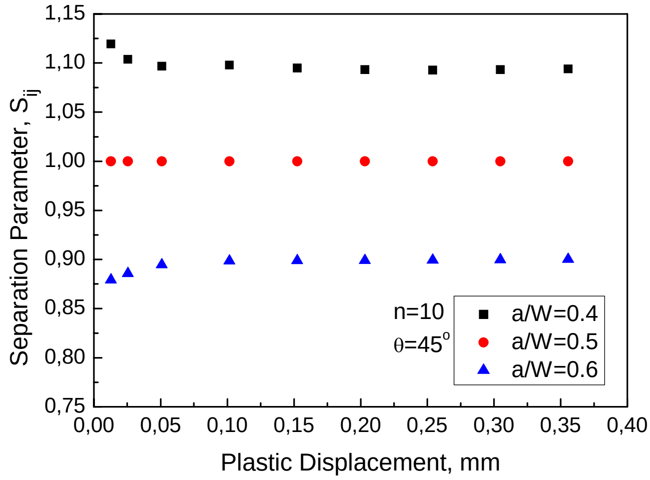

Figure 4. Variation of separation parameters S_{ij} of the tension plate with an inclined centre crack versus plastic displacement for n=10 and the crack orientation angle θ=45^o.

The separation constants S_{ij} versus the uncracked ligament b_i / W were estimated from the constant separation parameter region in Fig. 4. The straight line fits through these points and the slope of the $\log(S_{ij}) - \log(b_i / W)$ curve for the tension specimen is the mixed mode plastic η_{pl} factor for the value of a / W =0.4…0.6 (Fig. 5).

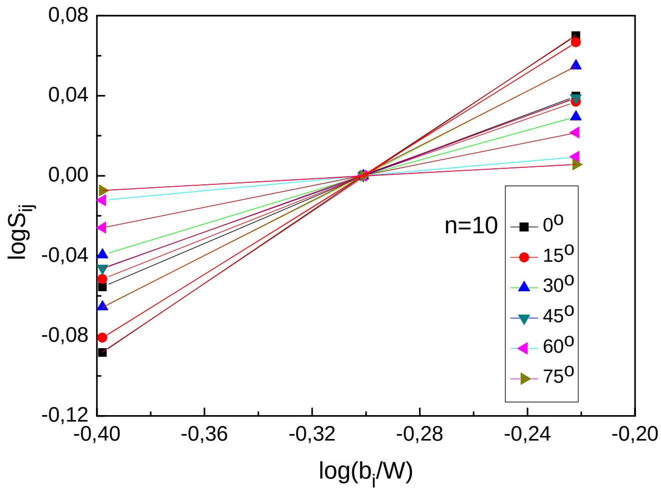

Figure 5. Variations of $\log(S_{ij})$ versus $\log(b_i / W)$ for different angles of an inclined crack under tension.

The calculated plastic η_{pl} and η_{pl}^{COD} factors decrease with the increase of the crack orientation angle θ (Fig. 6). The results obtained by Smith et al. [2] show similar trends of the mixed mode η_{pl} factor for the single edge notch tension specimens (SENT) of A508 Class 3 forged steel subjected to combination of modes I and II. It should be pointed out that the mixed mode plastic η_{pl} factor is a function of both the crack angle orientation and the strain hardening exponent. The calculated mixed mode plastic η_{pl}^{COD} factor is also changed with the crack orientation angle (not shown), but η_{pl}^{COD} exceeds the value of η_{pl} over the whole domain of the angle θ.

At the same time, to characterise the mixed I+II mode loading, it was recommended to employ the mixity ratio which was defined through the applied elastic parameter M_e [7]

$$M_e = \frac{2}{\pi} \arctan\left(\frac{K_I}{K_{II}} \right) \tag{1}$$

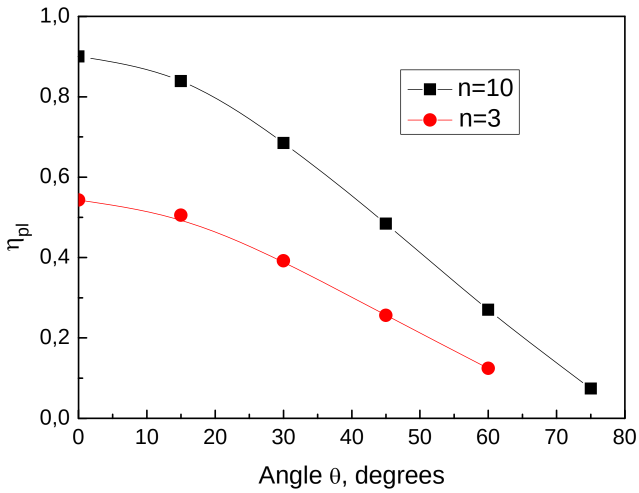

Figure 6. Mixed mode plastic factors for the tension plate with an inclined centre crack for power law hardening material

It is clear that this ratio is varied by changing the crack orientation angle, defined in Fig. 1. The applied elastic mixity ratio is adopted instead of the plastic mixity ratio at the crack tip because of its simplicity [7]. The stress intensity factors for the tension plate with an inclined centre crack are assumed in the following form: $K_I = \sigma \cos^2 \theta \sqrt{\pi a}$ and $K_{II} = \sigma \cos\theta \sin\theta \sqrt{\pi a}$. In this case, the value of M_e=1 corresponds to pure mode I loading (θ=0°), $M_e \neq 1$ corresponds to mixed mode ($\theta > 0$). The variation of the normalized mixed mode plastic factor $\eta_{pl}(\theta)/\eta_{pl}(\theta = 0)$ with M_e is summarized in table 1. It can be seen that effect of specimen configuration on the normalized mixed mode plastic factor is negligible. At the same time, the normalized mixed factor is slightly lower for the strain hardening exponent n=3 at the crack orientation angle 45°.

Table 1. Normalized mixed mode plastic factors $\eta_{pl}(\theta)/\eta_{pl}(\theta = 0)$ for the tension plate (CCT) with an inclined centre crack (n=10 and 3) and CTS specimens (steel StE 550)

θ	0°	15°	30°	45°
M_e	1	0.83	0.67	0.50
CCT (n=10)	1	0.932	0.761	0.538
CCT (n=3)	1	0.930	0.722	0.471
CTS [7]	1	0.921	0.735	0.560

CONCLUSIONS

The calculated mixed mode plastic η_{pl} and η_{pl}^{CTOD} factors in the case of the tension plate with an inclined centre crack for power law hardening material decrease with the increase of the crack orientation angle θ and the decrease of the strain hardening exponent. The mixed plastic η_{pl}^{COD} factor exceeds the value of η_{pl} over the whole domain of the crack orientation angle. Normalized representation of the mixed mode plastic factor $\eta_{pl}(\theta)/\eta_{pl}(\theta=0)$ allows avoiding the dependence of these values on specimen configuration.

The obtained the η_{pl} and η_{pl}^{CTOD} factors for mixed loading should be very useful to validate the J-integral criteria of a mixed crack initiation and crack re-orientation angles as well as to measure the mixed fracture toughness.

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