Non-local Method in Fatigue Life Assessment Using Critical Plane Concept

A. Karolczuk[1] and A. Cichański[2]

[1] Opole University of Technology, Faculty of Mechanical Engineering, ul. Mikolajczyka 5, 45-271 Opole, Poland, a.karolczuk@po.opole.pl
[2] University of Technology and Life Sciences in Bydgoszcz, Faculty of Mechanical Engineering, ul. Prof. S. Kaliskiego 7, 85-796 Bydgoszcz, Poland, arci@utp.edu.pl

ABSTRACT. *The aims of the paper are: (i) presenting a new non-local method, (ii) analysis of influence of weight function parameters in calculation of non-local damage parameter and fatigue lives. The non-local damage parameter is computed by the weighted integration process of normal strain amplitudes over the potential crack plane. Based on the experimental results and fatigue life calculation it is concluded that the 'non-local effect' of strain distribution depends on cyclic properties of strain-stress relation.*

INTRODUCTION

Existence of the inhomogeneous stress field in a material is a very common case in structural elements. In case of variable loading the inhomogeneous stress field additionally changes in time, which is the cause of a complicated fatigue failure mechanism. Experimental tests show that the local damage parameter applied in fatigue life calculation of elements with inhomogeneous stress distribution is not appropriate for every case [1]. The main aim of the present work is to analyse the non-local method which is based on the weighted average process of local damage parameter over a material plane. The analysis is performed using the experimental data taken from [2].

A SHORT REVIEW OF NON-LOCAL METHODS

Non-local methods devoted to fatigue life or fatigue limit calculations assume that local fatigue process does not respond for fatigue failure of element but fatigue processes having place over some geometrical space in material. Two fatigue mechanisms must be distinguished which separate fatigue non-local methods. The first mechanism concerns the situation when particular fatigue crack is initiated and it grows in area of inhomogeneous stress distribition. Physical (technical) crack length must be formed to define the failure of element and depending on some conditions (e.g. material properties) the critical crack length is formed with the influence of inhomogeneously stressed points. The first mechanism assumes interaction between neighbouring points which form the crack. The other mechanism concerns the situation when the crack starts from the weakest link (elementary subdomain) in material, for example from the biggest defect (probabilistic mechanism). Therefore, the population of defects in the analysed element, which depends on the element size, influences the fatigue failure. Under inhomogeneous stress distribution, the probability of the crack initiation in subsequent links changes within the material. Finally, the fatigue crack initiation depends on highly stressed area of the material in probabilistic matter in contrast to the first mechanism, where interaction between points inside the fatigue-damaged area is considered. The distinguished mechanisms were used as a main key to categorise the non-local approaches in fatigue calculations.

Deterministic methods

The so-called deterministic methods consider the interaction between locally damaged points located in the vicinity of the hot spot by normal or weighted integration process of local damage parameters. Depending on the considered geometrical space in the material, the non-local, deterministic methods are divided into: volume method, area method, line method and point method (Fig. 1).

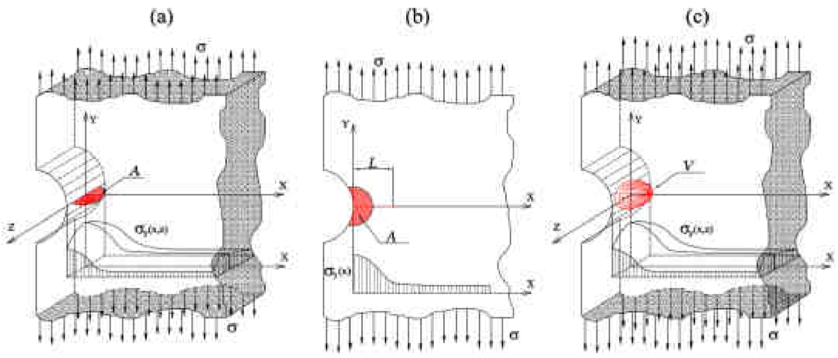

Figure 1. Domain of integration: (a) area *A*, (b) alternative orientation of area *A* and line *L*, (c) volume *V*

Formally, it seems that the point method does not belong to non-local methods because fatigue life is determined by a local parameter instead of the averaged process. However, the point method could be classified as a non-local method because location of the considered point does not coincide with point of the highest damage but with the point where the damage parameter properly estimates fatigue life.

Volumetric method

The method assumes that the fatigue failure of the elemen is due to damage processes having place in some volume of the material. The mean value of the representative local damage parameters averaged over the volume must reach the critical value to cause the failure of the structure.

Yao [3] proposed to calculate non-local damage parameter from weighted average process of stresses computed according to Huber-Mises-Hencky hypothesis. The local value of the weight function depends on location of the considered point and stress gradient of the equivalent stress. Yao assumes that integration volume *V* is a material constant which can be approximated as a sphere with the centre at the notch root (several grains).

Palin-Luc and Lasserre [4] gave physical meaning of integration volume V different from Yao's assumption. According to Palin-Luc and Lasserre, the integration volume V is not constant but it depends on stress/strain distribution. The volume V is defined by points in which the damage parameter is higher than the threshold damage parameter.

Area method

This method assumes that the fatigue failure is due to averaged damage parameter over some plane. The existing area methods consider two orientations of the integration plane A in respect to free surface area (Fig. 1a and 1b).

Seweryn and Mróz [5] proposed a non-local stress condition for crack initiation and propagation in area of stress concentration (Fig.1a). The criterion was proposed for brittle fracture with assumption that crack initiation or propagation occurs when the maximum value of the averaged failure function $\hat{R}_\sigma(\sigma_n, \tau_{ns})$ on a particular plane A reaches its critical value, where σ_n and τ_{ns} are normal and shear stresses on integration plane A. Location and orientation of the integration area are defined by the maximum value of the averaged failure function $\hat{R}_\sigma$.

In paper [6], Taylor and Susmel have analysed the area method under the reversed torsional loading. Orientation of the integration area A does not coincide with the potential crack plane (Fig. 1b). In case of torsional loading, the range of the fatigue limit of a notched element is computed by the averaged process of maximum principal stress $\Delta\sigma_1$. Integration area is limited by radius L_T which value depends on the threshold value of stress intensity factor for mode I.

Line method

This method assumes that the fatigue failure could be estimated by the stresses averaged over a line with the beginning at the notch root (Fig. 1b). Kukn and Hardraht (cit. for [7]) proposed to average one stress component σ_y (where y is a direction of the applied forces) over distance L from the notch root which was defined as a material constant depending on the ultimate strength of the material.

Qylafku et al. [7] based on the Yao model assumed that the damage zone V always contains a small plasticised zone, the effect of which could be described by effective distance L_{eff} from the notch tip. Therefore, the volumetric integration was replaced by the line integration of stress component σ_y using weight function $w = 1 - \chi x, \quad \chi = d\sigma_y / (\sigma_y dx)$, where: x is the direction over which the integration process is performed, χ is a relative stress gradient. The effective distance L_{eff} is measured from the notch root to the point where the relative stress gradient χ achieves the local minimum.

Probabilistic methods

It is well known that the fatigue mechanisms have a statistical nature. The identical (in macroscopic sense) specimens subjected to the same loading history exhibit different fatigue lives. This phenomenon may be explained by the weakest link concept, which was originally proposed for explanation of size effect on the tensile strength [8].

Application of the weakest link concept to fatigue calculations of elements with inhomogeneous stress field boils down to calculations of failure probability $P_f = 1 - \exp[(-1/\Omega_0)\int_\Omega f(\sigma_{eq}(x,y,z))d\Omega]$, where: $\sigma_{eq}(x,y,z)$ is the equivalent stress field computed basing on the selected fatigue criterion; Ω_0 is reference domain (volume or surface); f is the function of so-called 'risk of rupture', [9]. Existing probabilistic methods using the weakest link concept differ in form of function f, equivalent stress field and domain of integration: $\Omega=V$ or $\Omega=A$.

Summary

The point method is often applied because of its simplicity. However, diversity of stress distribution dependent on element geometry and loading for the same material results in discrepancy between point locations [10]. The line method has got similar disadvantages. The volume method assumes that crack initiation is created by connecting process of microcracks contained in the critical volume of material. Fatigue failure of elements is manifested by appearance of crack planes (crack paths). Planes, not volumes, reflect character of fatigue failures that suggests that the area method should be the most effective.

A NON-LOCAL METHOD BASED ON THE CRITICAL PLANE

A basis of the proposed method is the critical plane concept applied to elements with inhomogeneous stress distribution. The critical plane concept is widely used in reduction of multiaxial stress (strain) state into the equivalent one in respect to fatigue life [11]. The reduction assumes that only some stress or/and strain tensor components, that work on the physical plane in material with constant orientation during loading, are responsible for fatigue failure. In case of inhomogeneous stress distribution the multiaxial reduction should be performed in every point on the chosen physical plane (critical plane). The question is how damage parameters on the critical plane influence fatigue processes. More detailed discussion on this problem was presented in papers [12, 13], where two areas on the same plane orientation have been distinguished. In the first area a shearing process is dominant in contrary to the second area where tensile process of crack opening is the most important. If the fatigue criterion assumes that normal stress σ_n accelerates the crack initiation (e.g. Findley criterion: $\sigma_{eq,a}=\tau_{ns,a}+k\sigma_{n,max}$) then both areas overlap, but influence of shear components (τ_{ns}) is modulated by weight function w_{ns} which is much more 'local' than influence of normal components (σ_n) modulated by weight function w_n. A general form for the averaging process is as follows

$$\hat{\kappa}_{ns}(\mathbf{r}_0) = \frac{1}{\hat{w}_{ns}(\mathbf{r}_0)}\int_A \kappa_{ns}(\mathbf{r})w_{ns}(\mathbf{r}-\mathbf{r}_0)dA, \quad \hat{\kappa}_n(\mathbf{r}_0) = \frac{1}{\hat{w}_n(\mathbf{r}_0)}\int_A \kappa_n(\mathbf{r})w_n(\mathbf{r}-\mathbf{r}_0)dA, \tag{1}$$

where κ is damage parameter; ns, n are indices pointing the shear and normal components, respectively; $\mathbf{r}$ is vector of point location where values of $\kappa(\mathbf{r})$ are known; $\mathbf{r}_0$ is vector of basic point location (Fig. 2a); A is integration area coincided with the critical plane orientation; $\hat{w}_{ns}(\mathbf{r}_0)=\int_A w_{ns}(\mathbf{r}-\mathbf{r}_0)dA,\ \ \hat{w}_n(\mathbf{r}_0)=\int_A w_n(\mathbf{r}-\mathbf{r}_0)dA$. Expression (1) has general form and depending on: materials sensitivity to stress gradient effect, loading, geometry of element, the form of equation (1) could be reduced to averaging process of only normal components [12] or could remain in general form [13].

The aim of the weight function w_n (w_{ns}) is to reflect the influence of stress or strain (κ) located in some distance r from base point $\mathbf{r}_0$ on fatigue life. Application of only maximum stress (strain) from stress (strain) field results in the underestimated fatigue life [1]. This phenomenon called the stress gradient effect could be explained that the fatigue failure is not due to bonding failure in one point but bonding failure over some area. It is proposed that the weight function has the following form

$$w_i(r)=e^{-(2r/l_i)^2}, \tag{2}$$

where r is distance between the base point $\mathbf{r}_0$ and the point with $\kappa(\mathbf{r})$ value; l_i is the parameter that reflects the influence of normal ($i=n$) or shear ($i=ns$) components on fatigue life. Figures 2b present exemplary influence of parameter l on distribution of weight functions.

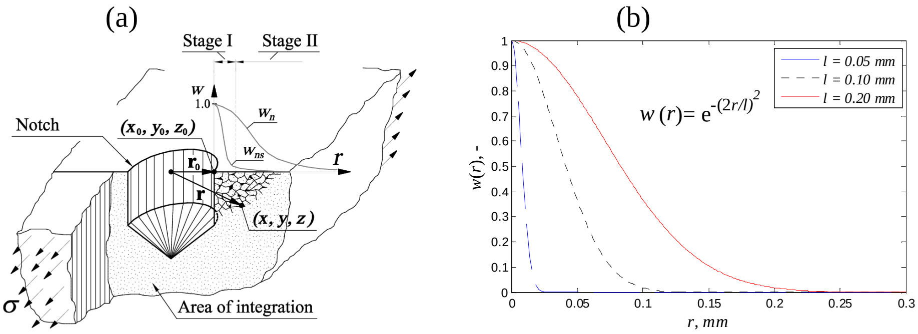

Figure 2. (a) Area of integration and exemplary distribution of weight functions w_n and w_{ns}; (b) examplary distribution of value of weight function w for different value of parameter l

EXPERIMENTAL RESULTS AND MODELLING OF STRESS-STRAIN DISTRIBUTIONS

For analysis of the proposed non-local method and particularly the influence of parameter l the experimental data published in [2] are used. A circumferentially notched round bar (Fig. 3a) made of vanadium-based micro-alloyed forged steel, in both the as-forged (AF) and quenched and tempered (QT) conditions were subjected to tension-compression loading. In (AF) condition, two notch radii R=0.529 mm or R=1.588 mm were tested which generated the following stress concentration factors in tension K_t=2.8 and K_t =1.8, respectively. Under (QT) condition only one specimen geometry with notch radius R=1.588 mm (K_t =1.8) was tested. The properties of the reference curve are presented in Tab. 1. The fatigue life of the notched and smooth (reference) specimens were defined as the number of cycles endured until the specimen failure in two parts.

(a) (b)

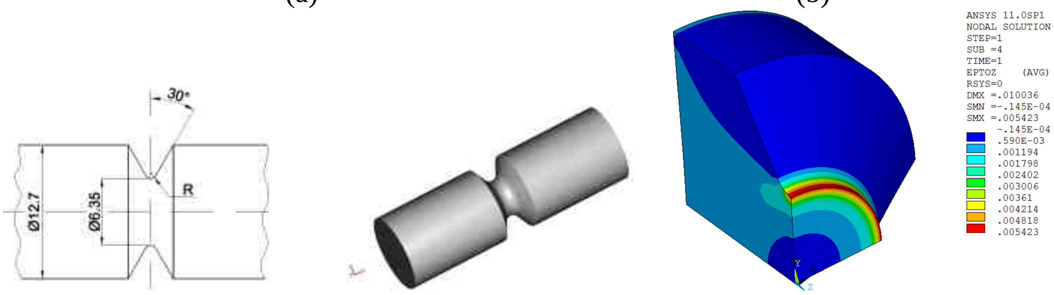

Figure 3. (a) Geometry of notched specimen made of AISI 1141 steel; (b) total strain amplitude distribution $\varepsilon_{zz,a}$ for notch radius R=0.529 mm and nominal stress amplitude S_a=400 MPa

Table 1. Cyclic properties of AISI 1141 steel

		$\varepsilon_a = \sigma_f'/E \cdot (2N_f)^b + \varepsilon_f'(2N_f)^c$				$\varepsilon_a^p = (\sigma_a/K')^{1/n'}$	
State	*E*, GPa	σ_f', MPa	ε_f', -	*b*, -	*c*, -	*K'*, MPa	*n'*, -
AF	200	1296	1.026	-0.088	-0.686	1205	0.122
QT	212	765	1.664	-0.041	-0.704	1133	0.134

The strain and stress distributions in the specimens were calculated using the 3D finite element analysis applied in ANSYS software. In computations a cyclic constitutive model with non-linear hardening was applied. The material hardening was identified from the cyclic hardening curve (from half-life hysteresis loops) expressed by the Ramberg-Osgood $\varepsilon_a^p = (\sigma_a/K')^{1/n'}$ equation (Tab. 1).

RESULTS OF COMPUTATIONS

The current paper focuses on the analysis of influence of weight function on the fatigue life. Because of this reason, for the analysis, the experimental data that concerns only push-pull loading of smooth and notched specimens were selected. The same state of loading applied to smooth and notched specimens allows narrowing the problem down to nonlocal influence of inhomogeneous stress (strain) distribution. Problem of reducing multiaxial stress (strain) state in notch specimens using fatigue criterion is omitted. Fatigue characteristic (ε_a-$2N_f$, Tab. 1) uses total strain amplitude ε_a in the plane normal to the applied forces. Thus, the same parameter is used in the notched specimens: $\varepsilon_{eq,a} = \varepsilon_{zz,a}$, where $\varepsilon_{zz,a}$ is total strain amplitude in direction of applied forces. The equivalent strain amplitude $\varepsilon_{eq,a}$ is computed over the area of integration *A* and then integrated, using weight function $w_n(r) = e^{-(2r/l_n)^2}$, to receive the non-local parameter $\hat{\varepsilon}_{eq,a}$. This non-local parameter is used to calculated number of cycles to fatigue failure using fatigue characteristic ε_a-$2N_f$ ($N_{cal} = N_f$). To estimate the goodness of fit between experimental N_{exp} and calculated N_{cal} fatigue lives the following estimators are used:

$$E_r^{(i)} = \log \frac{N_{cal}^{(i)}}{N_{\exp}^{(i)}}, \quad E_m = \frac{1}{j}\sum_{i=1}^{j} E_r^{(i)}, \quad E_{std} = \sqrt{\frac{1}{j-1}\sum_{i=1}^{j}(E_r^{(i)} - E_m)^2}, \quad E_{eq} = \sqrt{E_m^2 + E_{std}^2}, \tag{3}$$

where the upper index (i) points subsequent specimen; (j) is a total number of specimens. The estimator E_m determines general location of points in 2D space N_{exp} - N_{cal}. The estimator E_{std} determines magnitude of data scattering. The weight function parameter l_n is analysed in a wide range. For each value of l_n integration process is performed and fatigue estimators (3) are calculated. Figure 4a presents distribution of estimators (3) over the parameter l_n. The higher value of l_n the stronger non-locality effect is considered and as a result longer calculated fatigue life is obtained. The best estimation ($\min_{l_n}(E_{eq})$) for AISI steel in (AF) state for both notch radius is received for l_n=0.17 mm (Fig. 4b).

(a) (b)

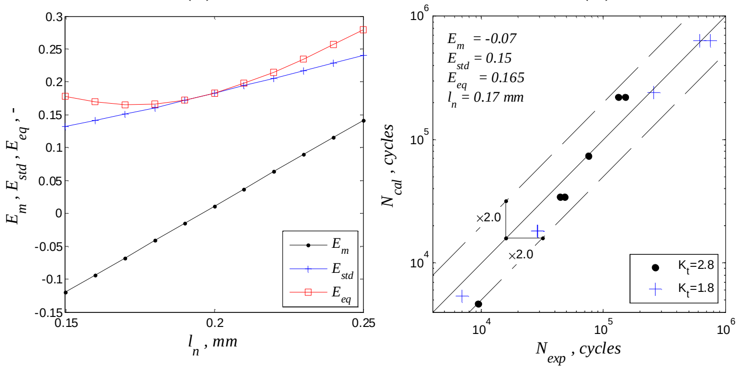

Figure 4. (a) Distribution of fitting estimators, (b) comparison of the experimental N_{exp} and calculated N_{cal} fatigue lives for AISI 1141 (AF) steel for l_n=0.17 mm

Whereas, for AISI steel in (QT) state the best estimation is received for l_n=0.43 mm (Fig. 5).

(a) (b)

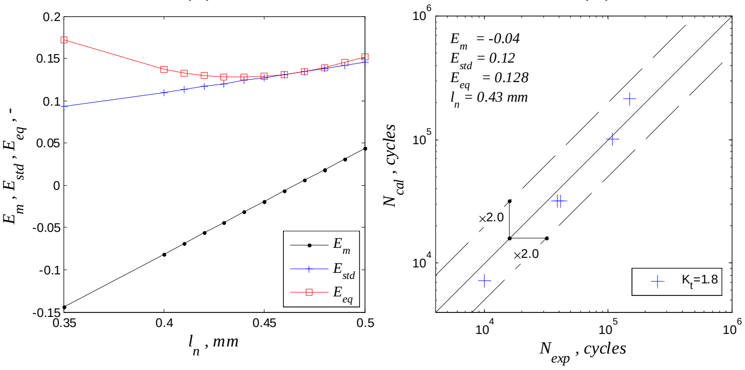

Figure 5. (a) Distribution of fitting estimators, (b) comparison of the experimental N_{exp} and calculated N_{cal} fatigue lives for AISI 1141 (QT) steel for l_n=0.43 mm

The differences in l_n parameter must be seen as a result of different material properties of cyclic plasticity (tab.1). Cyclic hardening curves of AISI steel are different for (AF) and (QT) states (Fig. 6). For the same stress amplitude σ_a AISI steel in (QT) state exhibits larger strains than in (AF) state. It means that AISI steel in (QT) state has larger ability to dissipate energy by plastic strains. The field of plastic strains slows down the crack propagation rate and causes that joining mechanism of micro-cracks is more significant. Thus, in such a case, the local damage parameter such as maximum strain is of less importance than the influence of strain distribution (non-locality). It is seen as a reason why for AISI steel in (QT) state the weight parameter l_n is larger than it is in (AF) state.

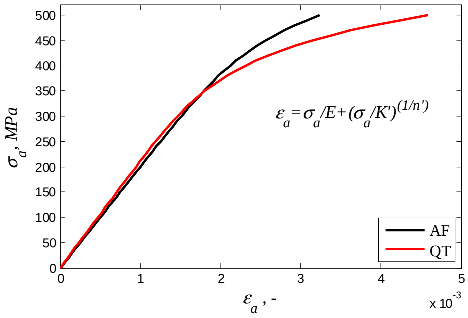

Figure 6. Cyclic strain-stress curves for AISI steel

CONCLUSIONS

The experimental and computed results show that the value of so called ‘the critical distance’ depends on cyclic plastic properties of a material. The larger ability of material to disperse energy by plastic strains the larger strain field must be considered to calculate the appropriate value of the non-local damage parameter.

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