# **Lifetime prediction of an expander impeller using 3-D mixedmode crack propagation algorithm**

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*ABSTRACT. An important task in turbomachinery design is the assessment of material flaws in machine components. Such flaws may occur during manufacturing process. Despite the modern forging techniques, a typical problem is that forged rotors and discs still suffer from crack-like material flaws in the form of segregation, non-metallic inclusions, shrink holes, cracks and cavities. The focus of this work is to perform lifetime predictions for a forged expander impeller containing a cluster of crack like indication in the central part of the impeller disc. For this purpose, a representative crack geometry has been defined for the detected cluster of flaws. A numerical study of growing mixed-mode internal cracks in the impeller is undertaken with the help of a finite element simulation. The model enables us to predict the lifetime of the impeller and the crack paths due to steady state and transient stresses during operation, including start up and turn down. The propagation of the crack is governed by the principle of maximum driving force which is a direct consequence of the variational principle of a cracked body in equilibrium. This criterion considers the effect of all three stress intensity factors in mixed-mode condition, and without any ad hoc assumption, the crack growth rate is calculated using its thermodynamic duality with the local maximum driving force.* 

# **INTRODUCTION**

To evaluate material flaws in machine components, lifetime approaches have been developed which are mainly based on fracture mechanics [1]. These can be for the sake of life extension of a machine component or for the assessment of material flaws in a new component. Based on such predictions, inspection intervals can be defined. Evaluation of lifetime of components containing flaws however requires the knowledge of the size, geometry, location and distribution of such flaws.

This requires reliable methods to determine the mentioned information. A popular method is the ultrasonic inspections of components. This method, however, does not result an exact picture regarding the size, orientation, shape and distribution of flaws. In order to obtain realistic data concerning flaw size, resulting flaw size from ultrasonic inspection is to be corrected. For this purpose, different criteria have been proposed in

literature (see e.g. [2]). Other questions arise as well, including whether detected flaws behave like cracks, how to treat a cluster of flaws and the interaction and shielding behavior of cracks during propagation.

A conservative method in lifetime assessment of low temperature components is applying linear elastic fracture mechanics with the assumption that the detected flaw is a crack like flaw with sharp crack tip. This assumption seems very conservative, as many material flaws have geometries other than cracks. But considering the fact that such defects under cyclic loads would grow to cracks, such conservative assumptions can simplify the assessment effort drastically. Using the concept of linear fracture mechanics, one assumes that crack like material flaws are susceptible to propagation if the variation of the effective stress intensity factor exceeds its material threshold value. With such information, it is possible to perform a lifetime analysis using 3-D crack propagation simulation.

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# **PROBLEM DEFINITION AND SIMULATION STRATEGY**

There are different types of operational stresses in the impeller of compressors and expanders. The first type is the steady state stresses, which in a rotating component occurs due to centrifugal forces, torques, axial forces from anchor bolts, pressure on the blades and gravity forces. During start-up and turn-down, turbomachinery components also experience transient thermal stresses which are in the order of magnitude of steadystate stresses [5].



a sample flat bottom hole indication

Figure 1. Indication of crack like discontinuity in an expander.

An indication of crack like discontinuity is considered in a forged high pressure impeller of an expander with given inlet and outlet temperatures. The considered flaw is located as shown in figure 1. To make a conservative estimation for the lifetime of flawed impeller, size of flaw is multiplied with a correction factors k according to [2]. This is indeed the effective crack size, as the ultrasonic results provide an approximate picture of flaw size and geometry only.

The matrix material of impeller is assumed to be a high strength steel and it is considered to be homogeneous, isotropic and linear elastic for the considered load level. Temperature dependent mechanical properties have been considered in order to describe the material behavior at different temperature levels. Hexahedral elements with quadratic displacement behavior are used to mesh the crack front region, where higher accuracy is needed (figure 2). The elements are refined towards the crack tip. These elements do not capture the stress singularity at the crack tip, as the quarter point elements do, but the results are still reliable if the first raw of elements at the crack tip are excluded in the calculation of crack tip parameters (stress intensity factors and Tstresses). The rest of impeller is meshed with the help of quadratic tetrahedral elements. A remeshing algorithm has been programmed to simulate the crack propagation inside the impeller.



Figure 2. Element design along the crack front.

It should be noted that relatively fine elements are to be generated to have accurate results. This is especially important since for an exact lifetime assessment, the stress intensity factors are to be calculated accurately. For this reason, crack front is embedded in a torus of radius  $R = a/20$  (figure 2). Inside this torus structured fine elements are generated. The size of the elements in the radial direction is  $R/n = a/(20 \text{ n})$ , n being the number of elements in the radial direction. The rest of the model is meshed with coarser tetrahedral elements to save computational effort. For the considered thermal boundary conditions figure 3-right shows the distribution of temperature in the impeller for the steady state situation. Figure 3-left shows a typical stress distribution in the impeller, which is a combination of the steady state and transient stresses as explained before. Red color indicates the high stress area and blue color corresponds to low stress values.



Figure 3. Temperature and stress distribution.

## *Numerical determination of the crack tip parameters*

The stress expansion near the tip of a 2-D straight crack is well known from the work of Williams [6]. For the most general 3-D case of an arbitrary shaped crack with an arbitrary curved front, the asymptotic stress expansion formula is given by [7]:

$$
\sigma_{ij} = K_{\alpha} f_{ij}^{\alpha}(\theta) r^{-1/2} + T_{\alpha} g_{ij}^{\alpha}(\theta) + O(r^{1/2}),
$$
\n(1)

where r,  $\theta$  correspond to the local polar coordinates measured from the periphery of the crack front in the plane perpendicular to it, and Greek indices range over I, II and III denote the three crack deformation modes, i.e. opening, sliding and tearing, respectively. The constants  $K_I$ ,  $K_{II}$  and  $K_{III}$  are the stress intensity factors corresponding to each mode, and  $T_I$ ,  $T_{II}$  and  $T_{III}$  are the constant non-singular terms in the stress expansion, the so called T-stresses [8].  $f_{ij}$ 's and  $g_{ij}$ 's are universal functions of  $\theta$ , and depend only on the Poisson's ratio [7]. To calculate stress intensity factors along the crack front, a direct method based on stress result is used. The method is based on rearranging equation (1) for a fixed  $\theta$ .

## **MIXED-MODE CRACK PROPAGATION AND LIFETIME ASSESSMENT**

Having evaluated the effective stress intensity factor along the crack front for the initial flaw size, the next steps for the lifetime assessment are listed below:

- 1. Verify whether the initial crack is safe or critical during the first machine start,
- 2. If safe, verify whether crack growth will occur during further starts,
- 3. In case of crack growth, determine the critical crack size for the calculated stresses,
- 4. Calculate the number of starts after which the crack reaches its critical size.

This method is based on the analyses methodologies given in [1] and [2]. The calculated effective stress intensity factors show that the initial crack is susceptible to growth. However, the initial crack is not critical as the resulting stress intensity factors are well below the considered material toughness. Such information can be summarized in a so called Failure Assessment Diagram (FAD), as presented in figure 4.



Figure 4. Failure Assessment Diagram.

The next step in the analysis is to determine the maximum crack size at the location of the detected flaw above which the operation of the flawed component is no longer safe. This is the so called critical crack size, which is a function of the applied stress level, wall thickness and mechanical material properties. Such information is gained by constructing a residual strength diagram, or the so called operating stress map . Figure 5 shows a sample operational stress map [1].



Figure 5. A sample operating stress map.

The last step in the lifetime analysis is to calculate the number of machine starts for which the detected flaw reaches its critical size. This is done by performing the simulation of crack propagation. For this purpose, simulation of quasistatic propagation of the mixed-mode crack under small strain assumptions has been performed, using a stepwise technique and finite element method. The model enables us to obtain the stress intensity factors along the crack front as the crack grows.

Propagation of the crack is governed by the criterion of maximum driving force [4], which is a direct consequence of the variational principle of a cracked body in equilibrium. According to this criterion, a crack grows in the direction of the local maximal driving force. With this, local propagation rate can be obtained considering a generalized Paris' law:

$$
\frac{da}{dN} = C \left( \Delta K_{eff} - \Delta K_{th} \right)^{\eta},
$$
\n
$$
K_{eff} = \sqrt{\left( (K_{I}^{*})^{2} + (K_{II}^{*})^{2} + \frac{1}{1 - \nu} (K_{III}^{*})^{2} \right)},
$$
\n(2)

where *a* and N are characteristic crack length and number of load cycles,  $K_{ef}$ represents the effective stress intensity factors which reflects the driving force acting along the crack front (J-integral concept) and  $\Delta K_{th}$  is the threshold value. C and  $\eta$  are the constants of Paris law. The procedure of simulating quasistatic propagation of cracks, using the stepwise method, can be summarized as follows. Stress intensity factors are evaluated numerically along the crack front using a direct method based on the extrapolation of stress field. Propagation angle and propagation rate for each point are then determined using the mentioned fracture criterion along with equation (2). Points on crack front are then propagated in planes perpendicular to crack front in a local manner. After propagating all points on crack front, model is remeshed according to new crack geometry, and solved. This procedure is repeated in further steps to propagate the crack. Figure 6 shows the growth of the detected crack in the considered impeller.



Figure 6. Simulation of propagation of crack in an impeller.

Calculating the crack propagation using the explained mixed-mode crack growth algorithm, the number of starts for which the crack size may become critical can be determined.

## **SUMMARY AND CONCLUSIONS**

A 3-D mixed-mode crack propagation algorithm was presented for the lifetime prediction of an expander impeller. Finite element simulation of growing mixed-mode internal crack is undertaken with the help of a stepwise method and using a remeshing algorithm. The model enables us to predict the lifetime of components and the path of growing crack. Propagation of crack is governed by the principle of maximum driving force. This criterion considers the effect of all three stress intensity factors in mixedmode condition, and crack growth rate is calculated using its thermodynamic dual.

### **AKNOWLEDGEMENTS**

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