Fifty years of crack path research

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ABSTRACT

The complete solution of a crack propagation problem includes determination of the crack path. At the present state of the art the factors controlling the path taken by a propagating crack are not completely understood. In the last five decades there have been substantial advances in the understanding and prediction of both macroscopic and microscopic aspects of crack paths, largely through developments in fracture mechanics, and in the application of modern computers and microscopes. As examples, 6 crack path topics, all of which have a long history and are still of interest, are described briefly. These are: fatigue crack paths in aircraft structures, fractography, fracture toughness testing, mixed mode fatigue thresholds, crack path stability, and the transition from flat to slant crack propagation sometimes observed in thin sheets. The first five of these are illustrated by examples taken from the author's experience. The papers presented at the International Conference on Crack Paths (CP 2009) show that understanding of crack paths is increasing.

INTRODUCTION

The complete solution of a crack propagation problem includes determination of the crack path. At the present state of the art the factors controlling the path taken by a propagating crack are not completely understood. Steel became widely available with the introduction of the Bessemer process in 1856, and by 1879 there were reports of mysterious brittle crack propagation in steel structures. A contemporary observation on the failure of a gasholder in 1898 states 'Fracture path curved, did not follow rivet holes.' [1]. Macroscopic aspects of fatigue crack paths have been of industrial interest since the earliest fatigue investigations, around 170 years ago [2]. The book by Cazaud published in 1948 includes an analysis of fatigue crack paths in both laboratory specimens and industrial components. An English translation includes additional material [3].

In the last five decades there have been substantial advances in the understanding and prediction of both macroscopic and microscopic aspects of crack paths, largely through developments in fracture mechanics, and in the application of modern computers and microscopes. As examples, 6 crack path topics, all of which are still of interest, are described briefly. These are: fatigue crack paths in aircraft structures, fractography, fracture toughness testing, mixed mode fatigue thresholds, crack path stability, and the transition from flat to slant crack propagation sometimes observed in thin sheets.. All have practical engineering applications. The first five of these are illustrated by examples taken from the author's experience. The transition from flat to slant crack propagation in thin sheets is well known, but until recently there has been no convincing explanation of the transition. It is an example of a crack path problem where the application of a modern computer has increased understanding.

FATIGUE CRACK PATHS IN AIRCRAFT STRUCTURES

The catastrophic accidents to Comet aircraft [4] meant that, by the mid 1950s, fatigue testing of aircraft structures was standard practice. At that time metal fatigue was well understood in a general sense [5]. Much effort was devoted to the experimental determination of fatigue crack paths, and rates of fatigue crack growth, in aircraft structures. Simulated service loading was used. Residual static strengths in the presence of fatigue cracks were measured. The data obtained were used to determine safe lives and inspection intervals, usually with a safety factor of two on fatigue crack propagation rates. A 3m long section of fuselage, being prepared for fatigue testing in 1957, is shown in Figure 1. The author is on the left.

Figure 1. Section of fuselage, 3 m long, being prepared for fatigue testing.

In the Second World War military aircraft had only a short service life, and they were designed on a static load basis. In peace time service lives became extended, and in the 1950s there were a number of catastrophic failures due to fatigue. Figure 2 shows crack path and propagation data obtained from a fatigue test on the centre section of a fighter aircraft [2]. Simulated service loading was used, and the times shown are equivalent flying hours. When two times are shown the first is when the crack is observed to reach a rivet, and the second when it was observed to leave the other side.

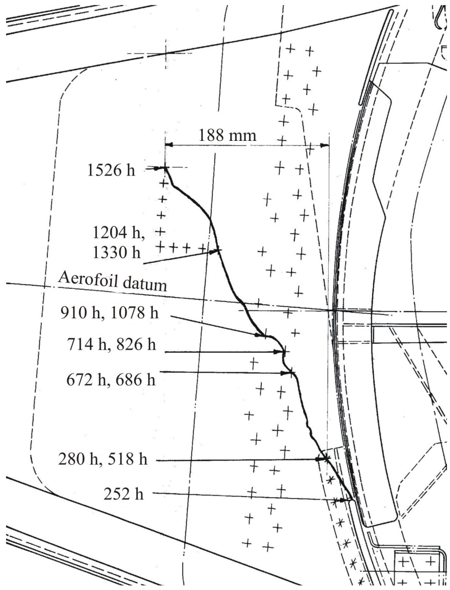

Figure 2. Fatigue crack path in a fighter aircraft centre section. Times are in flying hours.

George Irwin was the 'Father of Fracture Mechanics' [6]. His influence on the use of fracture mechanics in general, and stress intensity factors in particular, became widespread following publication of the Proceedings of the Crack Propagation Symposium which had been held at Cranfield in September 1961 [7]. Attendance at the Symposium was by invitation only. In the 1960s the use of stress intensity factors to correlate fatigue crack propagation data was regarded as not very helpful because of the lack of stress intensity factor solutions for cracks in components and structures.

Despite theoretical advances, fatigue crack paths in structures are still often obtained experimentally in a wide range of industries. Experimental validation of theoretical crack path and propagation data is sometimes a requirement of regulatory authorities [2].

FRACTOGRAPHY

The appearance of fatigue fracture surfaces in metals, at low magnification, had been of interest since the early days of service failure analysis and of fatigue testing [8]. At low magnifications crack path and crack propagation data can be derived from the programme markings on specimens subjected to programme load fatigue tests [2]. The microscopic examination of fatigue fracture surfaces started in the 1950s. The appearance of striations on fatigue fracture surfaces meant that crack propagation data could be derived by measurement of striation spacing. By 1962 the use of quantitative fractography in the reconstruction of crack path information was well developed [9].

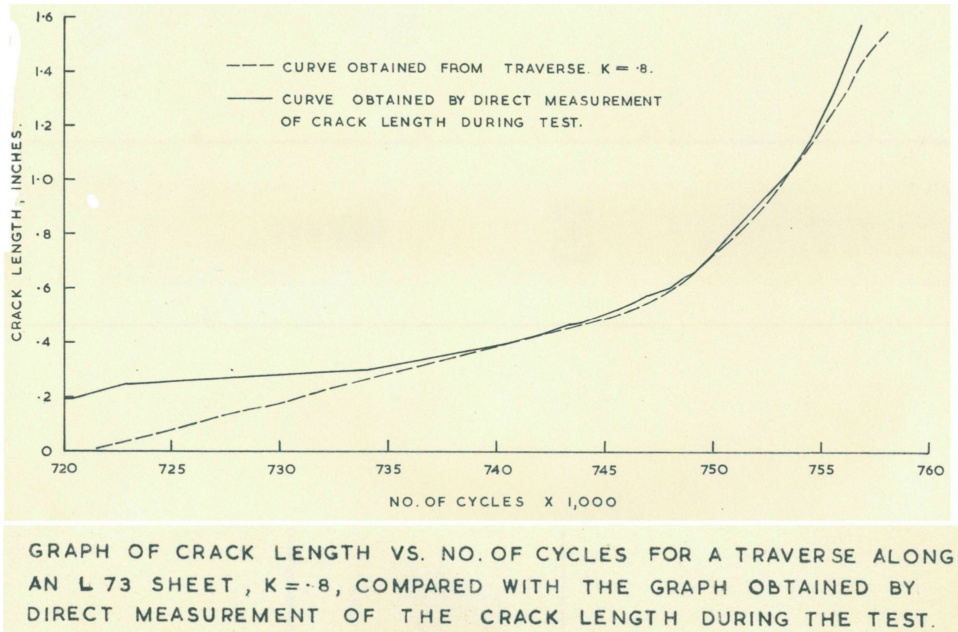

Figure 3. Validation of crack propagation data derived by measurement of striation spacing.

The use of striation counting in the post mortem determination of fatigue crack propagation data had been validated by comparison with measurement of crack lengths made during fatigue tests. Results obtained in 1961 for a fatigue crack in BS L73 aluminium sheet, tested under constant amplitude loading, are reproduced in Figure 3. Crack lengths are in inches (1 inch = 25.4 mm). Striation spacings were measured, using an optical microscope, during a traverse along the fracture surface. They were converted into crack propagation rates, and integrated numerically to obtain crack length vs number of cycles data. An empirically derived correction factor, $K = 0.8$, was applied to crack lengths to allow for local variations in crack propagation direction. Agreement with measured crack lengths is excellent at medium crack lengths. Crack growth rates are overestimated at short crack lengths because the finer striations were not resolved. They are underestimated at long crack lengths because no allowance was made for the brittle fracture jumps that sometimes occur in high strength aluminium alloys.

FRACTURE TOUGHNESS TESTING

The plane strain fracture toughness of a metallic material, K_{Ic}, is an appropriately defined critical value of the opening mode stress intensity factor, K_I, at which fracture takes place under a rising static load. Under certain conditions, the value of K_{Ic} is a geometry independent material property which, however, varies with temperature and, to some extent, with loading rate [10]. In 1968, in the conclusions to the results of numerous fracture toughness tests on various metallic materials it was noted that '...there is still no method of determining K_{Ic} which has received general acceptance...' [11]. This situation changed due to the influence of American work [12], and the successful completion of collaborative experimental programmes [13], in which the author participated. A British Standard Draft for Development was published in 1971 [14], and a full British Standard issued in 1977 [15].

Experimental work had shown, that for consistent results to be obtained, specimens must be precracked in fatigue. If sharp machined notches are used, then the fracture toughness is a function of the notch tip radius [10]. Achieving a satisfactory fatigue precrack is a difficult aspect of K_{Ic} testing. The author was the Chairman of the British Standards Institution Sub-committee on Toughness testing from 1974-1984, and fatigue precrack paths were a major issue. Experience had shown that the notch from which the precrack is grown, and the fatigue loading used must be carefully controlled.

Fatigue precrack fronts in specimens of constant thickness tend to be curved due to crack front constraints. Figure 4 shows the fracture surface of a 19 mm thick DTD 5050 aluminium alloy fracture toughness test specimen [11]. In the 1960s, when the photograph was taken, the reason for the occurrence of curved fatigue precrack fronts was unknown. It is now known to be due to constraints on permissible crack paths [16]. In particular, there is preferred angle at which a crack front intersects a free surface. This is because of a change in the nature of the crack tip singularity in the vicinity of a corner point where a crack front intersects a free surface [17].

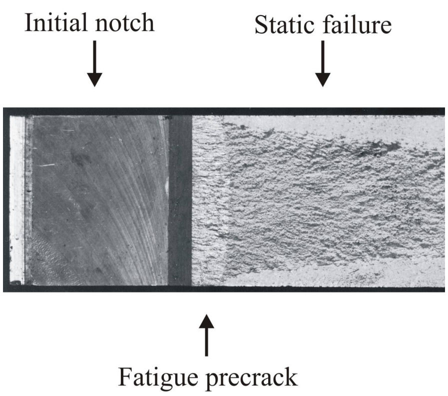

Figure 4. Fracture surface of a 19 mm thick aluminium alloy fracture toughness test specimen.

In order to achieve consistent K_{Ic} values the method of calculating an average crack length is specified in detail in standards, and a limit placed on precrack front curvature. For example, British Standard DD 3 [14] states 'After fracture, the crack length shall be measured to the nearest 0.5% W at the following positions: at 25, 50 and 75 %B. The average of the three measurements shall be the crack length used to calculate K_Q, but the crack is invalid if the difference between any two of these measurements exceeds 2.5 %W. The crack is also invalid if any two possible measurements differ by more than 5

%W or if any part of the crack front is closer to the machined notch than 2.5 %W or less than 1.25 mm.'. Here, W is specimen width, B is specimen thickness, and K_Q is a provisional value of K_{Ic}. These requirements were refined in later versions of the standard.

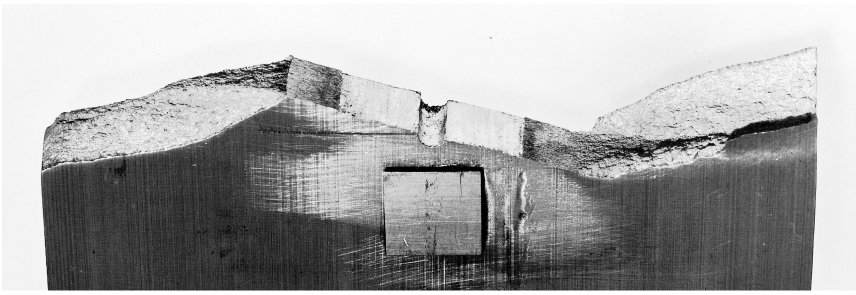

Figure 5. Fracture surface of a 19 mm thick DTD 5050 aluminium alloy mixed Modes I and II fracture toughness test specimen.

Fracture toughness tests to determine K_{Ic} are carried out with the precrack oriented so that it is in Mode I, and subsequent crack propagation is also, in general, in Mode I. In service, cracks may be loaded in mixed mode, that is with at least one mode, other than Mode I present. In mixed mode situations fracture takes place when a function of the corresponding stress intensity factors reaches some critical value. Crack propagation may not be in the plane of the initial crack, and the crack path has to be determined as part of the solution. Mixed mode fracture toughness testing started in the late 1960s [11]. As an example, Figure 5 shows the fracture surface of a DTD 5050 aluminium alloy mixed Modes I and II fracture toughness test specimen [18]. The fatigue precracks were introduced in Mode I. The crack paths change abruptly as crack propagation starts from the precracks. The precrack fronts are slightly curved, which complicates analysis of the fracture toughness test results by introducing some unwanted Mode III.

MIXED MODE FATIGUE THRESHOLDS

In an uncracked specimen, made from a metallic material, fatigue cracks initiate on slip planes subjected to high shear stresses. Initial crack propagation is on these planes, and is Stage I fatigue crack propagation. A Stage I fatigue crack becomes a Stage II fatigue crack when it reaches some length, changes direction and propagates normal to the maximum principal tensile stress, that is in Mode I. Stage II fatigue crack propagation data are usually analysed in terms of the range of Mode I stress intensity factor, ΔK_I. Stage II fatigue crack propagation does not take place from an initial Mode I crack unless a threshold value of ΔK_I, ΔK_{th}, is exceeded. Several formal definitions of ΔK_{th} are possible [2], and thresholds can be determined using different experimental techniques. Usually, all give essentially the same result [19].

If Mode I (Stage II) fatigue crack propagation is to take place from an initial mixed mode crack, then a Mode I branch crack (or cracks) must be initiated at the tip of the initial crack. Occasionally, Mode I branch cracks form behind the initial crack tip. Sometimes, Mode I branch crack formation is preceded by a limited amount of mixed mode Stage I crack propagation (up to about ½ mm) in the initial crack plane.

Investigation of fatigue crack propagation thresholds from an initial mixed mode crack started in the 1970s [20, 21]. Some early mixed mode fatigue crack propagation threshold data were obtained from a fracture mechanics analysis of fatigue tests on spot welded joints [20]; this Reference is a seminal paper that is still being cited. There are at least three events which could, in principle, be used to define a mixed mode fatigue threshold in terms of critical values of stress intensity factor ranges [2, 22]. These are: crack propagation in the plane of the initial crack, Mode I branch crack formation at or near the tip of the initial crack, and Mode I branch crack propagation.

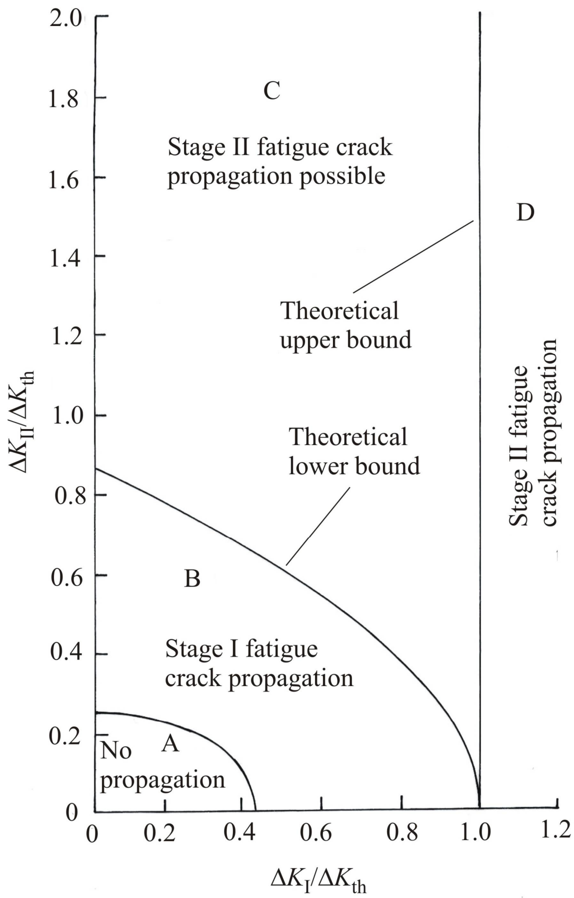

Figure 6. Failure mechanism map for mixed Modes I and II.

The precise definition of a mixed mode threshold can have a significant effect on numerical values, and there is some semantic confusion in the literature on what should be regarded as a ‘true’ mixed mode threshold. In particular, in metallic materials fatigue crack propagation threshold behaviour is sometimes controlled by Mode I branch crack formation, and sometimes by branch crack propagation. Confusion is easily resolved through the use of failure mechanism maps such as Figure 6, which is for mixed Modes I and II in phase fatigue loadings. The map was constructed by using 11 experimental mixed Modes I and II, and pure Mode II, threshold data sets taken from the literature [22]. In the figure, ranges of Modes I and II stress intensity factors, ΔK_{I} and ΔK_{II}, are normalised by the Mode I threshold, ΔK_{th}. Some of the data were obtained using specimens precracked in fatigue and then stress relieved, some using sharply notched specimens, and some by re-analysis of fatigue tests on spot welded joints. The occurrence of Stage I crack propagation was determined fractographically. Detailed criteria used to determine the acceptability of data are given in Reference [23].

The relatively large plastic zone in the presence of Mode II provides the plasticity needed for the limited amount of Stage I crack propagation which sometimes takes place before branch crack formation. This plasticity does not affect the validity of the use of stress intensity factors provided that the initial crack is ‘long’. In this context [23] long means longer than about ¼ mm. All the data used to construct Figure 6 satisfy this criterion.

In region C Stage II fatigue crack propagation is possible by Mode I branch crack formation and propagation. The experimental data for region C showed considerable scatter. It has been demonstrated that this is due to differences in Mode I branch crack formation. The conditions for Mode I branch crack formation are unclear, but it appears to be facilitated by precrack front curvature, as in Figure 4, which introduces unwanted Mode III, and also by metallurgical discontinuities. In chaos theory terms, Mode I branch crack formation is a chaotic event which is strongly dependent on initial conditions [24]. Once Stage II fatigue crack propagation starts it usually continues to complete failure. The theoretical lower bound for region C shown in Figure 6 was calculated by setting the range of Mode I stress intensity factor for a Mode I branch crack, $\Delta k_{\mathrm{I}}^{*}$, equal to ΔK_{th}. The stress intensity factor for a Mode branch crack, k_i^{*} is given, within 5 per cent, by [16]

$$k_{\mathrm{I}}^{*} = \frac{0.83K_{\mathrm{I}} + \sqrt{0.4489K_{\mathrm{I}}^{2} + 3K_{\mathrm{II}}^{2}}}{1.5} \tag{1}$$

The theoretical upper bound to region C was based on the assumption that when ΔK_{I} for the initial crack exceeds ΔK_{th} a Mode I branch crack must form and propagate (region D), irrespective of the value of ΔK_{II}. In region B, Stage I fatigue crack propagation has been observed, but fatigue cracks usually became non propagating. In region A, fatigue crack propagation does not take place. Any of the three boundaries between regions could be used to define a fatigue crack propagation threshold.

CRACK PATH STABILITY

Two dimensional linearly elastic analyses are normally used in the consideration of crack path directional stability. Related experimental work is usually carried out on sheets or plates of constant thickness, which are regarded as quasi two dimensional. A Stage II fatigue crack propagating in Mode I may be regarded as directionally stable if, after a small random deviation, it returns to its expected, ideal crack path, as shown in Figure 7. A directionally unstable crack does not return to the ideal path following a small random deviations; its path is a random walk, which cannot easily be predetermined. These ideas are not easily given rigorous mathematical form [16]. For example, arbitrary limits have to be placed on what is regarded as returning to the ideal crack path. Crack path stability is an important consideration in the design of fracture mechanics based fatigue crack propagation and fracture toughness test specimens.

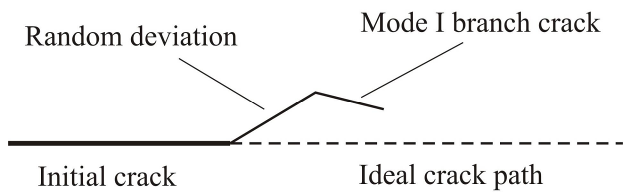

Figure 7. Directionally stable crack propagation.

Deciding whether or not a particular crack path is stable is a difficulty in the analysis of experimental crack path stability results. A practical definition of crack path stability needs to be associated with a finite amount of crack propagation. The British Standard for fatigue crack propagation rate testing [25], states that a crack path is acceptable only if it lies within a validity corridor defined by planes 0.05W on either side of the plane of symmetry containing the crack starter notch root. Here, W is the specimen width, or half width for a specimen containing an internal crack. A compact tension specimen, used for a fatigue crack propagation test, which did not meet this requirement is shown in Figure 8. The light fracture area on the left is static failure where the specimen was broken open for examination. The British Standard criterion may be adapted as a crack path stability criterion by defining a stable crack as one which remains within the validity corridor. This criterion is easy to apply, but has the disadvantage that it does not take into account changes in stability as a crack propagates.

In a two dimensional analysis of a cracked body, the elastic stress field may be expanded as a series [10]. The first term is the stress intensity factor, which dominates the crack tip stress field, and is a singularity. Other terms are non-singular. For a Mode I crack the coefficient of the first term is the Mode I stress intensity factor, K_I, and the second term is a stress parallel to the crack, usually called the T-stress. The third and higher terms can usually be neglected. It has been argued [26] that the directional stability of a Mode I crack in an isotropic material under essentially elastic conditions is governed by the T-stress. If the T-stress is compressive and there is a small random crack deviation, perhaps due to microstructural irregularity, then the direction of Mode I crack growth is towards the initial crack line (Figure 7). Repeated random deviations mean that the crack follows a zigzag path about the initial crack line, which has been called an ideal crack path. In nonlinear dynamics terms the ideal crack path is an attractor [16]. When the *T*-stress is tensile a crack is directionally unstable, and following small random deviations, it does not return to its initial line. The stability of a crack may change as it grows, and a stable Mode I crack may follow a curved path. Cracks tend to be attracted by boundaries, and are increasingly stable as a boundary is approached.

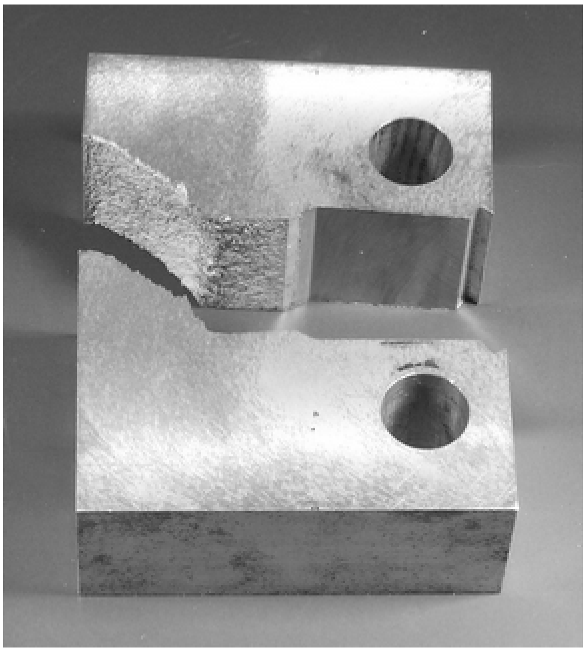

Figure 8. Unstable fatigue crack path in a 20 mm thick high strength aluminium alloy compact tension specimen.

It is sometimes found that cracks are directionally stable even when the *T*-stress is tensile. Hence, a non dimensional parameter is needed as a measure of crack path stability in a given material. This must only include parameters which describe the situation in the vicinity of a crack tip. The biaxiality ratio, *B*, is sometimes used, but this includes the crack length (half crack length for an internal crack). The biaxiality ratio is given by

$$B = \frac{T\sqrt{\pi a}}{K_{\mathrm{I}}} \tag{2}$$

An alternative approach [16], is to consider components of the direct stress parallel to the crack, and near the crack tip. The stress component due to the *T*-stress is simply

T. The stress component, σ_x, due to the Mode I stress intensity factor, on the crack line and ahead of the crack, is given by

$$\sigma_{\mathrm{x}} = \frac{K_{\mathrm{I}}}{\sqrt{2\pi r}} \tag{3}$$

where r is the distance from the crack tip. The T-stress ratio, T_R, may now be defined as the ratio of the T-stress to σ_x at some characteristic value of r, r_{ch}. Provided that r_{ch} is small T_R may be regarded as a crack tip parameter. Since the T-stress criterion is based on the idea of random crack path perturbations due to microstructural irregularities r_{ch} should be of the same order of size as microstructural features. Taking r_{ch} = 0.0159... mm leads, using Equations (2) and (3) and MN-m units, to the convenient expression

$$T_{\mathrm{R}} = \frac{0.01B}{\sqrt{\pi a}} \tag{4}$$

There is a size effect; for geometrically similar configurations T_R decreases in absolute value as the crack size increases.

For a particular material there should be a critical value of T_R, T_{Rc}, below which a crack path is directionally stable. Re-analysis of some biaxial fatigue tests on Waspaloy sheets indicate that the critical value of T_R, T_{Rc}, at which a fatigue crack path becomes unstable is about 0.022 [16]. Similarly, static load tests on PMMA indicate that T_{Rc} is about 0.013.

The compact tension specimen, in which the T-stress is tensile, is often specified in fracture mechanics based Mode I testing standards, such as References [15 and 25]. Specimen size limitations in standards mean that it is unusual to carry out tests on compact tension specimens with the specimen width, W < 50 mm. For W = 50 mm, T_R does not exceed about 0.022. Crack paths in compact tension specimens are usually stable; that shown in Figure 8 is an exception. It may therefore be deduced that for many materials T_{Rc} is at least 0.021. This is consistent with the value of 0.022 for Waspaloy.

T_{Rc} is therefore a convenient parameter for the characterisation of crack path stability in a particular material.

FRACTURE MODE TRANSITION IN THIN SHEETS

There are two fundamentally distinct classes of crack propagation under both fatigue loading and rising load static loading [27]. These are principal stress dominated crack propagation, and shear dominated crack propagation. Principal stress dominated crack propagation is associated with low stresses in the vicinity of a crack tip, and shear dominated crack propagation is associated with high stresses. Flat (Mode I) crack propagation in sheets is principal stress dominated, and the slant crack propagation sometimes observed in thin sheets is shear dominated. The transition from flat to slant crack propagation in thin sheets, as crack propagation proceeds, under both rising load static loading and fatigue loading, is well known. The main features of the transition are shown in Figure 9. The transition starts with the development of shear lips. These increase in width until they meet in the centre of the sheet, completing the transition.

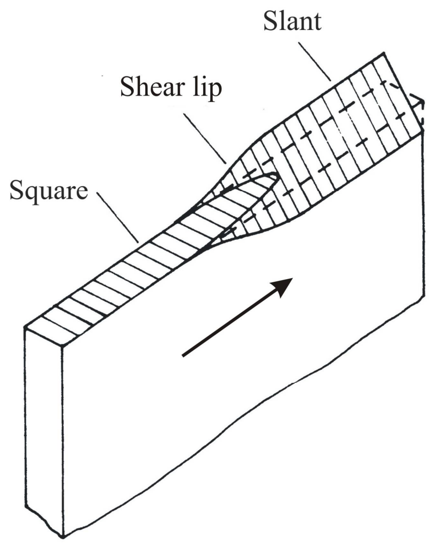

Figure 9 Transition from square to slant crack propagation in thin sheets. The arrow shows the direction of crack propagation.

Until recently, only qualitative explanations of the transition were available [2, 16]. However, a large amount of experimental crack propagation data has been accumulated, for example Reference [28], and practical engineering problems involving the transition can usually be solved. Experimentally observed features of the transition, under a rising static load, have recently been accurately reproduced numerically, for a compact tension specimen, by a series of finite element calculations using a newly developed theory of plasticity [29]. Factors taken into account included the Lode angle, which is a function of the third invariant of the stress tensor, the strain hardening properties of the material and its pressure sensitivity, and the specimen thickness. More than 100 000 elements were used, so the calculations would not have been possible without a modern computer.

CONCLUDING REMARKS

The very wide range of crack path topics covered in CP 2009 made it impossible to write a detailed review of reasonable length. The papers presented demonstrate the progress that has been made in recent years. A discernable trend is the increasing number of practical engineering applications.

The 6 crack path topics chosen for this closing paper all have a long history, and have practical engineering applications.

Results of research into crack paths are most easily transferred to engineering practice by their incorporation into standards and codes of practice of varying degrees of formality, for example the guide to methods for assessing flaws in metallic structures issued by the British Standards Institution [30].

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