Crack Growth Estimation of Steel Pipes with a pre-crack

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ABSTRACT. *The angled crack problem has been given special attention in the recent years by fracture mechanics investigators due to its close proximity to realistic conditions in engineering structures. In this paper, an investigation of fatigue crack propagation in steel pipes containing an inclined surface crack is presented. The inclined angle of the crack with respect to the axis of loading varied between 0^o and 90^o. A criterion based on the shape of the crack tip plastic zone is proposed in the present study. The direction of crack initiation coincides with the direction of the minimum radius of the plastic zone defined by the von Mises yield criterion. The threshold condition for non-growth of the initial crack was established and assessed based on the experimental data.*

INTRODUCTION

Accurate simulation of mixed mode crack propagation paths in engineering materials is what designers and engineers are always looking for. An important element of any simulation package is to define a criterion for crack initiation angles. Many criteria have been proposed to predict crack initiation angles under mixed mode loading. However, most of them assume a constant radius for the plastic zone at the crack tip. In practical, the stresses increase rapidly while approaching the crack tip, and soon surpass a limiting value, which is either the elastic limit, or the yield stress of the material, beyond which the relations expressing the stresses start to become progressively invalid. This means that the radius of the curve along which the suitable quantity for each criterion should be evaluated must be, at least, the elasto-plastic boundary, which is generally not a circle.

In this investigation, the behaviour of fatigue crack propagation of steel pipes, each consisting of an inclined semi-elliptical crack, subjected to axial loading was investigated both experimentally and theoretically. The inclined angle of the crack with respect to the axis of loading varied between 0^o and 90^o. A detailed analysis of the plastic zone is presented and the strong dependence of mixed mode crack initiation angles on the crack tip plastic zone shape is demonstrated. A three-dimensional radius criterion based on the shape of the crack tip plastic zone is proposed. It is assumed that the direction of crack initiation coincides with the direction of the minimum radius of the plastic zone defined by the von Mises yield criterion. The results obtained are compared with those obtained using the commonly employed mixed mode fracture criteria and the experimental data.

EXPERIMENTAL DETAILS

The specimens employed were steel pipes. The outer diameter of the pipe is 40 mm, the wall thickness is 10 mm and the pipe full length is 200 mm. Semi-elliptical surface notches were produced in the specimens using an electrical discharge machining (EDM) technique, introducing a surface flaw of aspect ratio $b/2a$ = 3/20, where b = 3 mm and $2a$ = 20 mm, as shown in Figs 1 and 2. Note that in the present investigation, the loading direction was defined by two angles, i.e., β and ω. The loading angle β was varied between 0^o and 90^o and the angle ω was set to 90^o for all the cases analyzed. For simplifying the test set up, the axial loading was fixed at $\beta = 90^o$ and the starter notches of different inclined angles were made instead of varying the loading angle β, as shown in two side elevations of Fig. 1.

The fatigue crack was initiated and grown from the starter notch by subjecting the notched specimen to a constant amplitude, sinusoidal, tension-to-tension loading, which was exerted by a 100 kN servo-hydraulic test machine operating at a stress ratio of $R = \sigma_{min} / \sigma_{max} = 0.1$, and a frequency of 2.0 Hz. The Model U10 ACFM Crack Microgauge [1] was used to monitor crack growth during the test. The Model U10 ACFM Crack Microgauge together with the tailored-made software can show the crack profile and the corresponding number of load cycles during the fatigue test.

EVALUATION OF CRACK TIP PLASTIC ZONE

Consider elasto-plastic fracture of a steel pipe with an inclined semi-elliptical crack. The local stresses, i.e., σ_n, σ_t, σ_z, τ_{nt}, τ_{tz} and τ_{nz}, on an element near the border of an elliptical crack can be expressed in terms of the spherical coordinates (r, θ, ϕ), as shown in Fig. 2, and they have been given by Sih [2]. For incompressible materials, the von Mises yield criterion can be written in the following form:

$$(\sigma_n - \sigma_t)^2 + (\sigma_t - \sigma_z)^2 + (\sigma_z - \sigma_n)^2 + 6(\tau_{nt}^2 + \tau_{tz}^2 + \tau_{zn}^2) = 2\sigma_{yld}^2 \quad (1)$$

where σ_{yld} is the yield stress in pure tension. Substituting the singular solution of the stresses given by Sih [2] into the above von Mises yield criterion and then solving for $r = r_p(\alpha, \theta, \phi)$ gives

$$r_P(\alpha,\theta,\phi) = \frac{1}{2\sigma_{yld}^2} f(k_1, k_2, k_3, \alpha, \theta, \phi) \quad (2)$$

where $r_p(\alpha, \theta, \phi)$ is the radius of crack-tip plastic zone, and f is a function of the angles α, θ and ϕ, which characterizes the shape of the plastic zone $r_p(\alpha, \theta, \phi)$. For the present problem, the stress intensity factors of mode I, mode II and mode III, i.e., k_1, k_2 and k_3 can be found in Sih [2].

For the special case of $\alpha = 90^o$ and $\omega = 90^o$, i.e., the case of mode I and II loadings, Eq. 2 can be expressed by

$$r_P(\theta) = \frac{1}{\sigma_{yld}^2}\left\{\begin{aligned}&\left(\frac{3}{8}k_1^2 - \frac{9}{8}k_2^2\right)\sin^2\theta + [\frac{1}{2}(1-2\nu)^2 k_1^2 + \frac{3}{2}k_2^2]\cos^2\frac{\theta}{2} \\ &+ 2(1-\nu+\nu^2)k_2^2\sin^2\frac{\theta}{2} + [\frac{3}{4}\sin 2\theta - \frac{1}{2}(1-2\nu)^2\sin\theta]k_1 k_2\end{aligned}\right\} \\ = \frac{1}{\sigma_{yld}^2} f(k_1, k_2, \theta) \tag{3}$$

where $r_p(\theta)$ is the radius of crack-tip plastic zone for the combined mode I and II, and f is a function of the angle θ, which characterizes the shape of the plastic zone $r_p(\theta)$.

It is important to note that for evaluation of the above elasto-plastic boundary, only the elastic dominant stress fields in conjunction with the von Mises yield criterion were utilized, based on the assumption that the perturbation due to the presence of crack tip plasticity was of no major significance. This is justifiable in the case of essentially small scale yielding, SSY. It can be shown that the radius of crack-tip plastic zone r_p is directly proportional to the applied stress, σ^2. The size of the plastic region increases with increasing loading on the body. In the present case, it is assumed that a plastic region at the edge of a crack is subject to plane strain conditions. Towards the plate surface, however, plane stress dominates, giving rise to a different shape of plastic region.

DIRECTION OF FATIGUE CRACK PROPAGATION

In the present investigation, for simplicity, we only present the results for the special case of $\alpha = 90^o$ and $\omega = 90^o$, i.e., the case of mode I and II loadings.

Minimum radius criterion

Based on the variable radius of the plastic zone $r_p(\theta)$ defined in Eq. 3, a minimum plastic zone radius criterion for the two-dimensional stress state can be established. It is assumed that the direction of fatigue crack initiation coincides with the direction of the minimum radius of the plastic zone defined by the von Mises yield criterion. This means that the angle θ_o of the fatigue crack growth direction can be determined by minimizing the $r_p(\theta)$ value, i.e.,

$$\left(\frac{\partial r_p}{\partial \theta}\right)_{\theta=\theta_o} = \frac{1}{\sigma_{yld}^2}\left(\frac{\partial f}{\partial \theta}\right)_{\theta=\theta_o} = 0, \qquad \left(\frac{\partial^2 r_p}{\partial \theta^2}\right)_{\theta=\theta_o} = \frac{1}{\sigma_{yld}^2}\left(\frac{\partial^2 f}{\partial \theta^2}\right)_{\theta=\theta_o} > 0 \tag{4}$$

By establishing $\partial r_p / \partial\theta = 0$ at θ_o, the direction of fatigue crack growth θ_o for the special case of $\alpha = 90^o$ can be determined.

Minimum strain energy density criterion

Sih [3] presented the minimum strain energy density criterion. It states that the direction of crack initiation coincides with the direction of minimum strain energy density, along a constant radius around the crack tip. For the case of mode I and II loadings, the strain energy density factor S was given by Sih [3] as follows:

$$S = a_{11}k_1^2 + 2a_{12}k_1k_2 + a_{22}k_2^2 \tag{5}$$

where the coefficients a_{ij}, which vary with the spherical angles (θ, ϕ) measured from the crack tip, were given by

$$a_{11} = \frac{\kappa+1}{16\mu\lambda\kappa^2\cos\theta}\left[2(1-2\nu)+\frac{\kappa-1}{\kappa}\right] \tag{6a}$$

$$a_{12} = \frac{(\kappa^2-1)^{1/2}}{8\mu\lambda\kappa^2\cos\theta}\left[\frac{1}{\kappa}-(1-2\nu)\right] \tag{6b}$$

$$a_{22} = \frac{1}{16\mu\lambda\kappa^2\cos\theta}\left[4(1-\nu)(\kappa-1)+\frac{1}{\kappa}(\kappa+1)(3-\kappa)\right] \tag{6c}$$

where μ and ν are the shear modulus of elasticity and Poisson's ratio, and the parameters λ and κ are given by Sih [3]. It is assumed that the crack will start to extend in a direction for which the strain energy density factor possesses a relative minimum value, i.e.,

$$\left(\frac{\partial S}{\partial\theta}\right)_{\theta=\theta_o} = 0, \qquad \left(\frac{\partial^2 S}{\partial\theta^2}\right)_{\theta=\theta_0} > 0 \tag{7}$$

Based on $\partial S/\partial\theta = 0$ at θ_o, the directions of fatigue crack growth θ_o can be determined. Note that the above total strain energy is evaluated along a constant circle with radius r around the crack tip based on the theory of elasticity.

Tensile stress criterion

In the present case, the tensile stress under the combination of mode I and II loadings was given by Sih [2] as the following equation,

$$\sigma_z = -\frac{k_1}{\sqrt{2r}}\sqrt{\frac{\kappa+1}{2\lambda\cos\theta}}\left(\frac{2-\kappa-3\kappa^2}{2\kappa^3}\right)+\frac{k_2}{\sqrt{2r}}\sqrt{\frac{\kappa-1}{2\lambda\cos\theta}}\left(\frac{2+\kappa-\kappa^2}{2\kappa^3}\right) \tag{8}$$

It is assumed that the crack growth direction coincides with the direction of maximum tensile stress along a constant radius around the crack tip. Therefore, the angle θ_0 of the fatigue crack growth direction can be determined by maximizing the σ_z value, i.e.,

$$\left(\frac{\partial\sigma_z}{\partial\theta}\right)_{\theta=\theta_o} = 0, \qquad \left(\frac{\partial^2\sigma_z}{\partial\theta^2}\right)_{\theta=\theta_o} < 0 \tag{9}$$

Note that the value of the above tensile stress, σ_z, is evaluated along a constant circle with radius r around the crack tip based on the theory of elasticity.

NON-GROWTH CONDITIONS

To define the non-growth conditions, it is necessary to first determine the critical case. It can be concluded that for a semi-elliptical crack, not all the points on the crack front fracture at the same time. Failure first occurs at the location of $\alpha = 90^o$ and $\beta = 90^o$ for all the loading conditions considered. Therefore, the critical case occurs when both angles β and α are equal to 90^o.

Tensile Stress Criterion

It is assumed that the onset of fatigue crack growth occurs when the maximum value of σ_z reaches a critical value, $(\sigma_z)_{cr}$, at $\theta = \theta_o$ and $\phi = \phi_o$. The value of the critical stress $(\sigma_z)_{cr}$ is now evaluated from the threshold value of Δk_I for the case of $\beta = 90^o$ and $\alpha = 90^o$. For the critical case, which occurs when $\alpha = 90^o$, $\omega = 90^o$ and $\Delta k_3 = 0$, the condition for non-growth of the fatigue crack is, thus, given by:

$$
\begin{aligned}
&-\frac{\Delta k_1}{\sqrt{2r}}\sqrt{\frac{\kappa+1}{2\lambda\cos\theta_o}}\left(\frac{2-\kappa-3\kappa^2}{2\kappa^3}\right)+\frac{\Delta k_2}{\sqrt{2r}}\sqrt{\frac{\kappa-1}{2\lambda\cos\theta_o}}\left(\frac{2+\kappa-\kappa^2}{2\kappa^3}\right) \\
&=-\frac{\Delta K_1}{\sqrt{2r}}\sqrt{\frac{\kappa+1}{2\lambda\cos\theta_o}}\left(\frac{2-\kappa-3\kappa^2}{2\kappa^3}\right)
\end{aligned}
\qquad (10)
$$

where ΔK_I is the threshold value for non-growth of the initial crack, and it can be determined based on the data presented in Table 1.

Strain Energy Density Criterion

It is assumed that the onset of fatigue crack growth occurs when the minimum value of $S_{\min}$ reaches a critical value S_{cr} at $\theta = \theta_o$ and $\phi = \phi_o$. The value of the critical strain energy density factor S_{cr} can now be evaluated from the threshold value of Δk_I for the case of $\beta = 90^o$ and $\alpha = 90^o$. For the critical case in which $\alpha = 90^o$, $\omega = 90^o$ and $\Delta k_3 = 0$, the condition at which no fatigue crack growth would take place is given by:

$$
a_{11}\Delta k_1^2 + 2a_{12}\Delta k_1 \Delta k_2 + a_{22}\Delta k_2^2 = a_{11}\Delta K_1^2 \qquad (11)
$$

where ΔK_I is the threshold value for non-growth of the initial crack, and it can be determined based on the data presented in Table 1.

Experimental Approach

For the critical case that occurs at $\alpha = 90^o$ and $\omega = 90^o$ where $\Delta k_3 = 0$, we assume that the condition for non-growth of the fatigue crack is in a quadratic form similar to that of Eq. 11, that is,

$$A_{11}\Delta k_1^2 + 2A_{12}\Delta k_1 \Delta k_2 + A_{22}\Delta k_2^2 = 1 \quad (12)$$

The coefficients, A_{11}, A_{12} and A_{22}, are determined from a best fit to the experimentally obtained threshold values of Δk_1 and Δk_2 for angles $\beta = 90^o$, 75^o, 60^o, 45^o, 30^o and 15^o, which are presented in Table 1. The final form of the equation is as follows:

$$0.00189\Delta k_1^2 + 0.000024\Delta k_1 \Delta k_2 + 0.0021\Delta k_2^2 = 1 \quad (13)$$

The stress intensifications corresponding to the stress range $\Delta\sigma = \sigma_{max} - \sigma_{min}$ are calculated by substituting $\Delta\sigma$ for σ in the equations of the stress intensity factors k_j (j=1, 2, 3). Thus, the ranges of the stress intensity factors computed are denoted by Δk_1, Δk_2 and Δk_3. For the present work, $b/a = 3/10$, $E = 230$ GPa, $\nu = 0.3$ and $\sigma_{yld} = 355$ MPa.

Table 1. Test results for the threshold condition of fatigue crack growth at α=90^o (ω=90^o).

Specimen No.	Loading angle β (degree)	SIF (MN/m$^{3/2}$) Δk_1	Δk_2	Δk_3		SIF (MN/m$^{3/2}$) Δk_1	Δk_2	Δk_3	
1	90	22.7	0.00	0.00	Non-growth	23.9	0.00	0.00	Growth
2	75	21.2	5.89	0.00	Non-growth	22.3	6.20	0.00	Growth
3	60	20.5	12.2	0.00	Non-growth	21.6	12.8	0.00	Growth
4	45	17.8	17.8	0.00	Non-growth	18.7	18.7	0.00	Growth
5	30	15.4	20.9	0.00	Non-growth	16.2	22.1	0.00	Growth
6	15	13.8	23.3	0.00	Non-growth	14.5	24.6	0.00	Growth

RESULTS AND DISCUSSION

Because the critical case, in general, occurs at the location of crack deepest point (center line of the crack), the experimental results of fatigue crack propagation at the location of $\alpha = 90^o$ along the crack periphery are presented in column 4 of Table 2. Note that the angle of fatigue crack growth with respect to the starter notch, θ_o, was measured from the crack surface by using an accurate angle measurement meter. In order to provide a better prediction, a minimum radius criterion for mixed mode cracks is proposed to determine the direction of the fatigue crack initiation. The angles θ_o presented in columns 5, 6 and 7 of Table **2** were derived based on the minimum radius criterion, strain energy density and tensile stress criteria, respectively. For the purpose of comparison, the growth angles predicted using the commonly employed criteria (elastic) and the proposed minimum radius criterion (elasto-plastic) are denoted by θ_o^e and θ_o^p, respectively.

It can be observed from Table 2 that the results obtained using the minimum radius criterion are in better agreement with the test data as compared with the corresponding fracture criteria. The results obtained from the strain energy density theory are closer to the test data in comparison to those obtained from the maximum stress theory. In Fig. 3, plots of k_2 versus k_1 are presented for three different non-growth conditions. Results obtained from the tensile stress and strain energy density criteria are indicated by the dashed and dot-dashed lines, respectively. The experimentally measured values corresponding to angles β = 90°, 75°, 60°, 45°, 30° and 15° fall between the two said curves in most of the cases studied. This means that the tensile stress criterion provides a conservative prediction for the present case, while the strain energy density criterion gives an over-optimistic prediction. The solid curve in Fig. 3 corresponds to Eq. 13. As can be seen from Fig. 3 the threshold condition for non-growth of the initial crack due to the combined fracture mode can be better predicted by Eq. 13 than Eq. 10 or 11.

Table 2. Test and theoretical results of the initial fatigue crack growth at α=90° (ω=90°).

Specimen No.	Loading angle β (degree)	SIF (MN/m$^{3/2}$) Δk_1	Δk_2	Test θ_o (degree)	r_p – criterion θ_o^p	S – criterion θ_o^e	σ – criterion θ_o^e
1	90.0	23.9	0.00	0.00	0.00	0.00	0.00
2	75.0	22.3	6.20	14.8	18.9	21.2	23.8
3	60.0	21.6	12.8	30.2	35.6	38.6	45.6
4	45.0	18.7	18.7	48.1	54.7	58.8	64.2
5	30.0	16.2	22.1	63.1	69.2	73.3	76.5
6	15.0	14.5	24.6	73.8	77.1	80.2	84.8

CONCLUSIONS

To develop a new minimum radius criterion, the variable radius of the plastic zone based on the von Mises yield criterion was defined. The value of radius r depends explicitly, on the material properties of the pipe, and also on the angular direction around the crack tip. The initiation angles predicted using the minimum radius criterion are in better agreement with the experimental data as compared with those predicted using the corresponding fracture criteria. The threshold condition for non-growth of the initial crack based on the test data was also derived. The tensile stress criterion provides a conservative prediction while the strain energy density criterion gives an over optimistic prediction.

ACKNOWLEDGEMENTS

The authors acknowledge the support provided by the Natural Sciences and Engineering Research Council of Canada (NSERC) and the Auto 21 Centre of Excellence.

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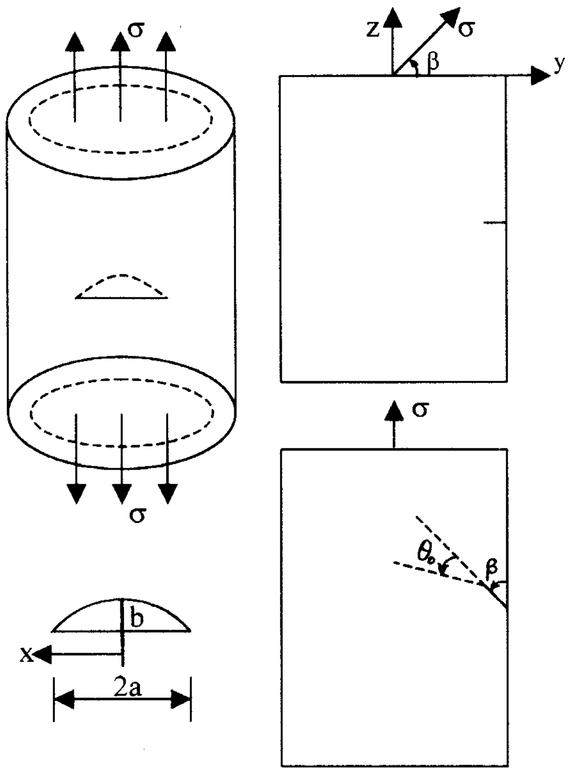

Fig. 1. Details of the specimen geometry.

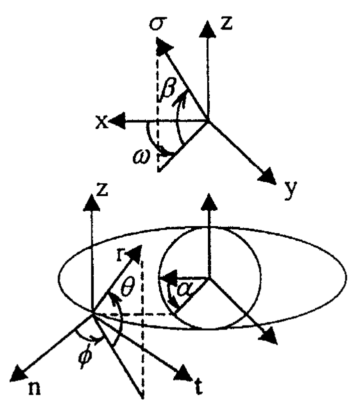

Fig. 2. Crack front coordinates and loading system.

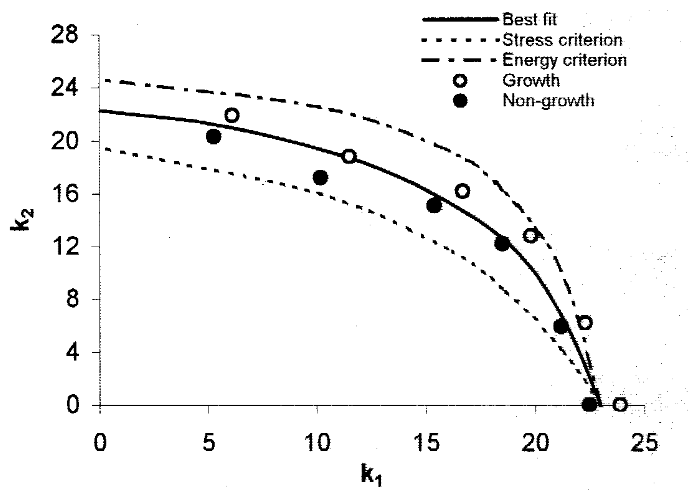

Fig. 3. Plot of k_2 versus k_1 using three different non-growth conditions.