Effect of shear forces on cracked beam deflections

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ABSTRACT *The influence of shear forces on the deflections of cracked beams loaded by bending and shear forces are analysed by means of several 3D non linear models of cracked beams. It is generally believed that the additional shear flexibility due to the presence of a transverse crack in rotating shafts is negligeable: this study aims to verify this assumption. The results show that the assumption is true for all deflections except for the torsion deflection, excited by a coupling effect, which is strongly influenced by the shear load.*

INTRODUCTION

The scope of this analysis is to evaluate the influence of shear loads in the cracked section on the deflections of a cracked beam. In other words the investigation aims to define the contribution in deflections of cracked beams of the additional shear flexibility introduced by the crack. This study originated by discussions among rotordynamics specialists on the influence of transverse cracks on the flexibility of rotating shafts.

This analysis is performed by considering a test beam with a diameter of 70 mm with transverse cracks with rectilinear tip and different depths, clamped at one end and loaded by shear forces and bending moments at the other end. Shear forces and bending moment values are selected in order to have always the same bending moment in correspondence of the crack. The deflections according to all the 6 degrees of freedom have been evaluated for the different angular positions of the loads with respect to the crack. For computational convenience a clamped beam and rotating loads have been considered instead of a rotating beam and fixed load as is the situation in real rotating cracked shafts.

The non-linear 3D model with a transverse crack with depth of 30%, 40%, 50%, 60% and 70% of the diameter has been meshed and analysed.

In order to emphasize the effect of the crack and to highlight the differences in its static behaviour attributed to the additional shear flexibility, the deflections of the un-cracked specimen have been subtracted from the deflections of the cracked specimens, and these differences are compared in the same diagrams for different values of the shear force. This way only the effect of the crack is shown.

DESCRIPTION OF THE 3D FINITE ELEMENT MODEL

Fig. 1 shows one of the different meshes which has been used for the cracked cylinder, with a relative crack depth of 50% of the diameter. Roughly 11000 elements respectively have been used for the analysis of the cracked cylindrical beams. The mesh has been chosen rather “dense” because not only deformations of the cracked specimen, but also stress intensity factors in correspondence of the crack tip have been calculated numerically and compared with those calculated by means of the classical fracture mechanics approach. This comparison allowed to evaluate the accuracy of the model as regards its capability of representing real crack behaviour in the region close to the crack.

A bi-linear stress-strain relationship has been taken into account according to Fig. 2 for the simulations, but the elastic limit was never exceeded in the simulations, except in a very small region close to the crack tip. This fact did not affect the calculation of the deflections. The consequence is that the elastic overall behaviour of the cracked beam can be considered linear.

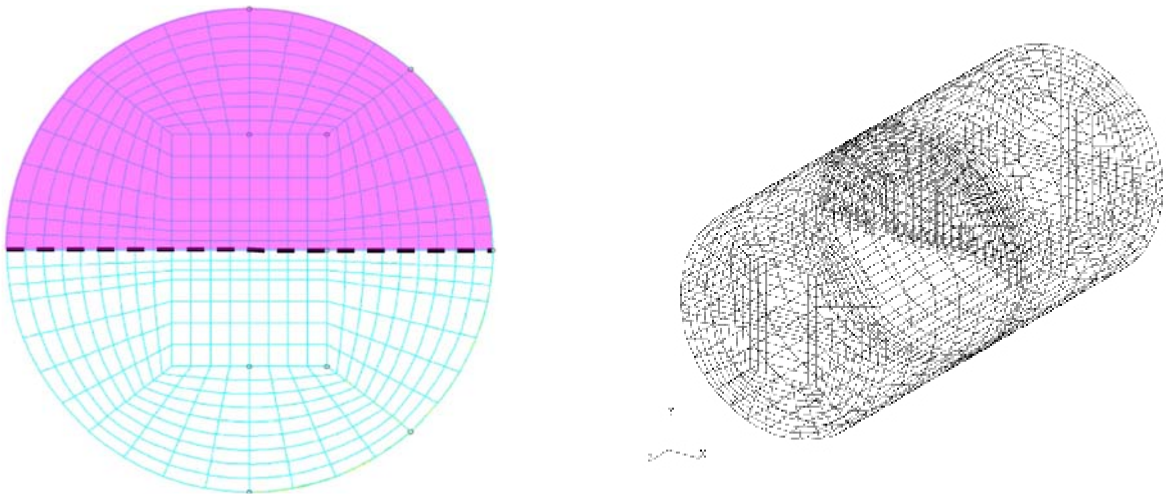


Figure 1 - Mesh of the section and isometric view of the model with a crack of 50%. The crack tip is indicated by the dashed line.

The contact model in the cracked surface is obviously non linear. Also a friction coefficient has been introduced in order to account for micro-slip conditions in the cracked area, due to shear forces. The value of the friction coefficient has been assumed equal to 0.4.

In order to avoid local deformations due to the application of loads, the model has been extended to a higher length, as shown in Fig. 3, where a pure bending load is applied to the specimen. This way in the cracked area and in the “measuring” section, where the deflections are evaluated, indicated by the dashed line, no local deformations are present, due to the application of loads.

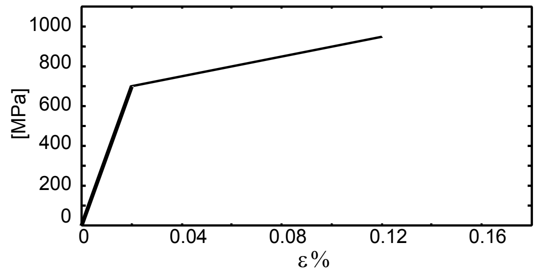


Figure 2 - Bi-linear stress-strain relationship used for simulations.

Fig. 3 shows also the axial stress distribution in the diametral longitudinal section in case of pure bending load, applied by means of two opposite shear loads: it can be seen that the axial stress distribution in the "measuring section" is unaffected by the crack and by the local application of the loads.

With respect to the traditional fracture mechanics approach, which allows to evaluate the additional flexibility introduced by the crack [1], the 3D non linear model takes into account also friction forces and microslipping conditions on the crack faces.

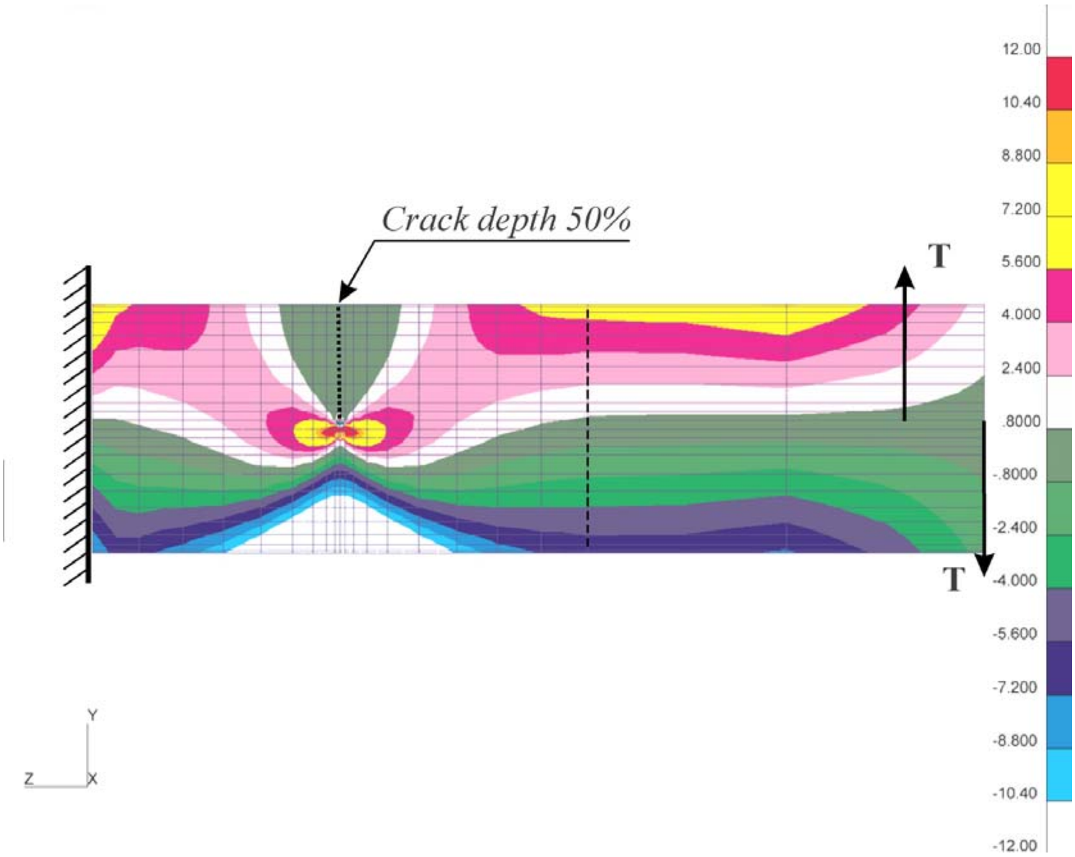


Figure 3 - Axial stress distribution, with a bending moment of 10^5 Nmm.

DEFINITION OF LOADS

The considered load cases are as follows: shear forces and bending moments applied to the extension length of the shaft have been selected in order to have always in correspondence of the cracked section the same bending moment and increasing shear forces. The shear force in load case T2 is double with respect to that one of load case T1, the shear force in case T3 is three times the load of case T1. Shear and bending loads are rotating with respect to the crack, in order to simulate the behaviour of a rotating shaft loaded with stationary bending and shear loads.

Load case C1	only bending moment Mb = 600 Nm
Load case T1	bending moment Mb = 600 Nm and shear force T = 4285 N
Load case T2	bending moment Mb = 600 Nm and shear force T = 8570 N
Load case T3	bending moment Mb = 600 Nm and shear force T = 12855 N

Figure 4 shows the cracked specimen along with the loads and the deflections according to the 6 degrees of freedom of the extremity node of the specimen.

In the initial position bending moment and shear force are directed towards the positive x axis: therefore the crack is closed. When the loads are rotated then the crack gradually opens. Axial and torsion deflections are due to so-called coupling effects, because axial and torsion loads are not applied to the beam. Since the measuring section is distorted (non-planar), the deflections have been evaluated as mean values over the section of the deflection of all nodes.

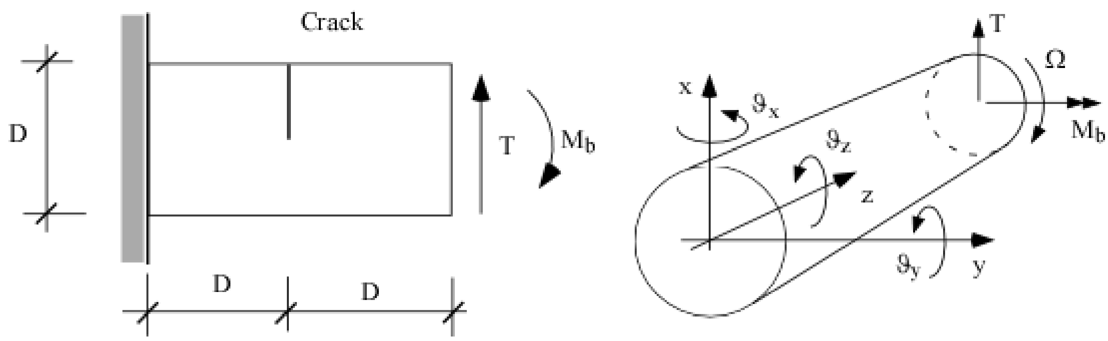


Figure 4 Cracked specimen with loads and associated deflections.

MAIN RESULTS OF THE MODEL

Figure 5 shows the deflections due to a crack with a depth of 50% of the diameter, according to the 6 degrees of freedom specified in Fig.4, for the 4 different loading cases. All displacements are given in mm, and the rotations in mrad. The effect of increasing shear loads can be seen in the horizontal deflections which increase as the shear force increases. But the deflections according to the other degrees of freedom are

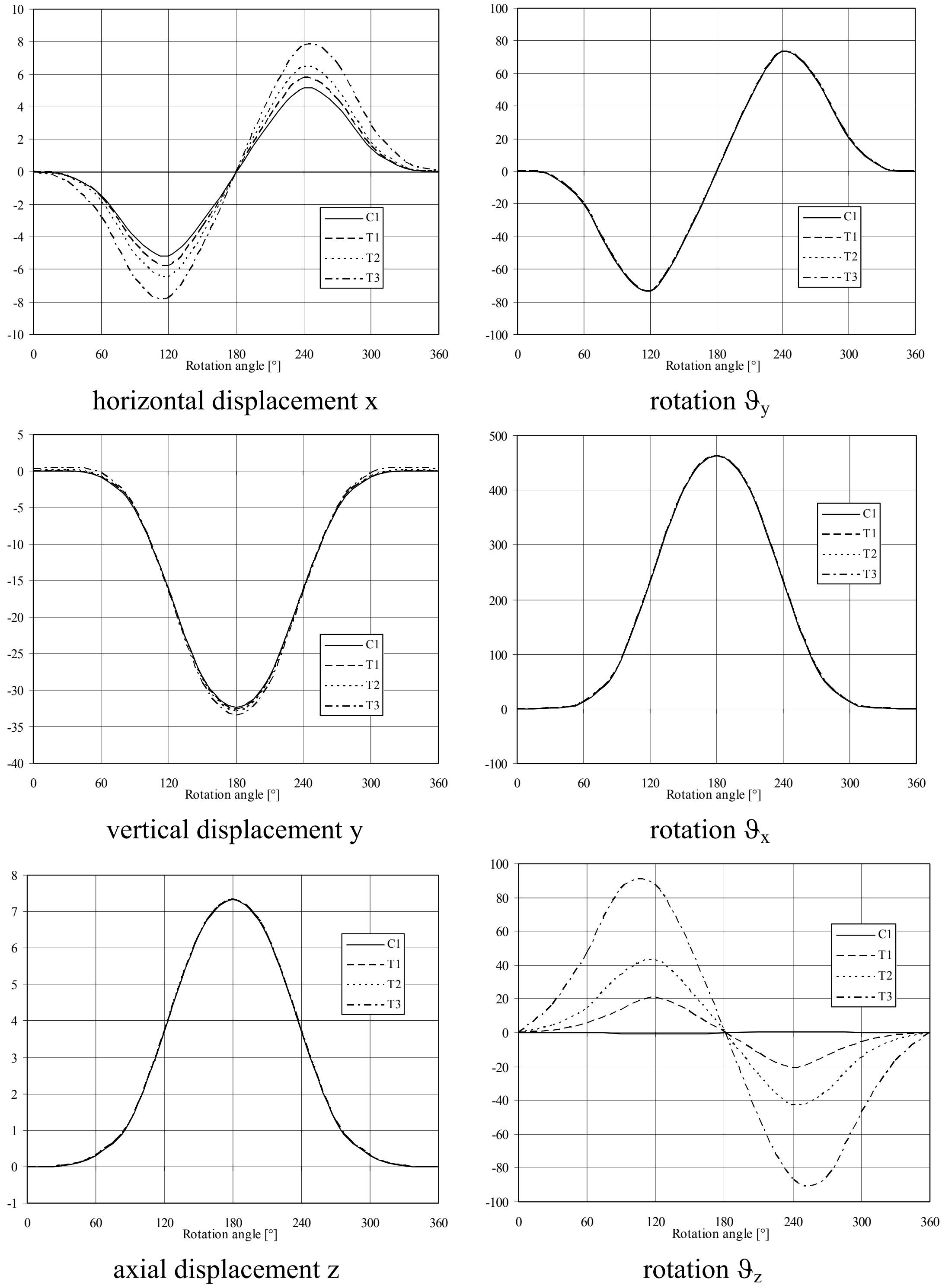


Figure 5 Effect of increasing shear forces on the deflections of a beam with 50% crack

rather unaffected by the shear force, except for the torsion deflection. This behaviour can be explained by following consideration.

The shear force is applied to a resisting section of the crack which, considering an open or partially open crack or a crack in which microslipping occurs, is eccentric with respect to the beam axis and with respect to the shear force direction, producing eccentric torsion due to a so-called “coupling effect”. Torsion on the cracked beam produce torsional deflections and also horizontal deflections due again to a “coupling effect”. It should be noted that this effect on horizontal deflections is related to the situation of fixed beam and rotating load; it will be shown that when the results are converted to rotating beam and fixed load the effect of shear forces on the lateral deflections is reduced and spread over both horizontal and vertical directions. Rotations ϑ_x and ϑ_y and axial displacements are completely unaffected by the shear forces. The main effect is the excitation of torsional rotation ϑ_z.

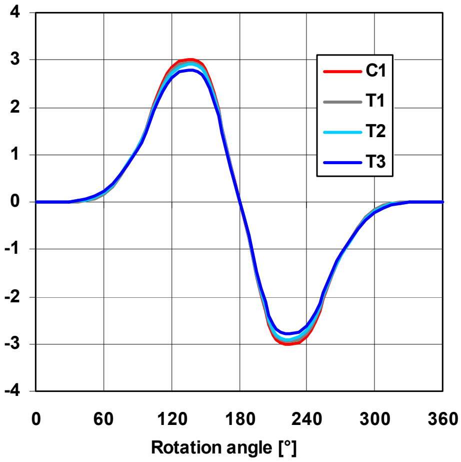
The same behaviour has been found for all different crack depths analysed. Obviously the deflections are magnified by high crack depths and reduced for small crack depths

The excitation of torsional deflections has an important consequence considering the application to rotordynamics: the excitation of torsional vibrations in rotating shafts is due to the same coupling mechanism which causes statically the torsion deflection mechanism. Therefore torsional vibration excitation is directly proportional to the shear forces in correspondence of the crack position, in a similar way as the lateral vibrations are proportional to the static bending moment in correspondence of the crack. [2]

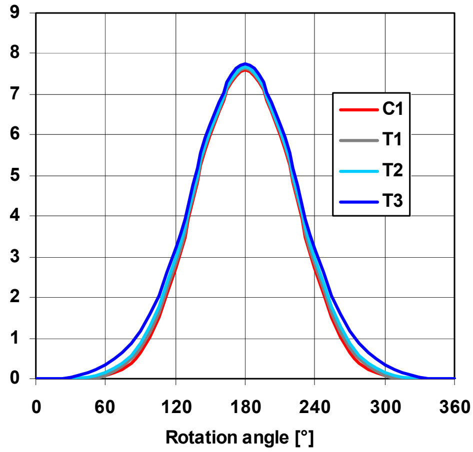
RESULTS OBTAINED WITH ROTATING CRACKED BEAM

The results obtained with clamped beam and rotating load can be processed for obtaining results related to rotating cracked beam loaded by fixed bending and shear loads, as it occurs in cracked shafts of rotating machinery. This has been done for the two extreme crack depths of 30% (minimum depth) and 70% (maximum depth). Fig. 6 show some results for the small crack: rotations ϑ_x and ϑ_y are not shown for brevity. As already pointed out the axial displacement (as well as the rotations ϑ_x and ϑ_y) are completely unaffected by the shear load, and horizontal and vertical displacements are only slightly affected by the value of the shear force.

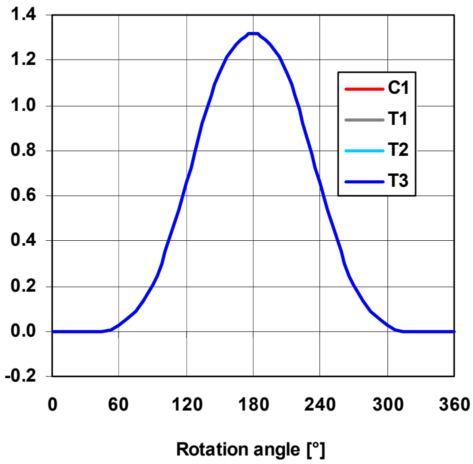
The effect of friction forces can be seen mainly in the torsion rotation. Without friction forces the deflection will have a pure sinusoidal law according to the component of shear parallel to the crack tip which is sinusoidal with the angle of rotation of the crack. This shear component is responsible for the torsion generation due to the eccentricity of the cracked resisting section with respect to the direction of shear load. The orthogonal component instead is directed towards the beam axis and does not generate any moment.Taking instead into account friction the torsional deflection is prevented in all points of the cracked area where tangential stresses are lower than the compressive stresses multiplied by the friction coefficient. Therefore in positions between 0° and 90° (and between 270° and 360°) torsion deflections are considerably lower with respect to the sinusoidal law.



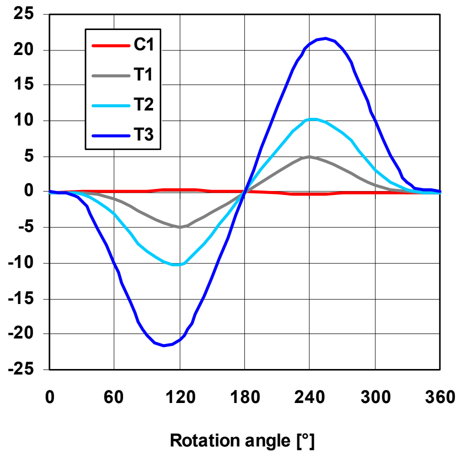
horizontal displacement x



vertical displacement y



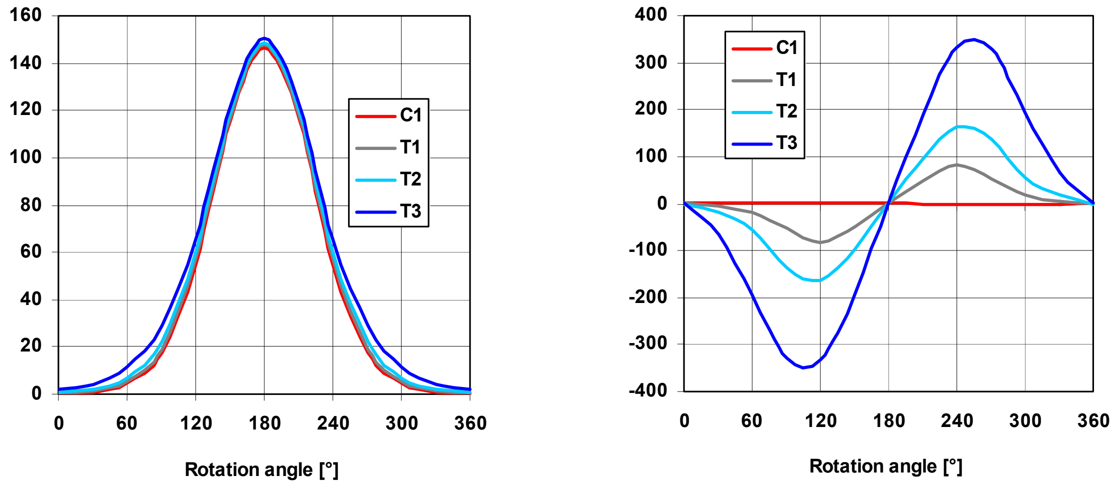
Axial displacement z



torsion rotation ϑ_z

Figure 6 Deflections obtained with 30% deep rotating crack and different shear loads.

Fig. 7 shows the results obtained with a 70% deep crack: only the vertical displacement and the torsion rotation are shown for a comparison with the small crack. Displacements are magnified more than 10 times, as could be seen also from other degrees of freedom, but the behaviour is exactly the same as for the small crack. The torsion excitation due to shear forces is now rather consistent. Displacements are increasing more than proportionally with the increasing shear load. The huge excitation of torsional vibrations of a rotating shaft with such a deep crack could constitute an important symptom of the presence of a crack in positions where high shear loads combined to low bending loads are responsible for weak excitation of lateral vibrations.



Vertical displacement y

Torsion rotation ϑ_z

Figure 7 Deflections obtained with 70% deep rotating crack and different shear loads.

CONCLUSIONS

3 D non linear models of cracked beams have been described and used for calculating deflections of cracked beams with different depths of crack loaded by constant bending moment and increasing shear loads. From the analysis following results can be drawn:1) only small effects can be seen in the horizontal and vertical deflections of the rotating cracked beam. These effects are so small that they could also be neglected. 2) no effects at all are recognized in axial displacements and in rotations about horizontal and vertical axis. 3) Strong effects have been shown in torsional rotation displacement, which are nul when shear forces are not present.

Therefore following conclusion can be stated:shear flexibility can be neglected when only lateral deflections (in a static situation) or lateral vibrations (in a dynamic situation) are of interest. But shear effect is predominant and cannot be neglected when torsion deflections or vibrations due to coupling effect should be evaluated.

This last consideration is important for rotordynamic analyses where vibrations are evaluated excited by bending and shear loads.

REFERENCES

1. Bachschmid N., Vania A. and Audebert, S., *A Comparison of Different Methods for Transverse Crack Modelling in Rotor Systems*, Proc. of *ISROMAC-8 Conference*, 26-30 March 2000, Honolulu, Hawaii,1056-1063.

2. Bachschmid N., Tanzi E., *Excitation of Torsional Vibrations in Cracked Rotating Shafts*, *ISROMAC-10 Conference*, 7-11 March 2004, Honolulu, Hawaii, 1-10.