Computational Simulation of Crack Propagation Trajectories by Means of the Contour Element Method

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ABSTRACT. *The article discusses a contour element method applied to numerical simulations of crack propagation trajectories in elastic structures. Because the boundary integral equation degenerates for a body with two crack-surfaces occupying the same location one of the forms of the displacement discontinuity method is implemented. According to the implemented method, resultant forces and dislocation densities, which are placed at mid-nodes of contour segments on one of the crack surfaces, are characterized by the indirect boundary integral equation. Contrarily to internal crack problems, for edge crack problems an edge-discontinuous element is used at the intersection between a crack and an edge to avoid a common node at the intersection. New numerical formulations that are built up on analytical integration are implemented. Therefore, all regular and singular integrals are evaluated only analytically. Tractions and resultant forces at a mid-node of any contour segment are regularized by a nonlocal characterization function. Hence, values of their components are obtained from the modified form of Somigliana's identity that embraces nonlocal elements and standard elements of kernel matrices used in the boundary element analysis.*

INTRODUCTION

The boundary integral equation for elastostatic problems can be derived from Betti's reciprocal work theorem (see Ref [1]) for two self-equilibrated states of displacement $\mathbf{u} \mid \mathbf{u}^*$, tractions $\mathbf{t} \mid \mathbf{t}^*$ and volume forces $\mathbf{b} \mid \mathbf{b}^*$. If Hooke's body is exposed to two different systems of volume and surface forces, then the actual work done by the forces of the first system along the displacements of the second system is equal to that work done by the forces of the second system along the displacements belonging to the first system:

$$\int_{\Omega} b_i^* u_i \, d\Omega + \int_{\Gamma} t_i^* u_i \, d\Gamma = \int_{\Omega} b_i u_i^* \, d\Omega + \int_{\Gamma} t_i u_i^* \, d\Gamma \tag{1}$$

In Eq. 1 the displacements, u_i, tractions, t_i (i.e., the stress vectors, $t_i = \sigma_{ij} n_j$, related to the outward normal vector, n_j), and body forces, b_i, are respectively determined on the boundary $\Gamma = \partial(\Omega)$ and in the domain Ω.

For the *i*-direction at any field point $\mathbf{r} = (x, y)$ due to the unit force e_j in the *j*-direction applied at the load point $\mathbf{r}' = (x', y')$, fields of $\mathbf{u}^*$, $\mathbf{t}^*$ and $\mathbf{b}^*$ corresponding to the governing solution of elasticity theory can be expressed as:

$$u_i^*(\mathbf{r}) = U_{ij}(\mathbf{r}, \mathbf{r}') e_j(\mathbf{r}') \quad (\text{displacement field}) \ , \tag{2a}$$

$$t_i^*(\mathbf{r}) = T_{ij}(\mathbf{r}, \mathbf{r}') e_j(\mathbf{r}') \quad (\text{traction field}) \ , \tag{2b}$$

$$b_i^*(\mathbf{r}) = \delta(r) e_i(\mathbf{r}') \quad (\text{body force field}) \ , \tag{2c}$$

where $U_{ij}(\mathbf{r}, \mathbf{r}')$ and $T_{ij}(\mathbf{r}, \mathbf{r}')$ are the fundamental solutions for linear elastic problems, $\delta(r)$ is the Dirac delta function and $r = \|\mathbf{r} - \mathbf{r}'\|$ is the distance between $\mathbf{r}$ and $\mathbf{r}'$. The lower case of $\mathbf{r}$ or $\mathbf{r}'$ represents a point located in $Int(\Omega)$, while the upper case of $\mathbf{R}$ or $\mathbf{R}'$ represents a point placed on Γ. For two-dimensional elastostatic problems, $U_{ij}(\mathbf{r}, \mathbf{r}')$ and $T_{ij}(\mathbf{r}, \mathbf{r}')$ are given by

$$U_{ij}(\mathbf{r}, \mathbf{r}') = \frac{\chi_C}{2\mu}\left[(3 - 4\nu)\ln(r)\delta_{ij} - r_{,i} r_{,j}\right] , \tag{3a}$$

$$T_{ij}(\mathbf{r}, \mathbf{r}') = \frac{\chi_C}{r}\left\{\left[(1 - 2\nu)\delta_{ij} + 2 r_{,i} r_{,j}\right]\frac{\partial r}{\partial n} - (1 - 2\nu)\left(r_{,i} n_j - r_{,j} n_i\right)\right\} \ , \tag{3b}$$

where $\chi_C = -1/[4\pi(1-\nu)]$, δ_{ij} is the Kronecker delta function, μ is the shear modulus of elasticity and ν is the Poisson's ratio. Note that components of the gradient of the one-form dA are denoted by the comma derivative: $A_{,k} = \partial A / \partial x_k$.

According to the expressions 2 in the absence of body forces, b_i, Eq. 1 can be rewritten for two-dimensional elastostatic problems as:

$$u_i(\mathbf{r}) + \int_{\partial(\Omega)} T_{ij}(\mathbf{r}, \mathbf{R}') u_j(\mathbf{R}') d\Gamma(\mathbf{R}') = \int_{\partial(\Omega)} U_{ij}(\mathbf{r}, \mathbf{R}') t_j(\mathbf{R}') d\Gamma(\mathbf{R}') \\ : \forall\, \mathbf{r} \in Int(\Omega) \quad (i, j = 1, 2) . \tag{4}$$

MODELING COPLANAR CRACK SURFACES

The straightforward application of Eq. 4 to crack problems leads to mathematical degeneration when upper and lower crack surfaces of a body occupy the same location.

In the presence of a crack in the domain Ω, the boundary of a two-dimensional body, $\Gamma = \partial(\Omega)$, can be divided into parts: $\Gamma = \Gamma_B + \Gamma_C^{\{+\}} + \Gamma_C^{\{-\}}$, where $\Gamma_C^{\{+\}}$ and $\Gamma_C^{\{-\}}$ represent the upper and lower crack surfaces and Γ_B represents the remaining boundary. From fundamental solutions 3, the displacement and traction on the lower and upper crack surfaces have the properties that

$$U_{ij}\left(\mathbf{r}, \mathbf{R}_C'^{\{+\}}\right) = U_{ij}\left(\mathbf{r}, \mathbf{R}_C'^{\{-\}}\right), \tag{5a}$$

$$T_{ij}\left(\mathbf{r}, \mathbf{R}_C'^{\{+\}}\right) = -T_{ij}\left(\mathbf{r}, \mathbf{R}_C'^{\{-\}}\right). \tag{5b}$$

The change in sign in tractions in expression 5b is because the direction of the normal is opposite on the two crack surfaces. A simple description of a crack is two coplanar surfaces that are closed, e.g., $\Gamma_C^{\{-\}} \to \Gamma_C^{\{+\}}$ (see Ref. [2]). Because of $\Gamma_C^{\{-\}} = \Gamma_C^{\{+\}}$ and expressions 5, Eq. 4 becomes

$$\begin{aligned} u_i(\mathbf{r}) + \int_{\Gamma_B} T_{ij}(\mathbf{r}, \mathbf{R}') u_j(\mathbf{R}') d\Gamma(\mathbf{R}') \quad + \\ \int_{\Gamma_C^{\{-\}}} T_{ij}\left(\mathbf{r}, \mathbf{R}_C'^{\{-\}}\right) \Delta u_j\left(\mathbf{R}_C'^{\{-\}}\right) d\Gamma\left(\mathbf{R}_C'^{\{-\}}\right) \quad = \quad \int_{\Gamma_C^{\{-\}}} U_{ij}\left(\mathbf{r}, \mathbf{R}_C'^{\{-\}}\right) \Sigma t_j\left(\mathbf{R}_C'^{\{-\}}\right) d\Gamma\left(\mathbf{R}_C'^{\{-\}}\right) \quad + \\ \int_{\Gamma_B} U_{ij}(\mathbf{r}, \mathbf{R}') t_j(\mathbf{R}') d\Gamma(\mathbf{R}') \end{aligned} \tag{6}$$

where

$$\Delta u_i\left(\mathbf{R}_C'^{(-)}\right) = u_i\left(\mathbf{R}_C'^{(+)}\right) - u_i\left(\mathbf{R}_C'^{(-)}\right), \tag{7a}$$

$$\Sigma t_i\left(\mathbf{R}_C'^{(-)}\right) = t_i\left(\mathbf{R}_C'^{(+)}\right) - t_i\left(\mathbf{R}_C'^{(-)}\right). \tag{7b}$$

For traction free cracks, or when the crack is loaded by equal and opposite tractions $\Sigma t_j\left(\mathbf{R}_C'^{\{-\}}\right) = 0$. Hence, Eq. 6 can be rewritten in the form:

$$\begin{aligned} u_i(\mathbf{r}) + \int_{\Gamma_B} T_{ij}(\mathbf{r}, \mathbf{R}') u_j(\mathbf{R}') d\Gamma(\mathbf{R}') \quad + \quad \int_{\Gamma_C^{\{-\}}} T_{ij}\left(\mathbf{r}, \mathbf{R}_C'^{\{-\}}\right) \Delta u_j\left(\mathbf{R}_C'^{\{-\}}\right) d\Gamma\left(\mathbf{R}_C'^{\{-\}}\right) \\ = \int_{\Gamma_B} U_{ij}(\mathbf{r}, \mathbf{R}') t_j(\mathbf{R}') d\Gamma(\mathbf{R}'). \end{aligned} \tag{8}$$

Equation 8 has the form as the standard integral equation 4 with an additional integral along the lower crack surface, $\Gamma_C^{\{-\}}$. The boundary form of this equation is indeterminate (i.e., when $\mathbf{R} \in \Gamma_C^{\{-\}}$ the number of unknown variables in the system of linear equations is greater than the number of linear equations, which represent the boundary form of the integral equation 8 with discrete distributions of u_i, t_i and Δu_i along the boundary parts Γ_B and $\Gamma_C^{\{-\}}$, respectively). Therefore, to determine the displacement discontinuity, $\Delta u_i\left(\mathbf{R}_C^{(-)}\right)$, along $\Gamma_C^{\{-\}}$ an additional integral equation for the crack surface tractions (i.e., $t_i\left(\mathbf{R}_C^{(-)}\right)=\sigma_{ij}\left(\mathbf{R}_C^{(-)}\right)n_j\left(\mathbf{R}_C^{(-)}\right)$) must be derived. Such equation can be obtained by means of Hooke's law:

$$\sigma_{ij}(\mathbf{r})=\frac{2\nu\mu}{1-2\nu}u_{k,k}(\mathbf{r})\delta_{ij}+\mu\left[u_{i,j}(\mathbf{r})+u_{j,i}(\mathbf{r})\right]\quad(k=1,2)\;. \tag{9}$$

By differentiating $u_i(\mathbf{r})$ in Eq. (8) with respect to the source point $\mathbf{r}$ and then substituting the derivatives: $u_{k,k}(\mathbf{r})$, $u_{i,j}(\mathbf{r})$ and $u_{j,i}(\mathbf{r})$ into the expression 9, the additional integral equation for $\sigma_{ij}(\mathbf{r})$ is given by

$$\begin{aligned}\sigma_{ij}(\mathbf{r}) \;=\; &\int_{\Gamma_B} D_{kij}(\mathbf{r},\mathbf{R}')t_k(\mathbf{R}')d\Gamma(\mathbf{R}') \;-\; \int_{\Gamma_B} S_{kij}(\mathbf{r},\mathbf{R}')u_k(\mathbf{R}')d\Gamma(\mathbf{R}') \;+\\ &-\int_{\Gamma_C^{\{-\}}} S_{kij}\left(\mathbf{r},\mathbf{R}_C'^{\{-\}}\right)\Delta u_k\left(\mathbf{R}_C'^{\{-\}}\right)d\Gamma\left(\mathbf{R}_C'^{\{-\}}\right),\end{aligned} \tag{10}$$

where

$$D_{kij}=\frac{2\nu\mu}{1-2\nu}U_{kl,l}(\mathbf{r})\delta_{ij}+\mu\left[U_{ki,j}(\mathbf{r})+U_{kj,i}(\mathbf{r})\right]\quad(l=1,2)\;, \tag{11a}$$

$$S_{kij}=\frac{2\nu\mu}{1-2\nu}T_{kl,l}(\mathbf{r})\delta_{ij}+\mu\left[T_{ki,j}(\mathbf{r})+T_{kj,i}(\mathbf{r})\right]. \tag{11b}$$

Multiplying both sides of Eq. 10 by $n_j(\mathbf{r})$ gives

$$\begin{aligned}t_i(\mathbf{r}) \;=\; &\left[\int_{\Gamma_B} D_{kij}(\mathbf{r},\mathbf{R}')t_k(\mathbf{R}')d\Gamma(\mathbf{R}')\right]n_j(\mathbf{r}) - \left[\int_{\Gamma_B} S_{kij}(\mathbf{r},\mathbf{R}')u_k(\mathbf{R}')d\Gamma(\mathbf{R}')\right]n_j(\mathbf{r}) \;+\\ &-\left[\int_{\Gamma_C^{\{-\}}} S_{kij}\left(\mathbf{r},\mathbf{R}_C'^{\{-\}}\right)\Delta u_j\left(\mathbf{R}_C'^{\{-\}}\right)d\Gamma\left(\mathbf{R}_C'^{\{-\}}\right)\right]n_j(\mathbf{r})\,.\end{aligned} \tag{12}$$

The boundary forms of the integral equations 8 and 12 define the problem to be solved. However, the fundamental drawback of the boundary form of Eq. 12 is that this equation contains the r^{-2} singularity that is difficult to handle in numerical calculations.

TREATMENT OF THE STRONGLY SINGULAR INTEGRALS

To avoid the numerical difficulties with the strongly singular integrals in Eq. 8 the idea of integration by parts developed by Ghosh et al. [3] and Zang [4] is adopted.

A single integration by parts starts with

$$\frac{\partial}{\partial s}\left[-W_{ij}(\mathbf{r},\mathbf{R}')u_j(\mathbf{R}')\right]=\frac{\partial}{\partial s}\left[-W_{ij}(\mathbf{r},\mathbf{R}')\right]u_j(\mathbf{R}')-W_{ij}(\mathbf{r},\mathbf{R}')\frac{\partial u_j(\mathbf{R}')}{\partial s} \tag{13}$$

and integrates both sides of the equality 13,

$$\begin{aligned}\int_{\Gamma_{\mathbf{AB}}}\frac{\partial}{\partial s}\left[-W_{ij}(\mathbf{r},\mathbf{R}')u_j(\mathbf{R}')\right]ds(\mathbf{R}') \quad &= \quad \int_{\Gamma_{\mathbf{AB}}}\frac{\partial}{\partial s}\left[-W_{ij}(\mathbf{r},\mathbf{R}')\right]u_j(\mathbf{R}')ds(\mathbf{R}') \; + \\ &- \quad \int_{\Gamma_{\mathbf{AB}}}W_{ij}(\mathbf{r},\mathbf{R}')\frac{\partial u_j(\mathbf{R}')}{\partial s}ds(\mathbf{R}')\,.\end{aligned} \tag{14}$$

Rearranging gives

$$\int_{\Gamma_{\mathbf{AB}}}T_{ij}(\mathbf{r},\mathbf{R}')u_j(\mathbf{R}')ds(\mathbf{R}')=\left[-W_{ij}(\mathbf{r},\mathbf{R}')u_j(\mathbf{R}')\right]_{\mathbf{A}}^{\mathbf{B}}+\int_{\Gamma_{\mathbf{AB}}}W_{ij}(\mathbf{r},\mathbf{R}')\frac{\partial u_j(\mathbf{R}')}{\partial s}ds(\mathbf{R}')\;, \tag{15}$$

where $T_{ij}(\mathbf{r},\mathbf{R}')=\frac{\partial}{\partial s}\left[-W_{ij}(\mathbf{r},\mathbf{R}')\right]$. Applying integration by parts for the upper and lower crack surfaces gives

$$\begin{aligned}\int_{\Gamma_C^{\{+\}}}T_{ij}\left(\mathbf{r},\mathbf{R}_C'^{\{+\}}\right)u_j\left(\mathbf{R}_C'^{\{+\}}\right)d\Gamma\left(\mathbf{R}_C'^{\{+\}}\right) \quad &= \\ \left[-W_{ij}\left(\mathbf{r},\mathbf{R}_C'^{\{+\}}\right)u_j\left(\mathbf{R}_C'^{\{+\}}\right)\right]_{\mathbf{B}_C}^{\mathbf{A}_C} \quad &+ \quad \int_{\Gamma_C^{\{+\}}}W_{ij}\left(\mathbf{r},\mathbf{R}_C'^{\{+\}}\right)\frac{\partial u_j\left(\mathbf{R}_C'^{\{+\}}\right)}{\partial s_C^{\{+\}}}d\Gamma\left(\mathbf{R}_C'^{\{+\}}\right),\end{aligned} \tag{16a}$$

$$\begin{aligned}\int_{\Gamma_C^{\{-\}}}T_{ij}\left(\mathbf{r},\mathbf{R}_C'^{\{-\}}\right)u_j\left(\mathbf{R}_C'^{\{-\}}\right)d\Gamma\left(\mathbf{R}_C'^{\{-\}}\right) \quad &= \\ \left[-W_{ij}\left(\mathbf{r},\mathbf{R}_C'^{\{-\}}\right)u_j\left(\mathbf{R}_C'^{\{-\}}\right)\right]_{\mathbf{A}_C}^{\mathbf{B}_C} \quad &+ \quad \int_{\Gamma_C^{\{-\}}}W_{ij}\left(\mathbf{r},\mathbf{R}_C'^{\{-\}}\right)\frac{\partial u_j\left(\mathbf{R}_C'^{\{-\}}\right)}{\partial s_C^{\{-\}}}d\Gamma\left(\mathbf{R}_C'^{\{-\}}\right).\end{aligned} \tag{16b}$$

For the internal crack problems when the displacements of the upper and lower crack surfaces at both crack tips **A** and **B** coincide (i.e., $u_i\left(\mathbf{R}_C'^{\mathbf{A}_C^{\{+\}}}\right)=u_i\left(\mathbf{R}_C'^{\mathbf{A}_C^{\{-\}}}\right)$ and $u_i\left(\mathbf{R}_C'^{\mathbf{B}_C^{\{+\}}}\right)=u_i\left(\mathbf{R}_C'^{\mathbf{B}_C^{\{-\}}}\right)$), the lower-order singular integral can be expressed by

$$\int_{\Gamma_C^{\{-\}}} T_{ij}\left(\mathbf{r}, \mathbf{R}_C'^{\{-\}}\right) u_j\left(\mathbf{R}_C'^{\{-\}}\right) d\Gamma\left(\mathbf{R}_C'^{\{-\}}\right) + \int_{\Gamma_C^{\{+\}}} T_{ij}\left(\mathbf{r}, \mathbf{R}_C'^{\{+\}}\right) u_j\left(\mathbf{R}_C'^{\{+\}}\right) d\Gamma\left(\mathbf{R}_C'^{\{+\}}\right) =$$
$$\int_{\Gamma_C^{\{-\}}} T_{ij}\left(\mathbf{r}, \mathbf{R}_C'^{\{-\}}\right) \Delta u_j\left(\mathbf{R}_C'^{\{-\}}\right) d\Gamma\left(\mathbf{R}_C'^{\{-\}}\right) = \int_{\Gamma_C^{\{-\}}} W_{ij}\left(\mathbf{r}, \mathbf{R}_C'^{\{-\}}\right) \frac{\partial\left[\Delta u_j\left(\mathbf{R}_C'^{\{-\}}\right)\right]}{\partial s_C^{\{-\}}} d\Gamma\left(\mathbf{R}_C'^{\{-\}}\right). \tag{17}$$

However, for edge cracks problems when the displacements of the upper and lower crack surfaces only at the one of the crack tips coincide (i.e., $u_i\left(\mathbf{R}_C'^{\mathbf{A}_C^{\{+\}}}\right) = u_i\left(\mathbf{R}_C'^{\mathbf{A}_C^{\{-\}}}\right)$ and $u_i\left(\mathbf{R}_C'^{\mathbf{B}_C^{\{+\}}}\right) \neq u_i\left(\mathbf{R}_C'^{\mathbf{B}_C^{\{-\}}}\right)$), the lower-order singular integral can be expressed by

$$\int_{\Gamma_C^{\{-\}}} T_{ij}\left(\mathbf{r}, \mathbf{R}_C'^{\{-\}}\right) \Delta u_j\left(\mathbf{R}_C'^{\{-\}}\right) d\Gamma\left(\mathbf{R}_C'^{\{-\}}\right) =$$
$$W_{ij}\left(\mathbf{r}, \mathbf{R}_C'^{\mathbf{B}_C^{\{-\}}}\right) \Delta u_j\left(\mathbf{R}_C'^{\mathbf{B}_C^{\{-\}}}\right) + \int_{\Gamma_C^{\{-\}}} W_{ij}\left(\mathbf{r}, \mathbf{R}_C'^{\{-\}}\right) \frac{\partial\left[\Delta u_j\left(\mathbf{R}_C'^{\{-\}}\right)\right]}{\partial s_C^{\{-\}}} d\Gamma\left(\mathbf{R}_C'^{\{-\}}\right). \tag{18}$$

Therefore, for edge cracks problems the edge-discontinuous element is used at the intersection between a crack and an edge to avoid a common node at the intersection.

Reduction of the order of the strongly singular integrals in the equality 12 can be obtained by integration of both sides of this equality with respect to the field point, $\mathbf{R}_C^{\{-\}}$, along the lower crack surface, $\Gamma_C^{\{-\}}$, that consists of smooth straight segments from one crack tip $\mathbf{A}_C$ to $\mathbf{R}_C^{\{-\}}$

$$F_i\left(\mathbf{R}_C^{\{-\}}\right) = \int_{\mathbf{A}_C}^{\mathbf{R}_C^{\{-\}}} t_i\left(\mathbf{R}_C'^{\{-\}}\right) d\Gamma\left(\mathbf{R}_C'^{\{-\}}\right) \quad : \forall\, \mathbf{A}_C \mathbf{R}_C^{\{-\}} \subset \mathbf{A}_C \mathbf{B}_C \ . \tag{19}$$

For the internal crack problems, an additional constraint equation for the dislocation densities, $g_i\left(\mathbf{R}_C'^{\{-\}}\right) = \partial\left[\Delta u_i\left(\mathbf{R}_C'^{\{-\}}\right)\right] / \partial s_C^{\{-\}}$, along the lower crack surface is given by $\int_{\Gamma_C^{\{-\}}} g_i\left(\mathbf{R}_C^{\{-\}}\right) d\Gamma\left(\mathbf{R}_C^{\{-\}}\right)$.

NUMERICAL TREATMENT OF THE BOUNDARY INTEGRAL EQUATIONS

For a given source point, $P_{(\alpha)}$, the boundary forms of Eqs 8 and 19 can be discretized into N_B boundary contour segments and N_C crack contour segments as follows:

$$u_{i(\alpha)} + \chi_c \sum_{\beta=1}^{N_B} \sum_{j=1}^{2} \left(\mathrm{H}_{ij(\beta)}^{F(\alpha)} u_{j(\beta)}^{F} + \mathrm{H}_{ij(\beta)}^{L(\alpha)} u_{j(\beta)}^{L} \right) + \chi_c \sum_{\gamma=1}^{N_C} \sum_{j=1}^{2} \mathrm{IH}_{ij(\gamma)}^{(\alpha)} g_{j(\beta)}^{\{-\}} = \frac{\chi_c}{2\mu} \sum_{\beta=1}^{N_B} \sum_{j=1}^{2} \mathrm{G}_{ij(\beta)}^{(\alpha)} t_{j(\beta)} \quad (20a)$$

$$F_{i(\alpha)}^{\{-\}} - 2\mu\chi_c \sum_{\beta=1}^{N_B} \sum_{j=1}^{2} \left(\mathrm{CH}_{ij(\beta)}^{F(\alpha)} u_{j(\beta)}^{F} + \mathrm{CH}_{ij(\beta)}^{L(\alpha)} u_{j(\beta)}^{L} \right) - 2\mu\chi_c \sum_{\gamma=1}^{N_C} \sum_{j=1}^{2} \mathrm{CIH}_{ij(\gamma)}^{(\alpha)} g_{j(\beta)}^{\{-\}} = \chi_c \sum_{\beta=1}^{N_B} \sum_{j=1}^{2} \mathrm{CG}_{ij(\beta)}^{(\alpha)} t_{j(\beta)} + \mathbb{C}_i \quad (20b)$$

where $u_{i(\alpha)} = \frac{1}{2}\left(u_{i(\alpha)}^{F} + u_{i(\alpha)}^{L}\right)$, $u_{i(\beta)}^{L} = u_{i(\beta+1)}^{F}$ $\left(\text{for } \beta = 1, 2, \ldots, N_B - 1\right)$, $u_{i(1)}^{F} = u_{i(N_B)}^{L}$ and $\mathbb{C}_i$ are arbitrary constants of integration.

On each boundary contour segment, $\Delta\Gamma_{B(\beta)}$, displacement components: $u_{\xi(\beta)}^{(\alpha)}$ and $u_{\eta(\beta)}^{(\alpha)}$, are approximated by the linear interpolation function. However, traction components: $t_{\xi(\beta)}^{(\alpha)}$ and $t_{\eta(\beta)}^{(\alpha)}$, are constant along $\Delta\Gamma_{B(\beta)}$ like dislocation density components: $g_{\xi(\beta)}^{(\alpha)\{-\}}$ and $g_{\eta(\beta)}^{(\alpha)\{-\}}$, and resultant force components: $F_{\xi(\beta)}^{(\alpha)\{-\}}$ and $F_{\xi(\beta)}^{(\alpha)\{-\}}$, along any crack contour segment, $\Delta\Gamma_{C(\beta)}$. Therefore, integrations of integrand functions of the discrete version of the integral equations 20 can be performed exactly. Exact integration is generally faster than numerical integration for a level of reasonable numerical accuracy. The transformation of integration results from the local co-ordinates system $\left(\xi_{(\beta)}^{(\alpha)}, \eta_{(\beta)}^{(\alpha)}\right)$ to the global one (x, y) is straightforward.

Due to proper shape functions for the displacement field, the strain field and the stress field along each contour segment a special treatment, used to circumvent the well-known corner problem of the boundary element method, is not required. The matrices: $\mathbf{H}_{(\beta)}^{F(\alpha)}$, $\mathbf{H}_{(\beta)}^{L(\alpha)}$, $C\mathbf{H}_{(\beta)}^{F(\alpha)}$, $C\mathbf{H}_{(\beta)}^{L(\alpha)}$, $\mathbf{G}_{(\beta)}^{(\alpha)}$, $\mathbf{CG}_{(\beta)}^{(\alpha)}$, $\mathbf{IH}_{(\beta)}^{(\alpha)}$ and $\mathbf{CIH}_{(\beta)}^{L(\alpha)}$, in Eqs 20 are assessed by integrating the fundamental solutions analytically (see Ref [5]) without the necessity to use numerical integration over each contour segment.

REGULARIZATION BY A NONLOCAL CHARACTERIZATION FUNCTION

According to the nonlocal theory of Eringen [6], the stress is computed by averaging the local stress that would be obtained from the local model. Thus, the nonlocal approach of Eringen can be characterized as averaging of the stress. To analyze the nonlocal mechanical behavior, the expressions that contain stress components in Eqs 20 are regularized by

$$\left\langle \sigma_{ij}(\mathbf{r}) \right\rangle = \int_{\Omega} \alpha\left(\left\|\mathbf{r}' - \mathbf{r}\right\|_{GD}\right) \sigma_{ij}(\mathbf{r}')\, d\Omega(\mathbf{r}'), \tag{21}$$

where $\sigma_{ij}(\mathbf{r}')$ are classical stress components, $\alpha\left(\left\|\mathbf{r}' - \mathbf{r}\right\|_{GD}\right)$ is a nonlocal characterization function and the pointed brackets $\langle\ \rangle$ denote the averaging operator.

Here, the concept of geodetical distance, $\left\|\mathbf{r}' - \mathbf{r}\right\|_{GD}$, suggested by Polizzotto et al. [7], is applied as the length of the shortest path joining $\mathbf{r}$ with $\mathbf{r}'$ without intersecting the boundary surface. The nonlocal characterization function can be expressed as a form of the Gauss distribution function. However, in the vicinity of the boundary of a finite body (what is typical for the boundary element analysis), it is assumed that the averaging is performed only on the part of the domain of influence that lies within the solid. Therefore, in this case the formula of averaging 21 is replaced by the more enhanced form:

$$\left\langle \sigma_{ij}(\mathbf{r}) \right\rangle = [1 - \gamma(\mathbf{r})]\sigma_{ij}(\mathbf{r}) + \int_{\Omega} \alpha\left(\left\|\mathbf{r}' - \mathbf{r}\right\|_{GD}\right) \sigma_{ij}(\mathbf{r}')\, d\Omega(\mathbf{r}'), \tag{22}$$

where $\gamma(\mathbf{r}) = \int_{\Omega} \alpha\left(\left\|\mathbf{r}' - \mathbf{r}\right\|_{GD}\right) d\Omega(\mathbf{r}')$.

CONCLUSION

The aim of the presented research is to improve and to develop the boundary element method applied to modeling of crack propagation trajectories (see Ref. [8]).

REFERENCES

1. Aliabadi, M.H. (2002) *The Boundary Element Method*, *Applications in Solids and Structures*, John Wiley & Sons, New York.
2. Cruse, T.A. (1988) *Boundary Element Analysis in Computational Fracture Mechanics*, Kluwer Academic Publishers, Dordrecht.
3. Ghosh, N., Rajiyah, H., Ghosh, S. and Mukherjee, S. (1986) *J. Appl. Mech.* **53**, 69-76.
4. Zang, W.L. (1990) *Int. J. Fracture* **46**, 41-55.
5. Jackiewicz, J. (2004) In: Advances in Computational & Experimental Engineering & Sciences, pp. 224-229, Atluri S.N., Tadeu A.J.B. (Eds), Tech Science Press.
6. Eringen, A.C. (1987) *Res. Mech.* **21**, 313-342.
7. Polizzotto, C., Fuschi, P. and Pisano, A.A. (2004) *Int. J. Solids Struc.* **41**, 2383-2401.
8. Jackiewicz, J. (1996) *Computer Assisted Mechanics and Engineering Sciences* **3**, 155-167.