Crack propagation path in lubricated rolling-sliding contact

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ABSTRACT. *The paper presents influence of asymptotic and constant term in Williams equation for stress distribution around the crack tip on crack propagation path relative to the pre-existing initial crack in lubricated rolling-sliding contact. Crack propagation path is predicted with modified maximum tangential stress criterion, which takes into account influence of stress intensity factor* K_I *and* K_{II}*, T-stress, stress on crack surface caused by lubricant pressure inside the crack and critical distance ahead the crack tip where fracturing process is initiated. The developed model is applied to a real spur gear pair. Results showed that T-stress have important role on crack propagation in lubricated rolling-sliding contact.*

INTRODUCTION

Rolling contact fatigue, which leads to surface cracking and consequently pitting, is one of the most common causes of failure in gears, bearings and railway tracks. The pitting phenomenon is strongly dependent on loading conditions, material of contacting parts, type of lubrication, surface roughness, etc., which all influence the appearance of initial surface cracks and later their propagation or arrest. The surface cracks could be initiated by the near-surface plastic deformation in the region of the maximum cyclic shear stress caused by repeated rolling-sliding contact, or alternatively at defects such as dents or scratches on the surface. It has been observed that the initial small cracks in gears appear at a characteristic shallow angle 20° to the surface and their orientation depends on the contact friction direction [1]. When they reach some critical length or depth, the cracks usually branch up towards the free surface, which results in separation of small patches of surface material, leaving behind a clear surface pit, as it is shown in Figure 1 [1]. Models for the description of the rolling-sliding contact fatigue phenomenon (e.g. pitting) are still being actively developed. One of the approaches is by employing the fracture mechanics for simulation of crack growth that leads to surface failure. A fluid lubricant between crack flanks, normal and traction force in the contact area cause tensile and shear loading around the crack tip. Several fracturing criteria have been developed in the past to describe brittle failure in linear elastic materials due to tensile and shear stresses occurring around the crack tip. The maximum tangential stress (MTS) criterion, proposed by Erdogan and Sih [2], is often used for crack propagation analysis in lubricated or dry contact area. Internal pressure inside the crack causes additional compressive stress, which can be captured with the stress intensity factors K_{I}, K_{II} and the T-stress [3]. Seweryn [4] discussed that higher terms in Williams [5] equation can be important in cases of short cracks (e. g. cracks on gear tooth flank). This paper investigates the influence of K_I, K_{II} and T-stress on the crack propagation angle relative to the pre-existing initial crack with use of the MTS fracturing criterion. The model also considers the influence of lubricant fluid trapped in a crack on its propagation path. Due to change of the stress intensity factors K_I, K_{II} and T-stress during motion of the contact-sliding loading, it is assumed that the maximum values of K_I, K_{II} and T, which occur when the crack mouth just enters the contact zone in every loading cycle, have the most significant influence on the crack propagation path [6]. The influence of contact temperature is presumed to be negligible in this investigation. The stated features are studied for the case of crack propagation on gear tooth flank by means of a two-dimensional computational model under plane strain conditions.

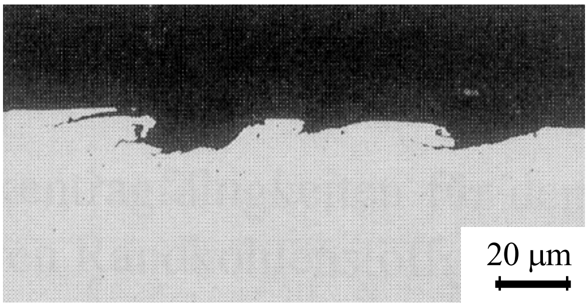


Figure 1: Surface micro cracks and pitting

MODIFIED MTS CRITERION

According to Erdogan and Sih [2], the crack extension starts along the radial direction in the plane perpendicular to the direction of the maximum tangential tension stress $\sigma_{\theta\theta}$, where the shear stress $\sigma_{r\theta}$ is zero. The tangential stress, which includes influence of stress intensity factor K_I, K_{II}, T-stress and constant internal pressure to the crack surfaces, can be written in polar co-ordinates as [7]:

$$\sigma_{\theta\theta}(\theta, r) = \frac{1}{\sqrt{2\pi r}} \cos\frac{\theta}{2} \left[K_I \cos^2\frac{\theta}{2} - \frac{3}{2} K_{II} \sin\theta \right] + T \sin^2\theta - \sigma_{xx}^c \sin 2\theta + \sigma_{yy}^c \cos^2\theta \quad (1)$$

where the tractions σ^c_{xx} and σ^c_{yy} are defined at the crack tip and their distribution is smooth enough along the crack surface.

The crack propagation angle θ_0 can be determined from the maximum condition

$$\left.\frac{\partial \sigma_{\theta\theta}}{\partial \theta}\right|_{r=r_c} = 0 \Rightarrow \theta = \theta_0 . \qquad (2)$$

where r_c is the critical length parameter.

By solving equation (1) according to (2) gives after rearrangement the following expression:

$$\left[K_{\mathrm{I}} \sin\theta_0 + K_{\mathrm{II}}(3 \cdot \cos\theta_0 - 1)\right] - \left(16\left(\left(T - \sigma_{yy}^c\right)\sqrt{2\pi r_c}\right)/3\right)\sin\frac{\theta_0}{2}\cos\theta_0 + \left(16\left(\sigma_{xx}^c\sqrt{2\pi r_c}\right)/6\right)\left(\cos\frac{\theta_0}{2}\right)^{-1}\cos 2\theta_0 = 0 \cdot \qquad (3)$$

Equation (3) represents a modified MTS criterion that can be used for determination of crack propagation angle θ_0 when the crack surfaces are loaded with constant pressure or sufficiently smooth traction distribution.
Physical length scale r_c presents the distance ahead of the crack tip where the fracturing process is actually initiated. The distance r_c is a material parameter, which is very difficult to determine. Several models have been proposed for its determination. The model of Larsson and Carlsson [8], which assumed that r_c is contained in the region of constrained yielding, was used to determine the critical distance r_c in this paper due to the availability of necessary material parameters.

NUMERICAL MODELLING OF CRACK PROPAGATION PATH

Crack propagation of initial surface crack was determined using generalized MTS criterion (eq. 3). Fracture mechanics parameters stress intensity factor K_{I}, K_{II} and T-stress were extracted using contour integral, which is implemented in commercial program ABAQUS [9]. During the analysis it was assumed that the crack surfaces are loaded with constant lubricant pressure. Experiments [10] showed that initial surface crack subjected to rolling-sliding contact and constant internal pressure propagates under steep angle θ_0 to the free surface (Figure 2).

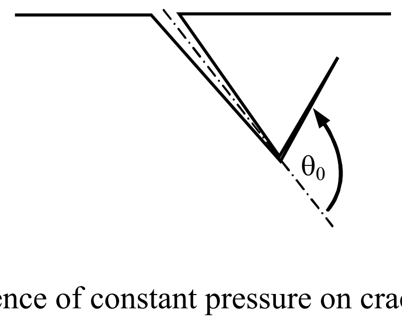


Figure 2: Influence of constant pressure on crack propagation [10]

Due to change of the stress intensity factors K_I, K_{II} and T-stress during motion of the contact-sliding loading it is assumed that the maximum values of K_I, K_{II} and T, which occur when the crack mouth just enters the contact zone in every loading cycle, have the most significant influence on the crack propagation path [6].

PRACTICAL EXAMPLE

Crack propagation was analysed with the model of initial surface crack subjected to lubricated rolling-sliding contact. The real contact geometry of gear tooth flanks can be transformed into a pair of equivalent contacting cylinders with the radii corresponding to curvature radii of analysed mechanical elements [11]. The two cylinders are then further transformed into equivalent contact cylinder of equivalent radius, for which the Hertzian normal contact pressure distribution $p(x)$ can be estimated with simple analytical relationships. The derived equivalent contact model has the following geometrical data: the cylinder of radius R_1 = 10.285 mm corresponds to the radius of pinion at the inner point of single teeth pair engagement with number of teeth $z_{1/2}$ = 16/24, gear module m = 4.5 mm, centre distance a = 91.5 mm, addendum modification coefficients $x_{1/2}$ = 0.18/0.17 and standard gear profile angle α_n = 20°. The pinion is made of carburised steel 16MnCr5 (according to the ISO standard) with Young's modulus E = 206 GPa and Poisson's ratio ν = 0.3, plane strain fracture toughness of 83.8 MPam$^{1/2}$ and yield stress of 900 MPa (assuming no cyclic hardening or softening). The Hertzian contact pressure distribution $p(x)$ with a maximum value p_0 = 1550 MPa, and the half-length of the contact area, b = 0.1987 mm, have been estimated by using the Hertzian contact theory [12].

Coefficients of friction μ = 0.04 was used in simulation which is representative for real gear meshing, depending on the roughness of the surface, lubricant viscosity, relative sliding, etc. [1] The tangential loading $q(x)$ has been determined using simply Coulomb friction law.

Initial length of the crack was equal to a_o = 20 μm, with the initial inclination angle towards the contact surface equal to β = 20°. Orientation and length of the initial crack follows from the metallographic examination of initial cracks appearing in gears made of the same material [1]. The constant pressure distribution along the crack surfaces was determined according to assumption that it is equal to that at the crack mouth, while the moving contact was simulated with five different loading configurations I to V (Figure 3).

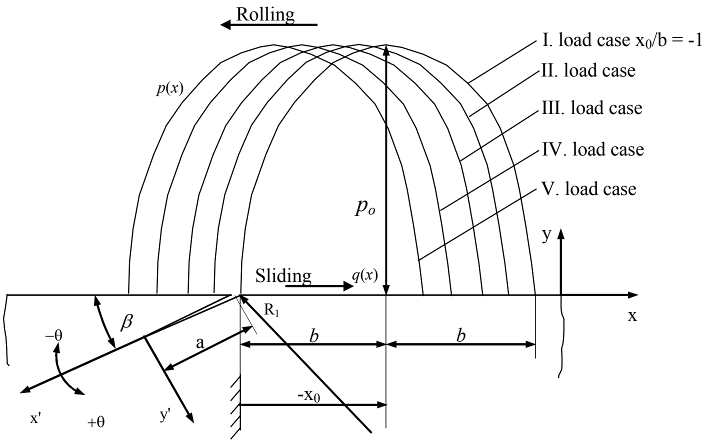


Figure 3: Simulation of the moving contact

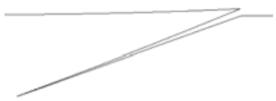
The equivalent contact model was discretised with standard quadratic quadrilateral isoparametric elements, while 24 collapsed quadrilateral quarter point finite elements were used around the crack tip to simulate $r^{-1/2}$ stress singularity and $r^{1/2}$ displacement variation at the crack tip.

There is no clear consensus on how to determine the critical distance r_c for cracks as small as 20 μm in a brittle material (e. g. cracks in flame hardened gear tooth flank layer). It was assumed that critical distance is constant due to crack propagation. Due to available data for plane strain fracture toughness of 83.8 $MPam^{1/2}$ and yield stress of 900 MPa (assuming no cyclic hardening or softening), the critical distance of r_c 2 μm was determined from the model of Larsson and Carlsson [8].

RESULTS AND DISCUSSION

Results in Table 1 show deformed initial crack with results for stress intensity factor K_I, K_{II}, T-stress and corresponding crack propagation angle for load case II, which was critical in all analyses.

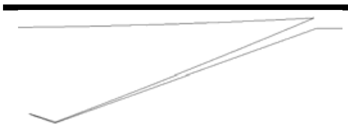
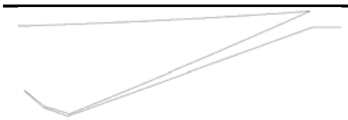
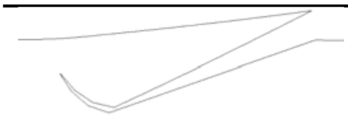
Table 1: Stress intensity factor K_I, K_{II}, T-stress and kink angle for initial crack

Crack shape	a [μm]	Load case	K_I [$MPam^{1/2}$]	K_{II} [$MPam^{1/2}$]	T [MPa]	θ_0 [°]
		I	0.1	0.03	-30	
		II	8.85	4.18	2158	-39 (-72)
	20	III	8.48	3.25	1638	
		IV	7.48	2.19	1121	
		V	6.11	1.35	815	

Results in Table 1 show that maximal values of K_I, K_{II} and T occur for load case II. Initial crack of 20 μm propagates in direction θ_0 = -39° in the local coordinate system x', y' if only stress intensity factors are considered (r_c = 0 μm) for determination of kink angle in eq. (3). Considering of T-stress and critical distance r_c = 2 μm in eq. (3) leads to the crack propagation angle of θ_0 = -72°.

Table 2 presents results for stress intensity factor K_I, K_{II}, T-stress and corresponding crack propagation angle θ_0 for load case II in case when crack propagation angle was determined regard to stress intensity factor K_I and stress intensity factor K_{II}.

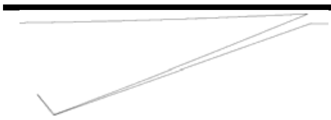
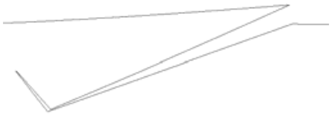
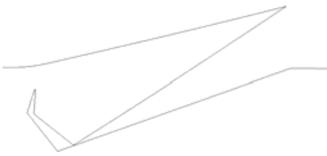
Table 2: Stress intensity factor K_I, K_{II}, T-stress and kink angle θ_0 for propagated crack

Crack shape	a [μm]	Load case	K_I [MPam$^{1/2}$]	K_{II} [MPam$^{1/2}$]	T [MPa]	θ_0 [°]
		I	0.08	-0.04	15	
		II	15.67	3.29	2066	-22
	22	III	14.28	2.20	1678	
		IV	11.93	1.13	1223	
		V	9.28	0.46	1090	
		I	0.04	-0.04	38	
		II	26.19	3.52	3441	-15
	24	III	23.58	2.25	2900	
		IV	19.43	1.07	2292	
		V	14.91	0.41	1860	
		I	0.01	-0.02	28	
		II	53.32	5.66	8133	-12
	26	III	47.91	3.96	7030	
		IV	39.40	2.3	5603	
		V	30.18	1.3	4339	

Results in Table 2 show that crack propagates to the free surface. Stress intensity factor K_I, K_{II} and T-stress increases with crack length except in case of first crack extension, where stress intensity factor K_{II} and T-stress are smaller than for initial crack length. This is due to influence of crack orientation.

Table 3 presents results for stress intensity factor K_I, K_{II}, T-stress and corresponding crack propagation angle for load case II when crack propagation angle was determined regard to stress intensity factor K_I, stress intensity factor $K_{II,}$ T-stress and critical distance r_c.

Table 3: Stress intensity factor K_I, K_{II}, T-stress and kink angle θ_0 for propagated crack

Crack shape	a [μm]	Load case	K_I [MPam$^{1/2}$]	K_{II} [MPam$^{1/2}$]	T [MPa]	θ_0 [°]
		I	0.03	-0.04	40	
		II	17.64	-0.02	1415	0
	22	III	16.63	-0.73	1398	
		IV	12.95	-1.18	1220	
		V	9.68	-1.08	1083	
		I	0.01	-0.03	33	
		II	32.3	3.28	4516	-55
	24	III	30.88	2.2	4101	
		IV	24.51	0.95	3149	
		V	18.43	0.49	2497	
		I	0	0	-1	
		II	82.58	-16.35	18642	68
	26	III	75.26	-16.15	17583	
		IV	56.96	-13.15	13791	
		V	42.68	-9.8	10326	

Results in Table 3 show that crack propagates to the free surface. It is seen from pictures in Table 3 that crack propagates steep to the free surface. Results for kink angle show that crack propagates in straight line after first kink. After 3rd extension (a = 26 μm) it tries to propagate anticlockwise (68°). This is due to short ligament which can not sustain the load, what leads to tearing of material.
Comparison with experimental results in Figure 2 show that crack propagation path in lubricated rolling-sliding contact is better determined when T-stress is considered in the analysis.

CONCLUSIONS

The paper is concerned with the influence of different terms in asymptotic stress field around the crack tip on crack propagation angle relative to the pre-existing initial surface breaking crack subjected to lubricated rolling-sliding contact conditions. Crack propagation angle was determined with generalized MTS criterion, which based on asymptotic stress field that comprises the stress intensity factors K_I, K_{II}, the T- stress, the critical distance r_c and tractions on crack surfaces caused by pressure trapped inside the crack. The developed criterion is valid only for crack faces loaded with constant pressure due to fluid trapped in the crack. The criterion is applied to a problem of short, surface breaking crack propagation on gear teeth contact surface of a real gear pair. The equivalent Hertzian contact model was used for determination of normal contact pressure distribution in the contact area. Tangential contact forces were simulated by frictional forces, while the influence of the fluid trapped inside the crack was modelled with a constant pressure distribution along the crack faces. The moving contact loading was simulated with five different load cases. Stress intensity factors K_I, K_{II} and T-stress were extracted with contour integral method, which is implemented in commercial program ABAQUS[9]. For the model, which considers that the surface traction is opposite to that of movement of contact pressure, the comparative computational analyses have shown that the largest crack propagation angles are estimated with generalized MTS criterion, which better agrees with available experimental data [10]. The fracture criterion developed in the paper depend on the critical distance r_c, which is a material parameter and presents the size of the fracture process zone. Critical distance r_c has a significant effect on the crack initiation angle [4]. For accurate determination of the crack propagation angle in metallic gears, appropriate value of parameter r_c should be evaluated from experimental tests.

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