Prediction of Crack Pattern in Reinforced Concrete Members Under in-Plane Stresses

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ABSTRACT. *A consistent constitutive model for two-dimensional nonlinear analysis of reinforced concrete structures is presented. The model is able to describe the behavior in uncracked stage, as well as in singly and doubly cracked stages. By schematizing the behavior of concrete and reinforcement between adjacent cracks as that of two springs working in parallel, while these and the phenomena which are generated in crack (as tension stiffening, dowel action, aggregate interlock, etc.) as springs working in series, secant stiffness matrix is obtained in direct mode. The reliability and capability of the proposed constitutive model are proved by analyzing results of well-known tests on plane stress elements, with particular reference to those which collapse when secondary crack forms.*

INTRODUCTION

The prediction of crack pattern, and its evolution as loading increases, in reinforced concrete (R/C) members subjected to in-plane stress field, with prevalence of tension and shear, is a very complex problem [1-3]. Moreover, a detailed and deep knowledge of it is fundamental in order to formulate an effective model for realistic structural analyses [4-10]. When the first cracks (primary cracks) form, crack pattern shows few cracks with orientation depending on stress field and on spacing and arrangement of reinforcing bars. Normal and shear stresses transfer across a cracks through complex phenomena, as aggregate interlock and confinement actions, aggregate bridging effect, dowel-action, tension-stiffening and kinking effects of steel bars, etc. Material discontinuities due to cracks cause a deep change of stress and strain fields in concrete and in reinforcing steel respect to those of uncracked R/C stage. So, as loading increases, new cracks (secondary cracks) can form oriented along any direction with respect to that of primary cracks, showing decrasing spacings.

In this paper a macroscopic model (PARC-2D) [10], based on realistic semi-empirical constitutive laws for concrete, for reinforcing steel and for their interaction at the crack, and which is able to simulate the evolution of the crack pattern of in-plane stress R/C members, is presented. Through a nonlinear analysis progressive up to failure, the model takes into account the parameters influencing the primary cracking, that is stress field, orientation and spacing of the reinforcing steel bars, and the parameters that govern the subsequent secondary cracking, that is, in addition to those already stated, bond between bars and concrete, dowel action, aggregate bridging, aggregate interlock, degradation of concrete between cracks. In order to verify the reliability and capability of the proposed approach, a comparison with observations of a well-documented experimental test program [2-4] is reported.

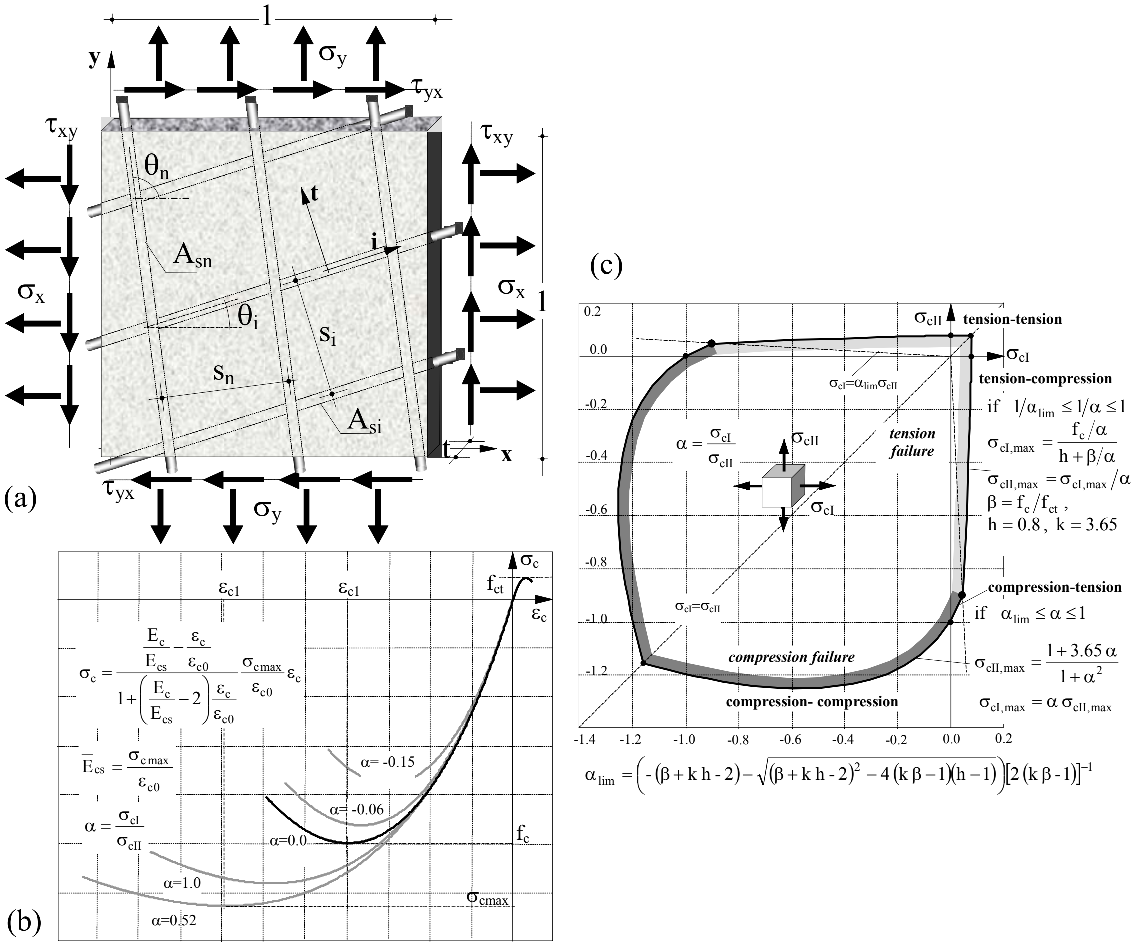

Figure 1. (a) R/C membrane element: geometry and notation, (b) equivalent uniaxial curves for tension and compression, (c) biaxial strength envelope.

MODELING OF REINFORCED CONCRETE

Uncracked stage

A concrete membrane element, with thickness t, reinforced by ordinary steel bars, arranged in i-series, characterised by direction θ_i , cross-section area A_{si} , spacing s_i , smeared through the geometric steel ratio $\rho_i=A_{si}/(s_i \cdot t)$, (i=1,n), is analysed (Fig.1a).

In uncracked stage, perfect bond is hypothesized between concrete and steel and so they are subjected to the same strain field. In the global x-y co-ordinate system (Fig.1a) it results:

$$\{\varepsilon_{cxy}\} = \{\varepsilon_{sxy}\} = \{\varepsilon_{xy}\} \quad , \tag{1}$$

where: $\{\varepsilon_{cxy}\}$, $\{\varepsilon_{sxy}\}$, $\{\varepsilon_{xy}\}$ are the strain field of concrete, of the steel reinforcement and of the global reinforced concrete (R/C), respectively. The total stress field in R/C, $\{\sigma_{xy}\}$, is evaluated by the addition of stress in the concrete $\{\sigma_{cxy}\}$ and that in the steel $\{\sigma_{sxy}\}$ and, by introducing the constitutive laws, where $[D_{cxy}]$ is the concrete stiffness matrix , $[D_{sxy}]$ is the steel stiffness matrix, the global uncracked R/C stiffness matrix $[D_{xy}]$ is obtained:

$$\{\sigma_{xy}\} = \{\sigma_{cxy}\} + \{\sigma_{sxy}\} = ([D_{cxy}] + [D_{sxy}]) \ \{\varepsilon_{xy}\} = [D_{xy}] \ \{\varepsilon_{xy}\} \ . \tag{2}$$

$[D_{cxy}]$ is proposed in [9,10] defined in the principal strain directions and then transferred into x-y coordinate system, $[D_{cxy}] = [T_\varphi]^T [D_{cI,II}][T_\varphi]$, $[T_\varphi]$ being the transformation matrix, function of the φ angle between the x-axis and the maximum principal strain direction, (Fig.5a), and $[D_{cI,II}]$ being the uncracked concrete matrix in principal directions.

In the local co-ordinate system of each i-th steel bar series (Fig.5b), $[D_{s_i}]$ is evaluated taking into account axial and shear stiffness and strength of steel bars, and mechanical behavior described by elastic-hardening constitutive laws. Reinforcing stiffness matrix is determined adding the contribution of each of n_s series of reinforcement steel bars:

$$\underset{(3\times3)}{[D_{sxy}]} = \sum_{i=1}^{n_s} [D_{s_i xy}] = \sum_{i=1}^{n_s} \underset{(3\times2)}{[T_{s_i}]^T} \ \underset{(2\times2)}{[D_{s_i}]} \ \underset{(2\times3)}{[T_{s_i}]}, \tag{3}$$

being $[T_{s_i}]$ the transformation matrix, function of the θ_i angle (Fig.5b).

Singly cracked stage

With reference to a singly cracked R/C membrane element (Fig.2a), by hypothesizing a stabilized cracking stage, cracks are assumed fixed and with constant spacing a_m, oriented at right angle with respect to the maximum principal stress direction corresponding to first cracking, individuated by the ψ_1 angle (Fig.5c). The behavior of R/C between two adjacent cracks, where the concrete is integer even if degraded, and that of the crack (that is all phenomena that occur in the crack process zone) are separately modeled: both the “materials” are subjected to stress fields, $\{\sigma'_{xy}\}$ and $\{\sigma^{cr1}_{xy}\}$

respectively, in equilibrium with the imposed external stress $\{\sigma_{xy}\}$. Total strain $\{\varepsilon_{xy}\}$ can be evaluated by the addition of the strain concrete between cracks $\{\varepsilon'_{cxy}\}$ and that of the concrete in the crack $\{\varepsilon^{cr1}_{cxy}\}$:

$$\{\varepsilon_{xy}\}=\{\varepsilon_{cxy}\}=\{\varepsilon'_{cxy}\}+\{\varepsilon^{cr1}_{xy}\} \tag{4}$$

where $\underset{(3\times1)}{\{\varepsilon^{cr1}_{xy}\}}=\underset{(3\times2)}{[T_{\psi_1}]^{-1}}\underset{(2\times1)}{\{\varepsilon^{cr1}_{12}\}}$ and $\{\varepsilon^{cr1}_{12}\}=\begin{Bmatrix}\dfrac{w_1}{a_{m1}} & \dfrac{v_1}{a_{m1}}\end{Bmatrix}^T$ where w_1 and v_1 are the opening and sliding of crack surfaces, respectively (Fig.2b).

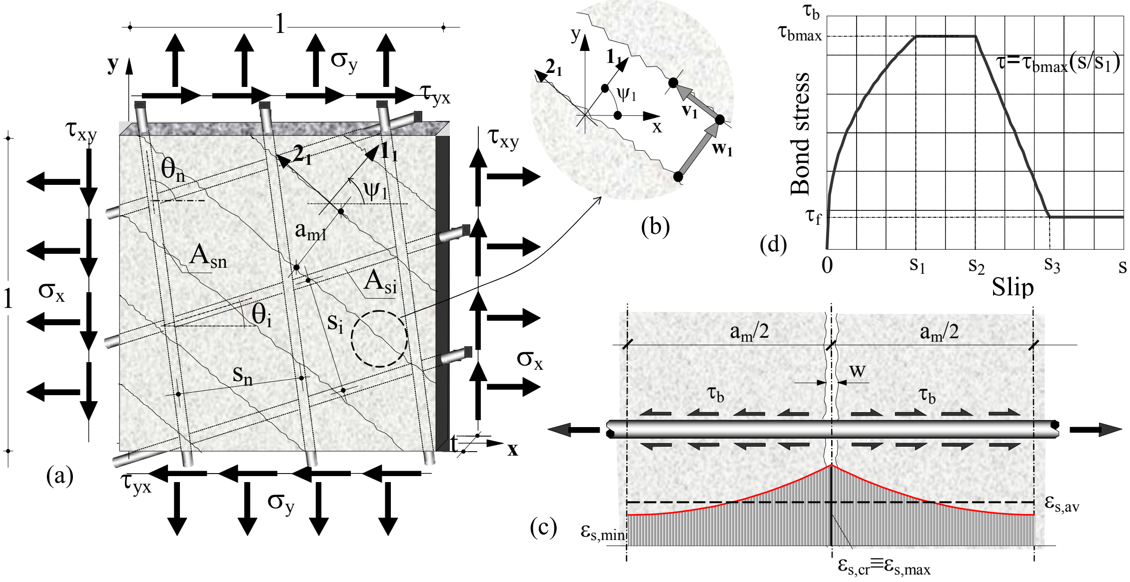

Figure 2. (a) R/C membrane element in the singly cracked stage: geometry and notation, (b) kinematical parameters of crack, (c) adopted bond relationship, (d) stabilized cracking stage: shear bond stress, tensile stress and strain of steel bar.

At the crack the stress field is transmitted by axial stiffness and dowel action of the steel bars crossing the crack, in addition to the bridging and interlock actions of concrete aggregate. R/C between cracks transmits the stress field by the concrete and by axial resistance of the steel bars embedded in the concrete. The strain of the steel in concrete between cracks $\{\varepsilon'_{sxy}\}$ is assumed equal to the global average strain of the steel $\{\varepsilon_{xy}\}$ bar evaluated for the whole element.

Therefore, it follows:

in the crack :

$$\{\sigma^{cr1}_{xy}\}=\{\sigma_{xy}\}=\{\sigma^{cr1}_{cxy}\}+\{\sigma^{cr1}_{sxy}\}=\left([D^{cr1}_{cxy}]+[D^{cr1}_{sxy}]\right)\{\varepsilon^{cr1}_{c}\}=[D^{cr1}_{xy}]\{\varepsilon^{cr1}_{c}\} \tag{5}$$

in the concrete between cracks:

$$\left\{\sigma'_{xy}\right\}=\left\{\sigma_{xy}\right\}=\left\{\sigma'_{cxy}\right\}+\left\{\sigma'_{sxy}\right\}=\left[D'_{cxy}\right]\left\{\varepsilon'_{cxy}\right\}+\left[D'_{sxy}\right]\left\{\varepsilon_{xy}\right\} \tag{6}$$

where $\left[D^{cr1}_{cxy}\right]$ and $\left[D^{cr1}_{sxy}\right]$ are the stiffening matrix due to bridging and interlock of concrete aggregate effects and due to axial and dowel actions of the steel bars crossing the crack, respectively, while $\left[D'_{cxy}\right]$ and $\left[D'_{sxy}\right]$ are the stiffening matrix of R/C between two adjacent cracks and of axial behaviour of steel bars embedded in uncracked concrete, respectively.

From the previous equations, the total strain is (being [I] the unitary matrix):

$$\left\{\varepsilon_{xy}\right\}=\left(\left[I\right]+\left[D'_{cxy}\right]^{-1}\left[D'_{sxy}\right]\right)^{-1}\left(\left[D'_{cxy}\right]^{-1}+\left[D^{cr1}_{xy}\right]^{-1}\right)\left\{\sigma_{xy}\right\}=\left[D'_{xy}\right]^{-1}\left\{\sigma_{xy}\right\}, \tag{7}$$

where the flexibility matrix assumes the form:

$$\left[D'_{xy}\right]^{-1}=\left(\left[I\right]+\left[D'_{cxy}\right]^{-1}\left[D'_{sxy}\right]\right)^{-1}\left(\left[D'_{cxy}\right]^{-1}+\left[D^{cr1}_{xy}\right]^{-1}\right), \tag{8}$$

and then the strain fields in the concrete between adjacent cracks and in the crack are:

$$\left\{\varepsilon'_{cxy}\right\}=\left[D'_{cxy}\right]^{-1}\left\{\sigma'_{cxy}\right\} \text{ and } \left\{\varepsilon^{cr1}_{cxy}\right\}=\left[D^{cr1}_{xy}\right]^{-1}\left\{\sigma^{cr1}_{xy}\right\}. \tag{9}$$

The behavior of concrete between cracks is similar to that assumed in uncracked stage, but it is degraded both in terms of strength both in terms of stiffness through the damage coefficient ζ [2-4], here assumed as: $\zeta=\left(1+200\cdot w/a_m\right)^{-1}$.

In the crack, mechanical phenomena, which provide strength and rigidity, related to opening and slip of the crack surfaces, are generated. Some of these contributions are due to concrete, in particular to the aggregates acting upon the crack, others are due to steel bars which cross the crack. So, the crack stiffness matrix $\left[D^{cr}_{xy}\right]$ is formed by addition of that $\left[D^{cr}_{cxy}\right]$ provided by aggregate contributions and that $\left[D^{cr}_{sxy}\right]$ due the reinforcement contributions:

$$\left[D^{cr}_{xy}\right]=\left[D^{cr}_{cxy}\right]+\left[D^{cr}_{sxy}\right]. \tag{10}$$

In the crack local 1-2 co-ordinate system (Fig.2a,5c), the contribution to crack stiffness matrix due to aggregate bridging and interlock is evaluated as:

$$\underset{(2\times 2)}{\left[D^{cr}_{c12}\right]}=\begin{bmatrix} c_t & -c_v \\ 0 & c_a \end{bmatrix}, \tag{11}$$

where the parameters c_t, c_a, c_v, functions of w and v, are defined in [7,9,10]. In x-y coordinate system it results:

$$\underset{(3\times 3)}{\left[D^{cr}_{cxy}\right]}=\underset{(3\times 2)}{\left[T_\psi\right]^T}\underset{(2\times 2)}{\left[D^{cr}_{c12}\right]}\underset{(2\times 3)}{\left[T_\psi\right]}. \tag{12}$$

The contributions related to steel bars crossing crack, evaluated in the i-t coordinate system (Fig.1a, Fig.5b), are tension stiffening and dowel action.

Being $\{\sigma_{sit}^{cr}\}$ the stress in the steel bars at the crack, it results:

$$\{\sigma_{sit}^{cr}\} = \{\sigma_{si}^{cr} \quad \tau_{sit}^{cr}\} = [D_{sit}^{cr}]\{\varepsilon_{it}^{cr}\}, \tag{13}$$

where: $\{\varepsilon_{it}^{cr}\} = \left\{\frac{\delta_i}{a_{mi}} \quad \frac{\eta_i}{a_{mi}}\right\}^T = [T_{s_i}]\{\varepsilon_{xy}^{cr}\} = [T_{s_i}][T_\psi]^T\{\varepsilon_{c12}^{cr1}\}$, $[D_{s_{it}}^{cr}] = \begin{bmatrix} \rho_{si}\bar{E}_{s_i}^{cr} g_i^{cr} & 0 \\ 0 & \rho_{si} d_i \end{bmatrix}$,

with $g_i^{cr} = \frac{\varepsilon_{s_i}^{cr}}{\delta_i} a_{mi}$ the ratio between steel axial strain ε_{si}^{cr} at the crack and the crack strain computed along i-direction by bond model previously introduced, and $a_{mi} = \frac{a_m}{\cos(\theta_i - \psi_1)}$.

Taking into account the contributions of all steel bar series, crack stiffness matrix due to steel reinforcement, evaluated in x-y coordinate system, assumes the form:

$$\underset{(3\times3)}{[D_{sxy}^{cr}]} = \sum_{i=1}^{n_s} \underset{(3\times2)}{[T_{si}]^T} \underset{(2\times2)}{[D_{s_{it}}^{cr}]} \underset{(2\times3)}{[T_{si}]}. \tag{14}$$

Doubly cracked stage

When a new crack forms, with direction at right angle to ψ_2 (Fig.5d), the flexibility matrix is:

$$[D'_{xy}]^{-1} = \left([I] + [D'_{cxy}]^{-1}[D'_{sxy}]\right)^{-1}\left([D'_{cxy}]^{-1} + [D_{xy}^{cr1}]^{-1} + [D_{xy}^{cr2}]^{-1}\right), \tag{15}$$

with the significance of the symbols and procedures for evaluation completely equal to the case of singly cracked stage.

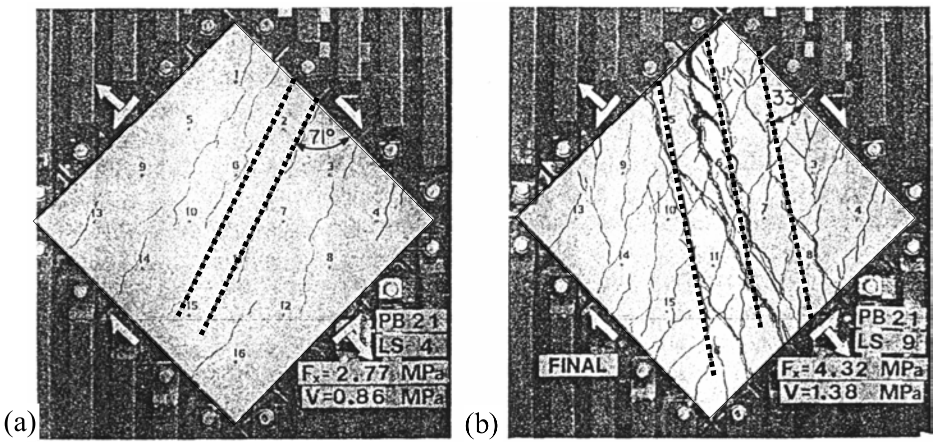

Figure 3. Reinforcement details for element PB21 subjected to combined shear and tension.

COMPARISON WITH EXPERIMENTAL OBSERVATIONS

The capability of the proposed model is highlighted through the analysis of PB21 panel, of a wide well-documented experimental program [4]. The reinforced concrete specimen, 890 mm square × 70 mm thick, containing only longitudinal reinforcement was subjected to combined shear and uniaxial tension with loading ratio 1:3.1. The concrete exhibits f_c=21.8 MPa, f_{ct}=2.4 MPa, ε_{c0}=-0.0018, and a maximum aggregate size of 9.5 mm; the reinforcement consisted of bars with diameter ϕ=6 mm, f_y=402 MPa and steel ratio ρ_s=0.022.

The panel showed a considerable capacity to carry loads in excess of the cracking load. The initial cracks formed close to the direction of principal stresses predicted, at about 71 deg to the reinforcement (Fig.3a). As load was increased, some cracks formed at about 50 deg and then others cracks formed at about 30 deg (Fig.3b); the latter cracks were characterized by a rapid widening that caused the failure of panel. The predicted response, compared with experimental observations, is shown in Fig.4, in terms of shear stress vs. shear strain (Fig.4a), vs. direction of principal stress (Fig.4b), vs. longitudinal strain (Fig.4c), vs. transversal strain (Fig.4d) relationships. By an observation of figures, experimental and numerical curves are in good agreement.

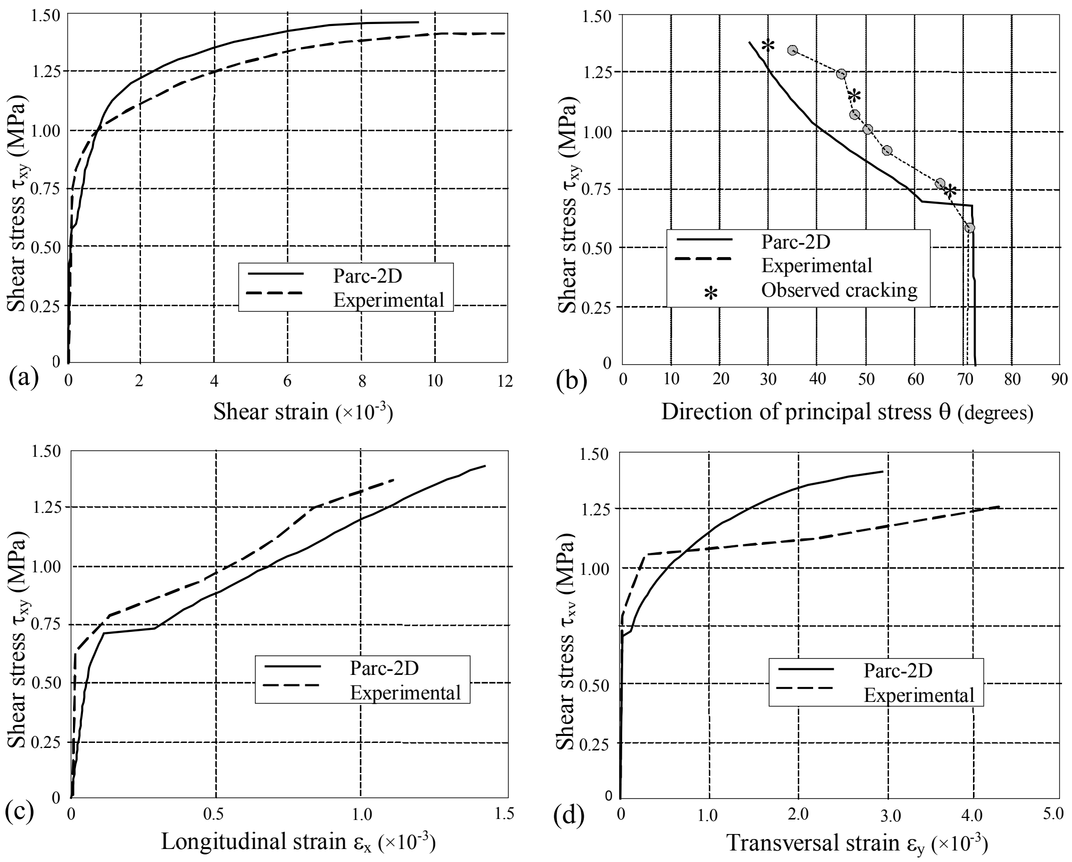

Figure 4. Comparisons between observed and predicted behaviour of specimen PB21.

CONCLUSION

The examined panel, subjected to combined shear and uniaxial tension, is in outlying conditions because reinforcement is arranged only along one direction and shear stress in the crack actives hardly aggregate interlock and dowel actions, producing wide slip of crack surfaces. The proposed model is capable of describing the real behavior of the structural element, not only regard to strain field but it is also able to evaluate the development of crack pattern, from the primary cracking to the ultimate one that yields the failure. This development is strictly connected with the change of principal stress directions in concrete between adjacent cracks.

NOTATION

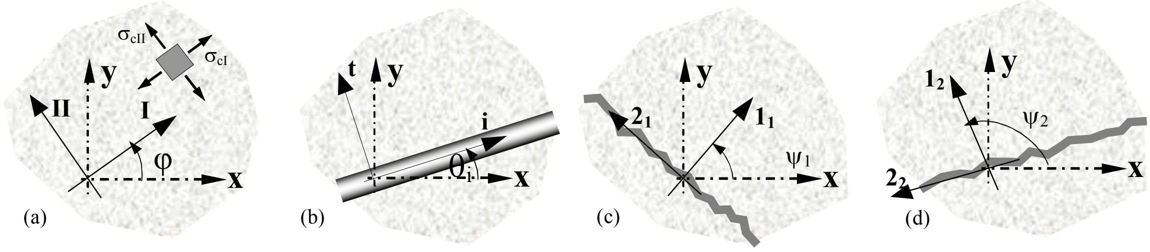

Figure 5. Local co-ordinate systems: (a) principal stress directions for concrete, (b) direction of i-th series of steel bars, (c) (d) at right angle and parallel directions of the first and secondary cracks, respectively.

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