Influence of Small Variations of Initial Defects upon Crack Paths in Creeping Plates

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ABSTRACT. *It has been shown in previous works by the authors that the entire process of failure of a structure can be described within the frame of Continuum Damage Mechanics. It consists of crack initiation, growth (including branching) and proliferation throughout a structure, which terminates structure's lifetime. These characteristic periods are especially well exposed when time dependent behavior of structure's material (e.g. creep) is taken into account. The results of numerical simulation of the process depend, however on a number of factors like assumed constitutive equations for material behavior, values of material constants, and boundary and initial conditions. Due to nonlinear behavior typical for metals in high temperature applications, the solutions are very sensitive to these parameters. In particular, times to reach consecutive stages of failure process can vary essentially.*

In the present contribution the influence of initial distribution of damage variable, which can result from material original heterogeneity or can be artificially introduced, is studied. It is demonstrated that even small variation in these conditions can influence crack paths and – consequently – the time to final failure of a structure, which can be very different from that when zero-value initial conditions are superimposed upon damage scalar parameter.

INTRODUCTION

It is well known that fracture process is of random character induced by different types of heterogeneity attributed to real structures. These heterogeneities can be related to material properties (inclusions, impurities, production flaws, etc.), process of structures assembling, as well as randomly changing environmental conditions (small loadings variation, temperature and chemical aggression). The randomness of the fracture process is sufficiently well recognized and has been already observed at the very beginning of systematical study of such phenomena like fatigue (Pamlgren, [1]) or material toughness (Weibull, [2]). The recent summary on a randomness of fracture can be found e.g. in [3]. However, relatively little is known when creep damage is concerned, and therefore this will be area of the present study.

In the present paper we will restrict ourselves to the study of the influence of initial condition set upon material virgin properties and represented by initial distribution of a state variable responsible for the description of material deterioration. Due to nonlinearity of creep process, even small initial variation can be expected to result in essential changes in such crack features like place of its origination, proliferation and morphology of cracks' set causing the whole structure to fail.

CRACK PATHS IN TERMS OF CDM

Continuum Damage Mechanics (CDM), the term coined by J.Hult in 1978 [4], originated from extensive studies of creep fracture process in the second half of the 20th century with a connection to the development of high temperature engineering applications (mostly – gas turbines and nuclear energy plants). Its fundamental concept is the introduction of damage parameter ω, as a counterpart to material continuity $\psi = 1\text{-}\omega$, as originally suggested by L.M.Kachanov, [5]. This parameter varies in the range of $(0,1)$, where $\omega = 0$ represents an absence of material deterioration, whereas $\omega = 1$ stands for a material failure in a given point. It can be also viewed as a probability of a material fault in a given point, as suggested in [6] with relation to the Weibull's strength theory. Further extensions of a damage concept went towards introduction of tensorial representation of damage of second (Murakami, [7]) and fourth (Chaboche, [8]) order. The extensive review of various damage representations can be found e.g. in [9], [10].

For the purpose of the present study, it will be sufficient to use just scalar damage representations, as directional phenomena will be represented by resulting cracks rather then local directional damage growth.

As crack propagation in structures is an objective of this study, it is useful to establish a global representation for structural damage. In its simplest form it can be represented as an integral of a local damage distribution taken over the structure's volume V and normalized:

$$\Omega = \frac{1}{V}\int_V \omega \, dV$$

with $\Omega = 0$ denoting a virgin, intact structure, and $\Omega = 1$ corresponding to the total strength exhaustion of all structure's points. One has to admit that neither of these extreme conditions may happen in a reality as structure always contains some initial imperfections and flaws, and it comes to the failure by formation of a crack network leading to the structure collapse much earlier than all material points lose its load bearing capacity.

Before we explain how crack can be defined in terms of CDM, let us put emphasis on the splitting the whole fracture process into three stages: crack initiation, propagation and formation of fatal cracks network. The subscript of *1*, *2* and *3* will be used throughout this paper to designate any values to theses stages, whereas the subscript *0* will be reserved for the beginning of the process. These characteristic features can be found in any fracture process but are better recognizable when time scale is expanded by true time-dependent processes like fatigue or creep. In the latter, in time $t = 0$ the values of local and global damage are set to ω_0 and Ω_0 , and the case when these values are set to 0 (flawless state) will be considered as an initial conditions for a reference solution of fracture process in a structure. The time t_1 will correspond to the situation when local damage parameter ω = 1 at least in one point of a structure's body. The value of Ω_1 will be greater then 0, but it can be as low as several percents. The time t_2 will be designated to denote the situation when the points at which ω = 1 along a line or on a surface is spanning a characteristic structure dimension (e.g. plate thickness). This surface can be viewed as initial macrocrack formed at time t_2, when global damage parameter takes the value of Ω_2 <1. At this time instant the crack begins to propagate throughout the structure (with the speed close to the sound speed in the given material) to form the crack network causing a structure to fail at time t_3, when $\Omega_3 \leq 1$. The value of Ω_3 =1 is reserved for rather abstractive situation when local damage reaches value of $\omega = 1$ in all points of a structure. This can be a case of uniaxial tension when localization is not taken into account, and which corresponds to Kachanov's original assumption of zero-value of initial damage in all point of a bar under uniaxial tension. Obviously, this assumption leads to a paradoxical simultaneous loss of load bearing capacity in all points of a structure.

The above systematization goes along that - proposed by Życzkowski [11] for plasticity - of splitting the analysis to the material point, characteristic cross section and the whole structure. It is also in accordance with original definition of so-called "rupture front" introduced already by L.M. Kachanov [5] as a surface on which $\omega = 1$. The propagation of this surface has been a subject of numerous investigations by the authors for different structures subjected to creep and fatigue damage process, but all with a zero-value of initial damage. The main result of these investigations was the observation that the time ratio t_2/t_1 can be dealt with as safety factor for a structure, as it characterizes the time span between first warning of a macro-defect appearance at time t_1 and time of macro-crack formation at time t_2. According to authors deep conviction these two first stages of fracture process can be covered only by damage mechanics, whereas the third stage is undoubtedly the area of fracture mechanics (FM) application, though some attempts of including it into CDM were made by authors also ([12]). For that reason the analysis performed in the preset paper will focus on the evaluation of characteristic time values t_1 , t_2, and its ratio t_2/t_1, corresponding values of global damage Ω_1 , Ω_2, and associated crack paths - as functions of initial local damage distribution ω_0 and Ω_0 .

Finally, let us mention that crack paths may not be necessarily the goal for itself, as the results of its propagation resulting in quantitative values of structures' life time evaluation certainly are of the most desirable by engineers to design safe structures.

CRACK PATHS IN CREEPING PLATE WITH ZERO INITIAL DEFECTS

In this chapter the results of calculation for a rectangular plate with clamped edges, subjected to uniformly distributed pressure on its upper surface will be considered.

The constitutive equations for the creeping plate, which undergoes also progressive deterioration, are:

$$\varepsilon_{ij} = \varepsilon^{e}_{ij} + \varepsilon^{c}_{ij},$$

$$\varepsilon^{e}_{ij} = D^{-1}_{ijkl}\,\sigma_{kl},$$

$$\frac{\partial \varepsilon^{c}_{ij}}{\partial t} = \gamma\left(\frac{\sigma_{eff}}{1-\omega}\right)^{n}\frac{\partial \sigma_{eff}}{\partial \sigma_{ij}},$$

$$\frac{\partial \omega}{\partial t} = A\left[\alpha\frac{\sigma_{\max}}{1-\omega} + \left(1-\alpha\right)\frac{\sigma_{eff}}{1-\omega}\right]^{m},$$

where: ε_{ij}, ε^{e}_{ij}, ε^{c}_{ij} - total, elastic and creep strain tensors, respectively, σ_{ij} - stress tensor, D_{ijkl} - elastic constants matrix, γ, n, A, m - steady-state creep and damage material constants, σ_{max}, σ_{eff} - main positive principal stress and Huber-Mises effective stress, respectively, ω - scalar damage parameter ($0\leq \omega \leq 1$), t - time.

The above constitutive equations can be referred to as non-stationary (including both elastic and creep deformations) creep theory coupled with scalar damage governed by a combination of main positive principal and effective stress. The parameter α in the damage evolution law defines different types of creep failure (α =0 for brittle case governed by main positive principal the stress, α=1 for ductile failure governed be the effective stress, with the intermediate values of α corresponding to mixed modes of failure) is introduced here after the propositions by Sdobyriev [13] and Hayhurst [14].

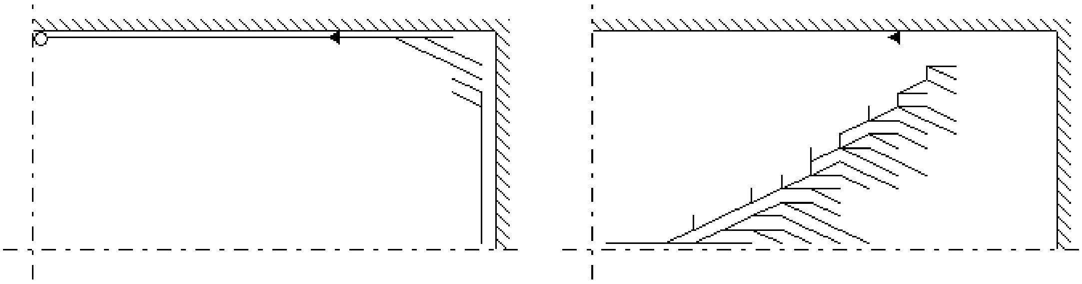
This plate has been studied previously by authors for zero-value initial conditions [15],] and the results are quoted here as a reference for the results with non-zero values of initial conditions reported in the next chapter. The geometry of the plate considered is: side length – 2.0m and 1.0m, plate thickness 0.1m, pressure of 12.51 MPa.

Material constants in the above equations were taken from Walczak [16] for Ti-6Al-2Cr-2M alloy at temperature 675 K, $E = 0.102\cdot 10^{6}$ MPa, $\nu = 0.33$, $n = 6.8$, $B = 1.38\cdot 10^{-21}$ (MPa)$^{-n}$ h^{-1}, $m = 5.79$, $A = 1.08\cdot 10^{-20}$ (MPa)$^{-m}$ h^{-1}, $\alpha=0.5$

Because of problem nonlinearity the calculations were performed numerically by FE method and forward Euler time integration; details of the integration procedures can be found elsewhere [15].

The results of calculation in quantitative terms can be summarized as follows: $t_{10} = 8.709\cdot 10^{4}$ hrs, $t_{20} = 8.709\cdot 10^{4}$ hrs, $t_{20}/t_{10} = 2.26$, $\Omega_{10} = 0.0122$, $\Omega_{20} = 0.3576$, (subscript 0 denotes here zero-initial damage conditions).

The crack paths on lower and upper plate surface are shown in Fig.1a and 1b, (only a quarter of a plate is shown because of symmetry), correspondingly. The lines representing cracks, viewed on the plate lower and upper surface, have to be understood as the lines joining adjacent integration point at which local damage $\omega = 1$. The point of macro-crack initiation at time t_1 is marked with an open circle, whereas the localization of thorough-thickness crack appearance is marked with a full triangle.



(a)

(b)

Fig.1. Crack patterns on upper (a) and lower (b) surfaces of a plate without initial damage

The main observation, which can be made from the analysis of the above example, is that the safety factor represented by t_{20}/t_{10} = 2.26 is rather high. Its value demonstrate an ability of the structure to sustain its load carrying capacity between the time of first warning (crack initiation) and forming of a macro-crack, which will start to propagate thorough structure's body. A characteristic to the most clamped plates analyzed by authors [14] is forming of two main cracks: along clamped edges on the upper plate surface and on the lower surface initiated at the plate center, proceeding alongside its mid-span and then branching towards main crack along longer clamped edge. The coalescence of this crack with the branched one results in forming a through-body crack at time t_2 (full triangle in Fig.1).

The great contrast between characteristic values of global damage parameter Ω_{10} = 0.0122, Ω_{20} = 0.3576, shows that crack initiation is rather localized phenomenon, whereas forming a crack requires a large of deterioration to be accumulated over the time in structure's body.

INFLUENCE OF INITIAL DEFECTS

A variation of both global damage and local one has to be related to their critical values. In the case of global parameter Ω the value of Ω_{20} can been consider as such, so small variations of Ω_0 will be of order of 0.01. In further analysis Ω_0 was set to:

Ω_0 = 0.0035 ; 0.0070 ; 0.00109 ; 0.0145 ; 0.0180

For local initial damage the five values of ω_0 were chosen as follows:

ω_0 = 0.1 ; 0.3 ; 0.5 ; 0.7 ; 0.9

which not necessarily can be viewed as small ones (especially these greater then 0.5), but the aim of the study was also to see how local extreme singularities might affect the final solution, which therefore will be represented by the values of following quantities: t_1 , t_2 , t_2/t_1 , Ω_1 , Ω_2.

The initial conditions were set in such a way that for any of chosen Ω_0 values, the constant value of ω_0 was randomly distributed over the structure's body. The results for all 25 combinations of initial parameters are illustrated by the following figures.

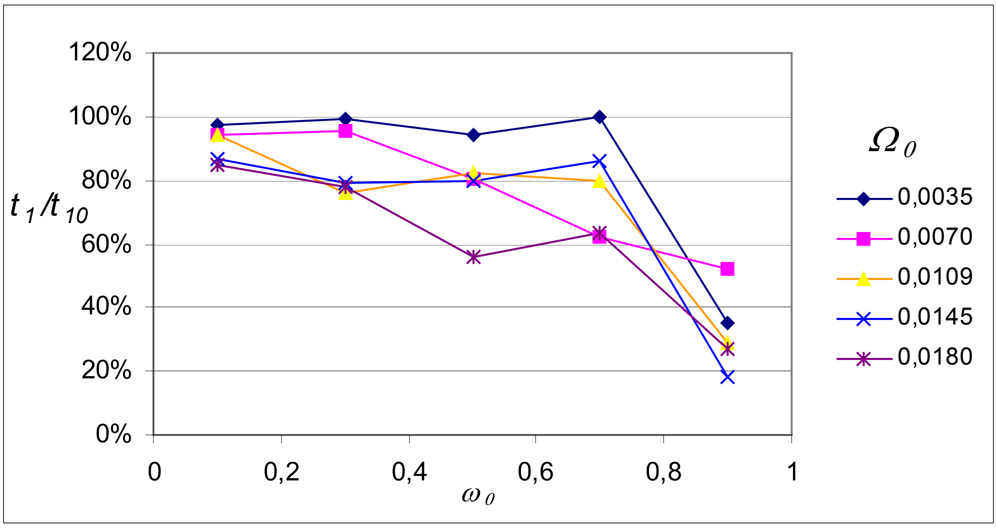


Fig. 2. Time to first crack appearance t_1 as a function of ω_0 for five initial values of Ω_0

Comment: reduction of t_1 in comparison with t_{10} for a plate of zero initial values is almost independent of ω_0 except its value of 0.9, for which reduction is dramatic (up to five-folds). Mean value of reduction is about 30%.

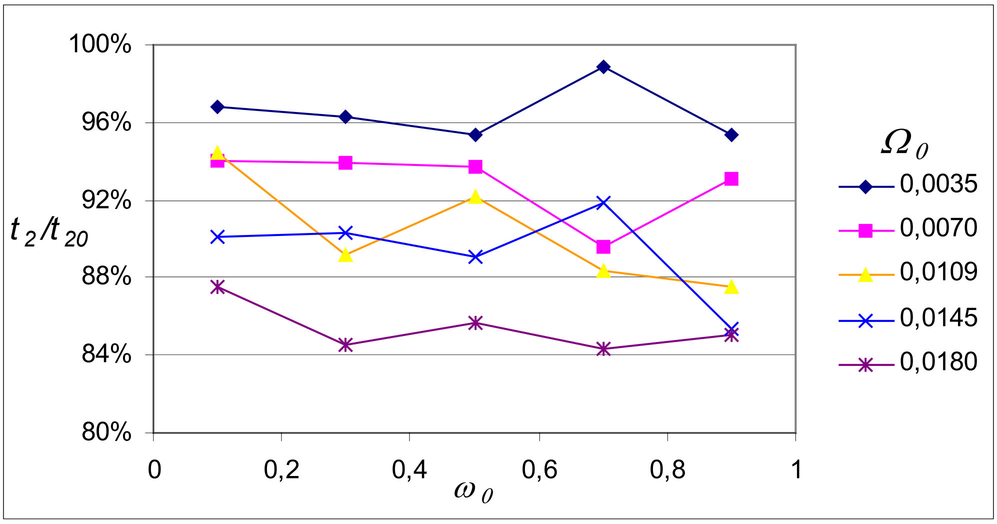


Fig. 3. Time to crack proliferation t_2 as a function of ω_o for five initial values of Ω_o

Comment: reduction of t_2 in comparison with a plate of zero initial values t_{20} is again almost independent of ω_0 , and depends mostly on Ω_0. An average reduction is of about 10%.

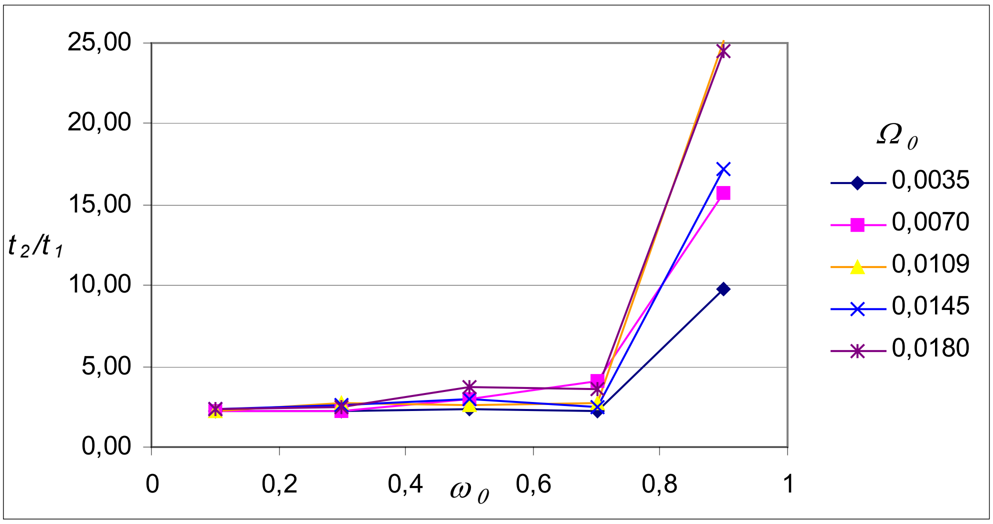


Fig. 4. Ratio of t_2/t_1 (safety factor) as a function of ω_0 for five initial values of Ω_0

Comment: as the result of much higher reduction of t_1 reduction than that of t_2 "safety factor" t_2/t_1 rises almost twofold, except for values of order 10-25 for ω_0 =0.9.

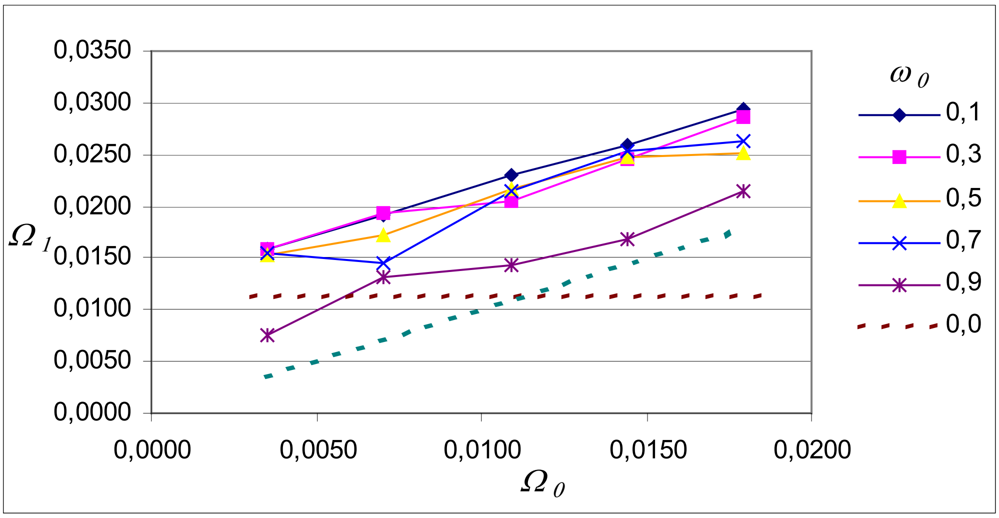


Fig. 5. Value of Ω_1 as a function of Ω_0 for five initial values of ω_0

Comment: It is seen that higher the initial value of Ω_0 higher is the value of global damage accumulated in time t_1 , but the value of accumulated global parameter over its initial value (dotted skew line) is almost constant for any value of ω_0 and lies within the range of its value of 0.01.

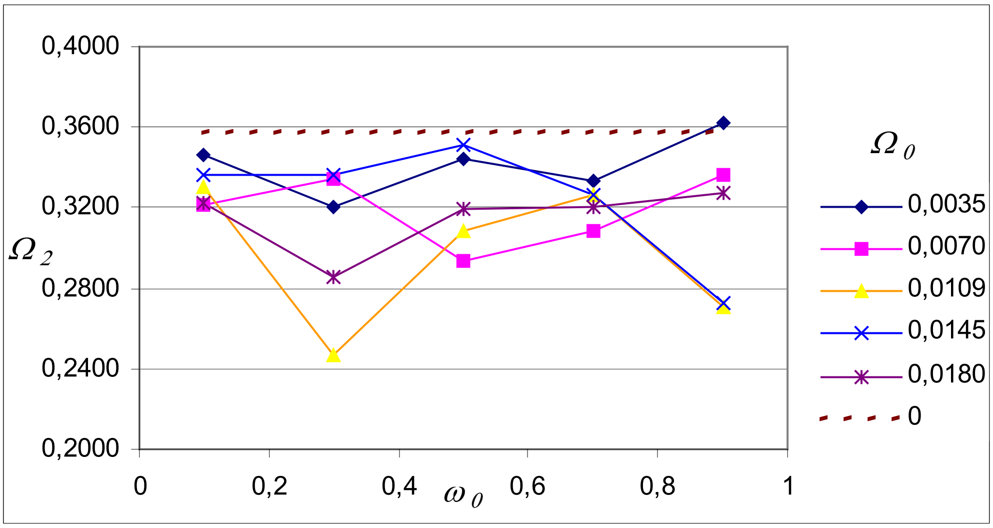


Fig. 6. Value of Ω_2 as a function of ω_0 for five initial values of Ω_0

Comment: For all cases (except that for ω_0 =0.9) Ω_2 is lower than in the case of zero initial values – in contrast to Ω_1 which was always higher. The reduction is also much more significant (from value of 0.3576 for zero-initial case shown by the dotted line – to about 0.28 and even lower) than in the case of Ω_1 which was higher by the amount of about 0.01 over initial value of Ω_0.

CONCLUSIONS

Detailed conclusions were already made by comments on diagrams which illustrated obtained results. The general conclusion can be set as follows:

Random distribution of initial local damages within small changes of initial global damage:

- reduces both times t_1 and t_2, though the reduction of t_1 is much higher then that of t_2. Consequently, the ratio of t_2/t_1, which can be viewed as "safety factor" for a structure, rises significantly.
- causes that the value of global parameter in time t_1 is always greater then that of zero-initial value problem, but only in amount of about 0.01 over initial value of Ω_0. This in contrast with the values of global parameter at t_2 is always less then that of zero-initial value case of order of 30%.

The final conclusion can be set also that the formation of first macro-damage at t_1 is of local character and requires overall deterioration to increase, whereas the first through-crack formation depends more on a global deterioration, and therefore is associated with the drop in the value of global parameter.

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