Determination of Bridging Stress from Bending of a Deep-Notched Specimen

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ABSTRACT. *A method to measure bridging stress by using bending test of a deep-notched specimen is developed in this paper. To obtain the bridging stress from the integral equation which relates the bridging stress to the load, a numerical inverse analysis method is also developed. By using the present method the authors evaluated the grain-bridging stresses of polycrystalline alumina ceramics with the mean grain size of 3 μm an 11 μm. The obtained results can be summarized as follows: (1) the mean grain size had no effect on the maximum bridging stress for alumina, (2) the correlation was not observed between the maximum bridging stress and the bending strength for alumina ceramics.*

INTRODUCTION

Bridging stress occurring in the wake of a crack is effective to increase the fracture toughness of structural materials. For instance, putting fibers into the material its bridging stresses increases and fracture toughness increases. Therefore it is important to have a reliable and easy method to evaluate bridging stress characteristics for the material development.

Several methods to evaluate bridging stress characteristics have been proposed in the past. There methods can be divided broadly into two types. One is to measure the crack concerned. Among this type of methods, bridging stresses can be determined either from crack opening displacement [1-4] or from compliance [5, 6]. This type of methods can be further divided into three different methods in terms of loading conditions [7-9].

In this study, an experimental method of measuring bridging stresses by using the bending test of a deep-notched specimen, shown in Fig. 1, and the inverse analysis of experimental data is developed. In the case of bending of deep-notched specimens, a crack passes through the ligament stably [10]. Bridging materials keep resistance against the load after a crack passed trough the ligament. Bridging stress characteristics can be evaluated from this resistance against the load.

However, this resistance is expressed by an integral from under various opening displacements in this method. Therefore, a numerical inverse analysis is developed to obtain bridging stress characteristics from the integral equation.

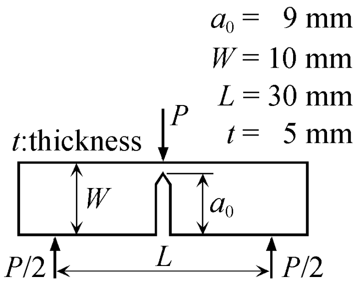


Fig. 1 Deep-notched specimen

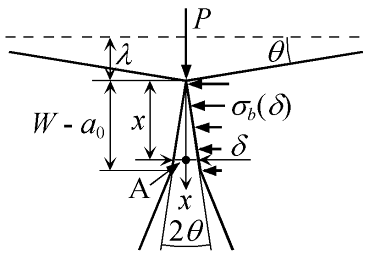


Fig. 2 Bridging stress

RELATIONSHIP BETWEEN LOAD AND BRIDGING STRESS

When a deep-notched specimen, as shown in Fig.1, is loaded, a crack passes through the ligament stably [10]. The elastic strain energy kept in an apparatus and specimen before a crack propagates in very small, because the load to propagate a crack from a deep notch is very small. The elastic strain energy released by propagation of a crack is smaller than the energy to make a crack propagate. Therefore a crack propagates through the ligament stably.

Bridging materials keep resistance against the load after a crack passes through the ligament. This resistance is from bridging stresses, namely, the moment due to the load equals the moment due to the bridging stress after a crack passes through the ligament. Thus, we can obtain the relationship between bridging stress $\sigma_b(\delta)$ and opening displacement δ from the relationship between load $P(\lambda)$ and displacement λ at the loading point.

Let L be the support distance of a specimen, the moment M due to the load becomes

$$M = \frac{P(\lambda)}{2} \cdot \frac{L}{2} = \frac{P(\lambda)L}{4} \tag{1}$$

The moment M due to the bridging stress $\sigma_b(\delta)$ can be expressed as follows:

$$M = \int_0^{W-a_0} \sigma_b(\delta) \cdot x \cdot t \cdot dx \tag{2}$$

where $\sigma_b(\delta)$ is the stress distribution along the fracture surfaces (see Fig. 2) , x is the distance from the loading point A. δ is the opening displacement of fracture surface at the point, and t is the thickness of a specimen.

From Eqs. (1) and (2), one can obtain the following relationship between the load $P(\lambda)$ and the bridging stress $\sigma_b(\delta)$:

$$P(\lambda) = \frac{4t}{L} \int_0^{W-a_0} \sigma_b(\delta) \cdot x \cdot dx \tag{3}$$

Let $\theta\ (= 2\lambda / L)$ be the inclined angle of the specimen as shown in Fig. 2, the opening angle of fracture becomes 2θ . The opening displacement δ at a point A $2\theta \cdot x$. The value of δ can be expressed by λ and x as follows:

$$\delta = 2\theta \cdot x = \frac{4\lambda}{L}x \tag{4}$$

Substituting Eq. (4) into Eq. (3), the relationship between $P(\lambda)$ and $\sigma_b(\delta)$ becomes

$$P(\lambda) = \frac{tL}{4\lambda^2}\int_0^{\frac{4\lambda}{L}(W-a_0)} \sigma_b(\delta)\cdot\delta\cdot d\delta \tag{5}$$

It is difficult, if not impossible, to solve Eq. (5) analytically to determine the bridging stress distribution $\sigma_b(\delta)$. Therefore we propose anumerical inverse analysis method to solve Eq. (5) for $\sigma_b(\delta)$ based on experimental data of $P(\lambda)$ in the bridging section.

INVERSE ANALYSIS METHOD

From bending test, we can obtain the loads $P(\lambda)$ and the displacement λ at the loading point. Using this information, we can obtain the bridging stress from Eq. (5) numerically. In what follows, the numerical inverse analysis used to determine the bridging stress is descrived.

As shown in Fig. 3, bridging stress characteristics is first approximated by a polygonal line function. The total element number is m and the opening displacement is zero when $\delta = \delta_C$. The magnitude of bridging stress at the point of δ_1, δ_2, ..., δ_m are s_1, s_2, ..., s_m, which are the unknowns to be obtained.

Polygonal lines as shown in Fig. 3 (a) are expressed by superposition of m number of trianglar function in Fig. 3 (b). Each triangular function is multiplication of magnitude of bridging stress at bending point and unit weight functions given by (see Fig. 4)

$$f_i(\delta) = \begin{cases} 0 & \left[\delta \le \frac{\delta_C}{m}(i-2)\,,\ \frac{\delta_C}{m}i \le \delta\right] \\ -\frac{m}{\delta_C}\delta + i & \left[\frac{\delta_C}{m}(i-1) \le \delta \le \frac{\delta_C}{m}i\right] \\ \frac{m}{\delta_C}\delta - i + 2 & \left[\frac{\delta_C}{m}(i-2) \le \delta \le \frac{\delta_C}{m}(i-1)\right] \end{cases} \tag{6}$$

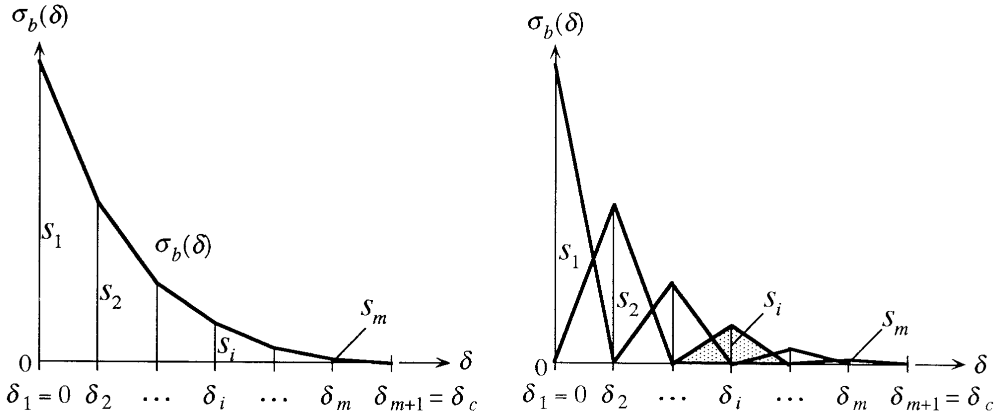
Hence the load $P(\lambda)$ in the experiment can be expressed apploximately as follows:

$$P(\lambda) = \sum_{i=1}^{m} \frac{tL}{4\lambda^2}\int_0^{\frac{4\lambda}{L}(W-a_0)} s_i \cdot f_i(\delta)\cdot\delta\cdot d\delta \tag{7}$$

Let $P_1(l_1)$, $P_2(l_2)$, ..., $P_n(l_n)$, $(n \ge m)$ be the measured data of $P(\lambda)$. From Eq. (7), one obtains

$$p_1(\lambda_1) = \sum_{i=1}^{m} s_i R_{1,i}, \quad p_2(\lambda_2) = \sum_{i=1}^{m} s_i R_{2,i}, \quad \cdots, \quad p_n(\lambda_n) = \sum_{i=1}^{m} s_i R_{n,i} \tag{8}$$

where $R_{j,i}$ is given by



(a) Bridging stress approximated by a polygonal line function   (b) Combination of unit weight functions

Fig. 3 Approximated expression of bridging stress

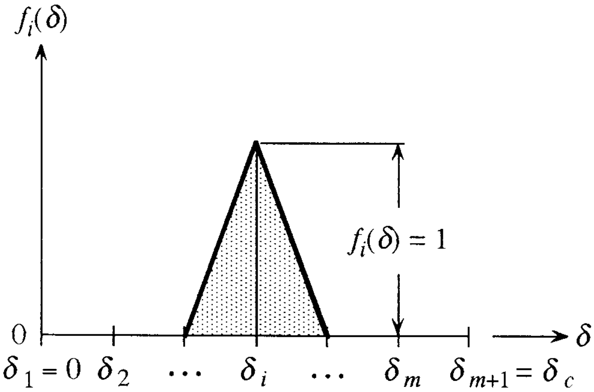


Fig. 4 Unit weight function $f_i(\delta)$

$$R_{j,i} = \frac{tL}{4\lambda_j^2} \int_0^{\frac{4\lambda}{L}(W-a_0)} f_i(\delta) \cdot \delta \cdot d\delta \tag{9}$$

The coefficients, $R_{j,i}$ can be obtained analytically.

Solutuons of least square approximation of s_i in Eq. (8) can be obtaioned by using Householder transformation [11].

EXPERIMENT

The specimen configuration is shown in Fig. 1. The specimen is ground into a shape of 5 mm × 10 mm × 35 mm, and is notched by grinding wheel with 60° point angle so that the ligament of the specimen is 1.0 mm.

The specimen is made of alumina ceramics with mean grain size of 3 μm or 11 μm. Different mean grain sizes in the specimens come from different sintering temperatures and sintering times. Sintering conditions for the alumina ceramics with of 3 μm and 11 μm are 1490 C° and 5 hours, and 1650 C° and 8 hours, respectively.

The experiments were performed in the room temperature. The crosshead speed was about 15 μm/min. In the three-point bending test, the measured displacement at the loading point after the crack has passed through the ligament consists of the displacement to open the fracture surface and the elastic deformations of the specimen and the apparatus. We have to remove the elastic deformations from the measured displacement at the loading point to find the opening angle of the fracture surfaces. Therefore the three-point bending test is performed by using the load-unloading method in this study.

RESULTS

Figure 5 shows the relationship between the load $P(\lambda)$ and displacement λ at the loading point measured by the loading-unloading method in the three-point bending test of the alumina ceramics with the mean grain size of 3 μm. In this figure, the sign ∘ represents points at whitch either the displacement starts to return or the load has returned to zero.

As can be seen from Fig. 5, after the crack propagates through the ligament, the load $P(\lambda)$ starts to decrease rapidly at point p_1. Then the displacement starts to return at point p_2. The pure displacement λ_2 corresponding to the load p_2 can be determined when the measured displacement are returned to a position with zero load, namely, $P(\lambda)=0$. The other pure displacement from λ_3 to λ_9 corresponding to the load from p_3 to p_9 can be obtained in the same manner.

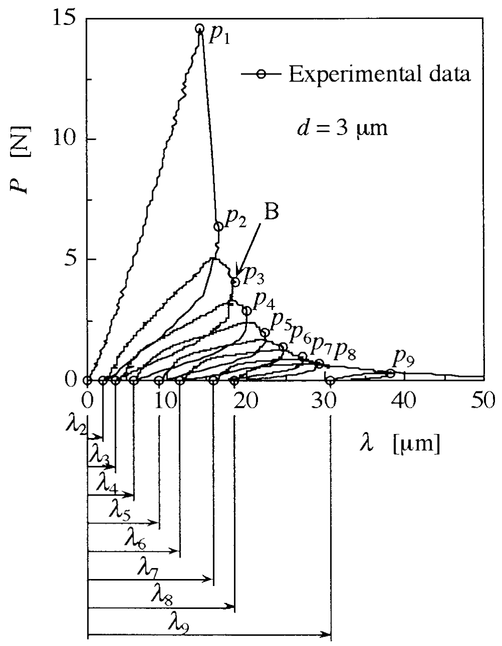


Fig. 5 Load $P(\lambda)$ of alumina ceramics with the mean grain size of $d = 3$ μm

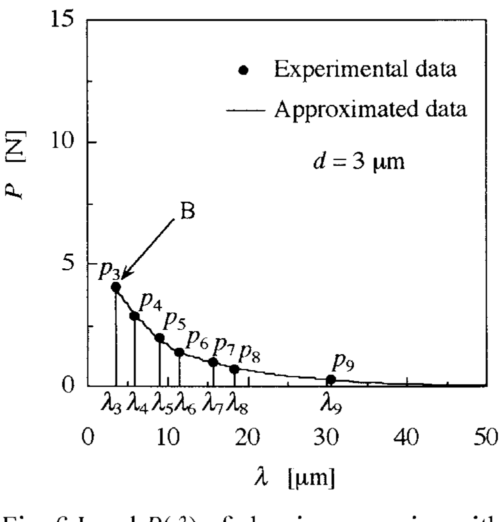


Fig. 6 Load $P(\lambda)$ of alumina ceramics with the mean grain size of $d = 3$ μm

The bridging stress characteristics are evaluated from the obtained relationship between p and λ. However before that, we must determine the point at which the crack has passed through the ligament, because the loads are fully supported by the bridging stresses only after the crack has passed through the ligament. In this study, a point at which the crack at each point from λ_1 to λ_9 in Fig. 5. Consequently, we found that at point B, the crack has passed through the ligament. Then the bridging stress characteristics $\sigma_b(\delta)$ are evaluated by using the relationship between p and λ starting from the point B.

Figure 6 shows the relationship between the pure displacement at the loading point and the load after the crack has passed through the ligament. This relationship is used to evaluate the bridging stress characteristics.

Figure 7 shows the relationship between the load $P(\lambda)$ and displacement λ at the loading point measured by the loading-unloading method on the three-point bending test of the alumina ceramic with the mean grain size of 11 μm. In this figure, the sign ∘ represents points at which either the displacement starts to return or the load has returned to zero, and the sign • represents the relation between the pure displacement and the load.

INVERSE ANALYSIS

In order to evaluate bridging stresses by using the present method discussed in the previous section, a critical opening displacement δ_C and a total element number m of polygonal line must be determined. If m is too large, solution of bridging stress s_i fluctuates badly. If m is too small, variations of bridging stress can not be expressed accurately. In this study, an initial value of 2 for m is chosen. Then m is increased until the evaluated bridging stress characteristic starts to fluctuate badly. Consequently, we have adopted $m = 7$ for mean grain size $d = 3$ μm, and $m = 6$ for $d = 11$ μm.

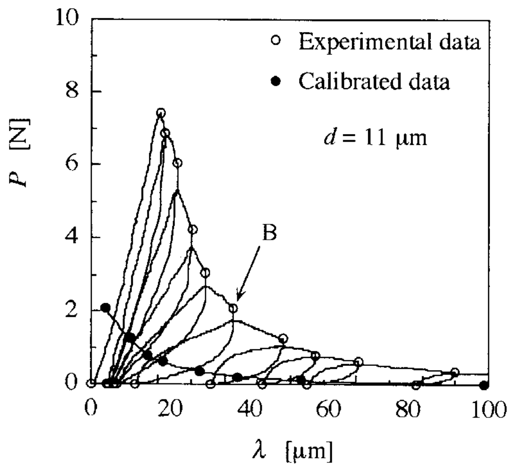


Fig. 7 Load $P(\lambda)$ of alumina ceramics with the mean grain size of $d = 11$ μm

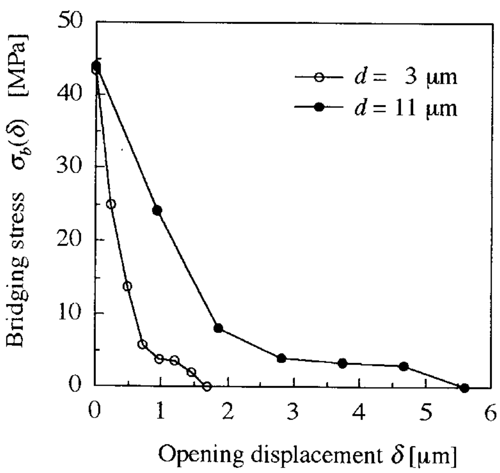


Fig. 8 Bridging stress for alumina ceramics with mean grain sizes of 3 and 11 μm determined by the inverse analysis

In initial value of δ_C is chosen to be a half of the mean grain size, and the critical opening displacement δ_C is determined by trail and error so that the bridging stress charactaristic was a smooth curve. In this study, we adopted $\delta_C = 1.7$ μm, for mean grain size $d = 3$ μm, and $\delta_C = 5.6$ μm for $d = 11$ μm.

Figure 8 shows the results of bridging stress characteristics obtaind by the present method. As can be seen from Fig. 8, there is no difference between the maximum bridging stresses of the specimens with different mean grain sizes.

DISCUSSION

The mean bending strength σ_B of alumina ceramics with the mean grain size 3 μm is 300 MPa, σ_B of alumina ceramics with 11 μm is 218 MPa. The tested alumina ceramics with different mean grain sizes has different the mean bending strength, but almost the same maimum bridging stress. This fact means that the mean bending strength is essentially determined by the size of the defect which is the origin of fracture and the fracture toughness.

In some previous studies, bridging stress characteristic was assumed to have the following expression:

$$\delta_b(\delta) = \sigma_{\max}\left(1 - \frac{\delta}{\delta_C}\right)^n \tag{9}$$

where $\sigma_{\max}$ is the muximum bridging stress, δ_C is the critical opening dispacement where bridging stress displacement, and n is the exponent. The methods to determine these parameter and discussed in references [12, 13].

Table 1 shows the parameters $\sigma_{\max}$, δ_C and n obtained by Rodel et al. [1], Hu et al. [5] and Hay-White [7].

In Fig. 9, bridging stress characteristics obtained by using the parameters in Table 1 and the result of this study are compared. The axis of abscissa δ / d is used in Fig. 9 to delete the effects of differennce of the mean grain sizes because the mean grain sizes in each evaluation method are different.

From Fig. 9, one can see that the opening displacement at the same bridging stress is quite different though the maximum bridging stresses are almost same.

Table 1 Parameters of bridging characteristics

	Mean grain size d (μm)	Maximum bending stress σ_{max} (MPa)	Critical opening displacement δ_C	Exponent n
Rodel et al [1]	11	70	$d/4$	2.5
Hu et al [5]	20	56	$d/4$	2.1
Hay, White [7]	16	46	$d/3$	11

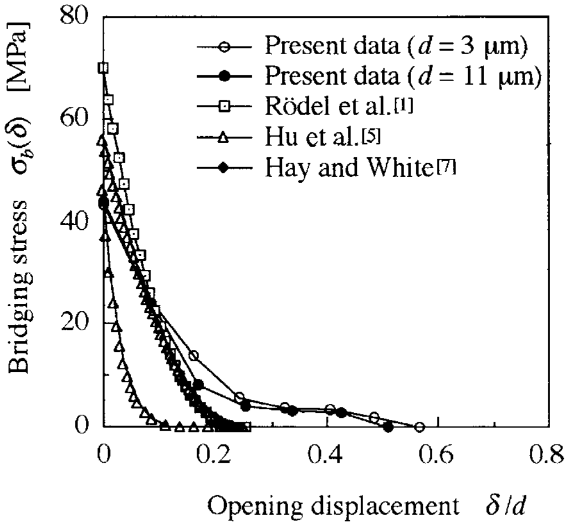


Fig. 9 Comparison with the results of other researchers [1, 5, 7]

CONCLUSIONS

1. An experimental method of evaluating bridging stress by the bending of a deep-notched specimen and an inverse analysis to determine bridging stress from measure measured loads and displacements have been developed.
2. Bridging stress characteristics of alumina ceramics with the mean grain size of 3 μm and 11 μm have been evaluated by using method.
3. The maximum bridging stress of alumina ceramics does not depend on the mean bending strength.

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