Modelling of the Transition from Stage I to Stage II Short Crack Propagation

P. Koester[1], B. Kuenkler[1], O. Dueber[2], U. Krupp[2], C.-P. Fritzen[1] and H.-J. Christ[2]

[1] Institute for Mechanics and Control - Mechatronics, University of Siegen, Germany
[2] Institute for Materials Technology, University of Siegen, Germany

koester@imr.mb.uni-siegen.de

ABSTRACT. *The propagation behaviour of short cracks under cyclic loading is simulated. Short cracks determine up to 90% of fatigue life and exhibit substantially non-uniform propagation kinetics as compared to the growth of long cracks due to their strong interactions with microstructural features. Experimental investigations on a duplex steel have been performed to characterise the different barrier effects of grain and phase boundaries on short crack propagation and to determine the mechanical properties of the individual components of the two-phase material [1]. The findings were implemented into a mechanism-based model for two-dimensional crack propagation in stage I (operating by single slip), which is capable to take the real microstructure into account. Crack growth simulations performed with the model have shown good agreement with experimental data. Based on this method, an algorithm for the transition of stage I crack growth to crack propagation on multiple slip systems is presented. Thereby, the crack changes its propagation direction from approximately 45° in stage I to a path perpendicular to the loading axis, which is the direction of crack growth in stage II. For propagation on multiple slip systems, the closure behaviour of the crack has been simulated and the findings were compared to experimental results. By means of using virtual microstructures based on Voronoi diagrams, it is possible to simulate the overall fatigue-crack propagation process starting from a microstructurally short crack in a single grain until the crack has crossed several (10-20) grains with just one model. It was shown that the propagation mechanism changes from stage I crack growth on single slip systems to the growth on alternating slip systems, which is the preliminary step to stage II crack propagation.*

INTRODUCTION

Cyclically loaded components in structural applications often undergo a stress amplitude, which is close to the fatigue limit of the material used. Under such conditions, crack initiation and short crack propagation is considered to play an important role. Therefore, new concepts and experimental methods have to be developed in order to provide a robust and reliable fatigue life assessment. Because of the strong interactions of short cracks with microstructural features (e.g. grain boundaries) and their substantially different growth mechanisms (stage I) compared to long cracks, the crack propagation rate da/dN of short cracks cannot be described by linear elastic fracture mechanics (LEFM).

In many cases, crack initiation in polished specimen starts trans- or intercrystalline after a relatively low number of cycles at locations in the microstructure exhibiting elevated stress due to stress raisers, like inclusions or grain boundaries. The following early crack propagation in stage I on single slip bands is determined by shear stresses on slip planes inclined by about 45° to the applied stress axis (under push-pull loading conditions), resulting in a zigzag-like crack path. The crack can propagate through several grains in stage I. Then, additional slip bands become activated and the crack propagates alternating on two different slip bands (Fig. 1). To distinguish between these two mechanisms the former one is termed stage Ia and the latter one stage Ib. The position and the length of the activated slip bands is determined by the microstructure.

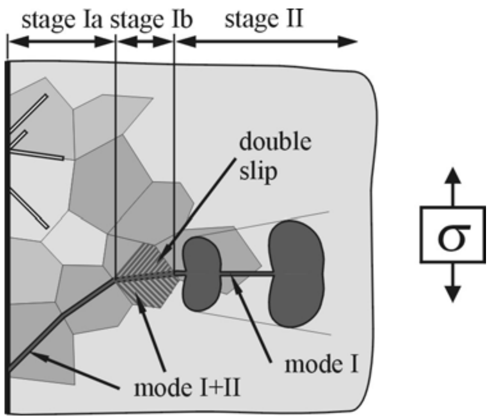

Figure 1. Different stages of crack propagation.

Due to the propagation on different slip systems (termed double slip mechanism in the following), the crack changes from its direction parallel to the maximum shear stress to a path perpendicular to the loading axis (mode I), which is the direction of crack growth in stage II. With further increase in crack length, the stress intensity in front of the crack increases and more slip bands become activated until a larger area at the crack tip is plastically deformed (Fig. 1). Then, the crack is no longer microstructurally short and can be described by elastic-plastic or linear-elastic fracture mechanics.

The prediction of the abnormal propagation kinetics of microstructurally-short cracks requires a micromechanical model, in which the microstructural parameters are taken into account. A very promising approach was proposed by Navarro and de los Rios [2]. Here, plastic slip ahead of a growing microcrack is blocked by the grain boundary until the stress on a dislocation source in the neighbouring grain exceeds a critical value. Then, slip is extended to the next grain and the crack propagation rate increases. In order to take arbitrary grain geometries and crystallographic misorientations of real microstructures into account, the analytical crack propagation model of Navarro and de los Rios was extended and solved by a two-dimensional numerical boundary-element approach, which is introduced in the next section of the present paper. The model has been applied to microcrack propagation in an austenitic/ferritic (γ/α) duplex steel (X2CrNiMoN 22 5 3), which depends not only on the strength of the $\gamma\gamma$ and $\alpha\alpha$ grain boundaries but also on the strength of the $\alpha\gamma$ phase boundaries. In the subsequent section, the model for the transition of crack growth on single slip planes (stage Ia) to crack growth on multiple slip planes (stage Ib) is presented and comparisons between the experimentally investigated crack closure behaviour and simulations of short cracks are shown.

SHORT CRACK MODEL

The model presented in this paper treats the crack and its plastic zone as yield strips. The plastic slip ahead of the growing microcrack is blocked by grain and phase boundaries. Once a critical stress intensity on a slip plane in the neighbouring grain is reached, the plastic deformation and the crack can propagate into the next grain. Thus, the crack decelerates when approaching a boundary and accelerates after passing this barrier, resulting in an oscillating crack growth rate. The advantage of the numerical model over analytical ones (e.g. [2]) is its ability to simulate two-dimensional crack paths in a randomly generated microstructure, taking geometrically crack closure into account.

The basic element of the model is a slip band consisting of a series of slip band pieces. The slip band allows tangential displacements of its two faces relative against each other. These displacements are modelled by means of mathematical edge dislocations (Hills et al. [3]). Plastic deformation resulting from the movement of these dislocations occurs if the shear stress on the slip band exceeds the resistance to dislocation motion τ_b. Thus, the behaviour of the plastic zone (yield strip) is elastic/perfectly plastic. A crack is defined as that part of the slip band, which is, contrary to the rest of the slip band, allowed to open. The opening of the crack is modelled by additional mathematical edge dislocations perpendicular to the slip band. Hence, the crack and its plastic zones are represented by an arrangement of dislocations (Fig. 2).

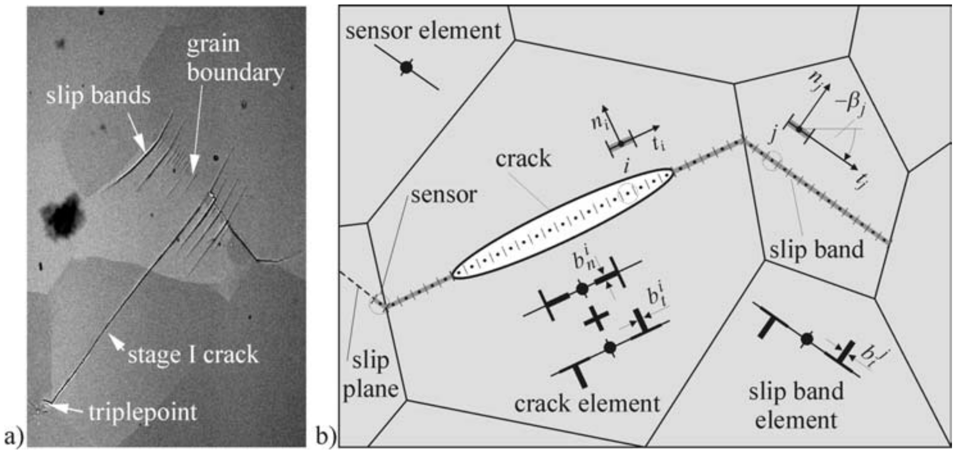

Figure 2. Stage I crack (a) and model with boundary elements (b).

The dislocation distribution is calculated numerically by a boundary element method, which assumes a constant displacement inside each element. Additionally, so-called “sensor elements” were introduced, which can be used to determine the state of stress at any position in the modelled structure. The influence of an element *j* on an element *i* is described by the influence function G^{ij} representing the geometric arrangement of the elements. Summation over all elements leads to an equation system, which together with the boundary conditions delivers a set of inequations for the normal and shear stresses:

$$\sigma_{nn}^{i} = \sum_{j=1}^{p} G_{nn,n}^{ij} b_n^j + \sum_{j=1}^{p+q} G_{nn,t}^{ij} b_t^j + \sigma_{nn}^{i\infty} \le 0 \qquad i = 1 \ldots p\,, \tag{1}$$

$$\left|\tau_{tn}^{i}\right| = \left|\sum_{j=1}^{p} G_{tn,n}^{ij} b_n^j + \sum_{j=1}^{p+q} G_{tn,t}^{ij} b_t^j + \tau_{tn}^{i\infty}\right| \quad \begin{cases} = 0 & i = 1 \ldots p \\ \le \tau_b & i = p+1 \ldots p+q \end{cases}, \tag{2}$$

$$b_n \ge 0 \qquad i = 1 \ldots p\,. \tag{3}$$

Here, *p* is the number of elements in the crack and *q* the number of activated elements in the plastic zone, where the shear stress has reached the critical shear stress. $\sigma_{nn}^{i\infty}$ is the external normal stress and τ_{tn}^{i} the resolved shear stress acting on an element *i*. Inequality (3) states that the crack faces must not penetrate each other, hence, the model accounts for roughness-induced crack closure effects. By use of the boundary element method, the relative displacements of the crack and slip-band faces can be calculated. Crack closure can be simulated by allowing only positive normal displacements along the crack. Analogously to the model of Navarro and de los Rios [2], the current crack propagation rate *da*/*dN* is calculated by means of a power law function

$$\frac{da}{dN} = C \cdot \Delta CTSD^{m}\,. \tag{4}$$

$\Delta CTSD$ denotes the range of crack tip slide displacement, C is a material-specific constant and m is an exponent ($m \approx 1$). The crack tip opening displacement $CTOD$ is equal to zero because the model does not allow normal displacements in the plastic zone. According to [4], Eq. 4 is based on the idea that plastic sliding due to external loads causes dislocation emission at the crack tip and that during reverse loading dislocations of opposite sign are emitted. Hence, vacancies are produced leading to crack advance. For a more detailed description of the model, see [5, 6]. In order to verify the crack propagation model, it was applied to crack geometries observed during fatigue experiments [5, 7].

TRANSITION FROM STAGE I TO STAGE II

The transition of crack growth on single-slip planes (stage Ia) to crack growth on multiple-slip planes is represented simulating a stage Ia crack inclined by about 45° to the applied loading axis (Fig. 3a). Fig. 3b shows the shear stress distribution around the crack tip in a constant radius for a linear elastic stage Ia crack (grey) of length d and a stage Ia crack with plastic deformation on one slip plane only (black) of length $8d$.

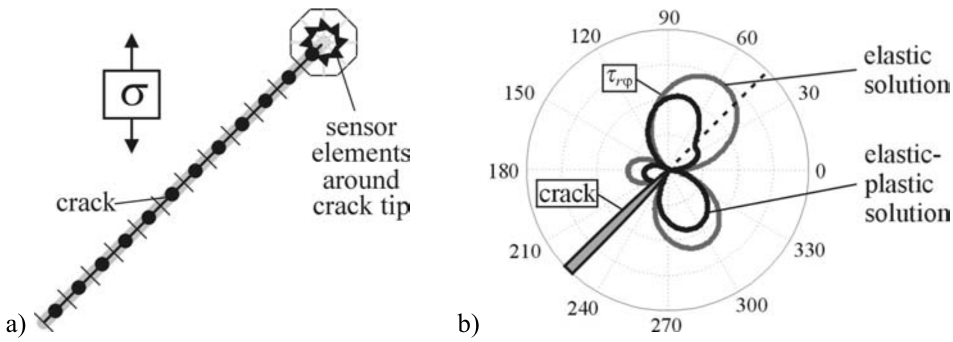

Figure 3. Elastic crack with sensor elements (a) and shear stress distribution around crack tip (b).

To calculate the shear stress distribution, the boundary element method introduced before with special sensor elements around the crack tip is used (Fig. 3a). The calculated elastic stress distribution is identically to the analytical solution (Fig. 3b). In the elastic-plastic calculation the shear stress on the slip plane is reduced to the critical shear stress resulting in a significant decrease of the shear stress near the slip plane. In a larger distance from plastic deformation, the shear stress is nearly as high as for the elastic crack. To identify the activation of a second slip band (beginning of crack propagation in double slip mechanism), additional sensor elements representing the other slip planes of the grain are positioned at the crack tip determining the shear stress on those slip planes (Figs. 4 and 5a). As soon as a critical stress value is reached at one of these sensor elements, the respective slip plane is considered to get "activated" and plastic deformation occurs on this second slip plane (Fig. 5b). This happens only above a certain crack length, because the elastic shear stress increases with crack length (Fig.

4). Accordingly, the shear stress in the elastic-plastic calculation outside the plastic zone increases. The activation of a slip band at the crack tip after a certain stress intensity is reached is in accordance to [8].

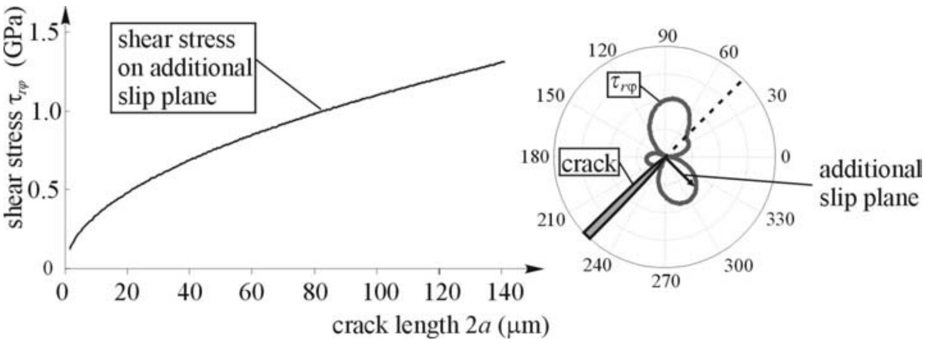

Figure 4. Increasing shear stress with increasing crack length on additional slip plane.

After the activation of the second slip plane, the new crack tip position results from the contribution of the crack growth on two slip bands according to Eq. 4 (Fig. 5c). New sensor elements are now positioned at the new crack tip representing new slip planes and again these elements are activated. Hence, with growing crack length, the crack becomes deflected on a path perpendicular to the loading axis (stage II, long cracks, Fig. 5d). If the orientations of the slip planes change (because the crack tip has reached a new grain) and the crack is still relatively short, the crack possibly grows again on a single slip plane because no adequate second slip plane is available. This is in accordance with our experimental observations [7].

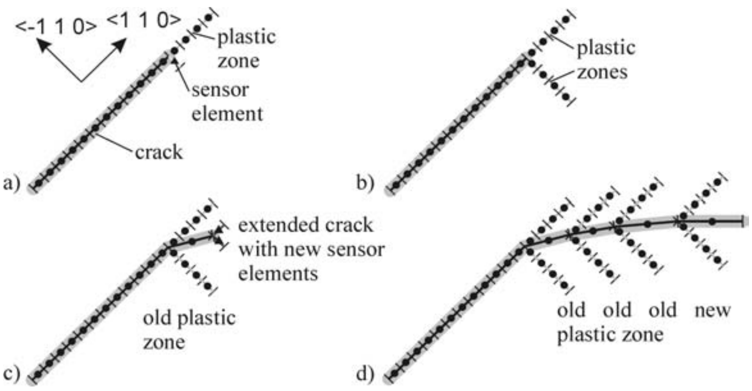

Figure 5. Transition from single slip to double slip mechanism.

RESULTS

An example of a crack growth simulation that accounts for alternate operating slip systems is given in Fig. 6. The crack growth rate as well as the crack path could be reproduced by the model. Thereby, it is important to mention that all parameters, also those for the crack growth law (Eq. 4), were the same as for simulations of cracks in stage Ia. Unfortunately the simulation could not be carried out for more cycles because the specimen broke due to faster propagation of a different crack.

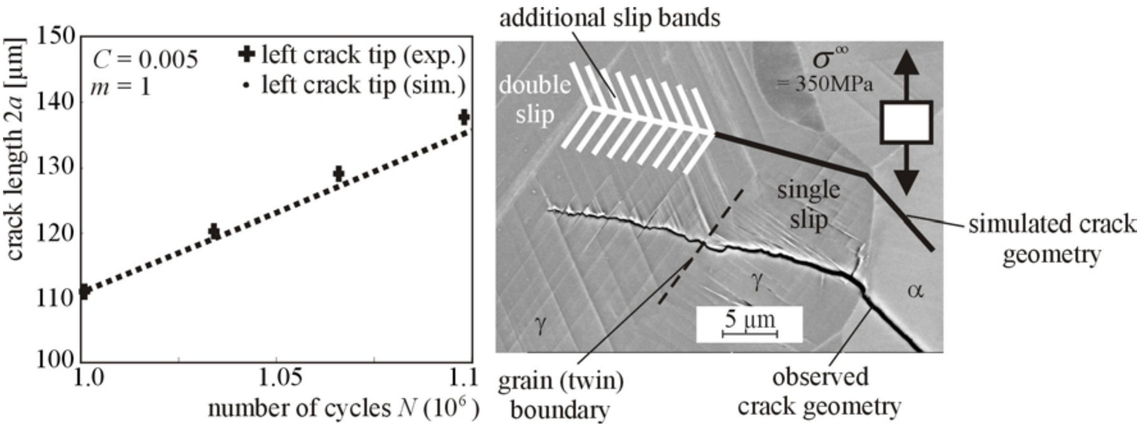

Figure 6. Simulation of crack advance in double slip mechanism.

An example for modelling crack closure is shown in Fig. 7. Here, the simulated crack grows in double slip mechanism with two activated slip planes at each crack tip, inclined by 45°. A comparison with Fig. 8b shows that the computed results confirm the experimental determined negative closure stress [9]. In the simulation, the closure stress increases with increasing crack length and approaches zero. The simulation was done with two different slip band lengths c and did not take into account the formation of a plastic wake (plasticity induced crack closure), hence the closure stress cannot reach positive values.

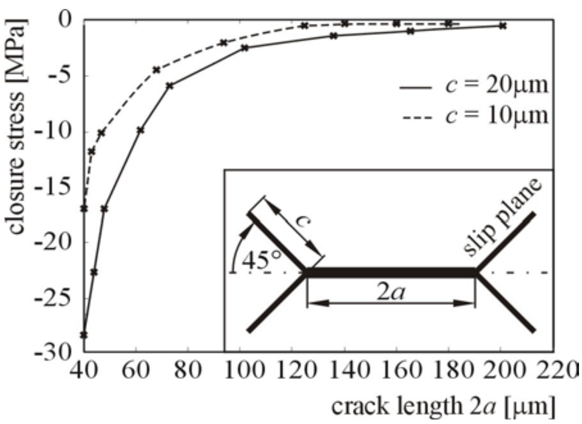

Figure 7. Simulation of the evolution of the closure stress for a growing double slip crack without plastic wake.

In addition to the simulation of cracks with defined geometry, it is possible to simulate cracks that find their path autonomously in a virtual single- or multiphase microstructure that is synthetically introduced using the Voronoi technique [10] (Fig. 8a). The crack starts to grow on a slip band in stage Ia until a critical stress on an additional slip plane is reached, which causes the activation of secondary slip planes and further crack growth in the double slip mechanism. This change yields a significant decrease of the crack growth rate (Fig. 8b), which agrees well with our experimental data.

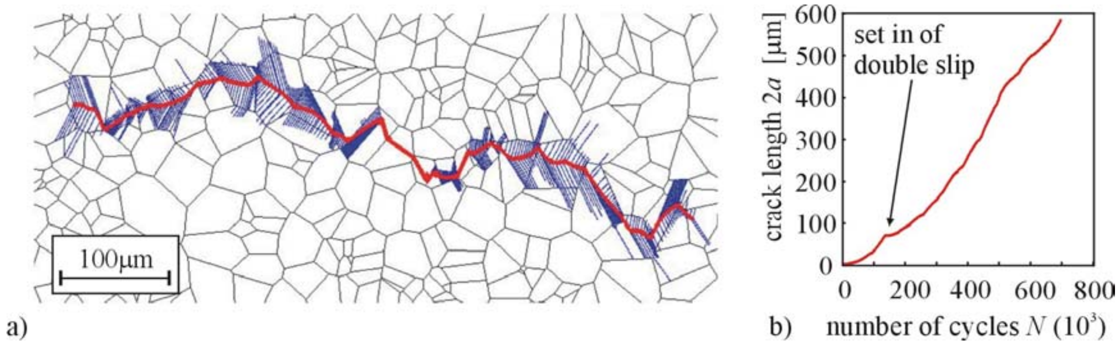

Figure 8. Simulation of crack growth in virtual microstructure (a) and the corresponding crack length over the number of cycles (b).

CONCLUSIONS

A two-dimensional model that is able to reproduce fundamental phenomena of short crack propagation (e.g. retardation of crack growth at grain boundaries, crack closure) and that can be used to investigate the influence of the microstructure (e.g. texture) on crack growth is introduced. The model has been extended to simulate the transition from stage Ia crack growth on single slip planes to crack growth on multiple slip planes. It was found that this transition depends on crack length and grain orientation. The crack propagation in double slip mechanism is the preliminary step to stage II crack propagation and represents the link between the drop of the closure stress to negative values after stage Ia and later to positive closure stresses, as they are well known for stage II crack propagation (long fatigue cracks). The examples demonstrate that short crack propagation and crack closure can be predicted in excellent agreement with the experimentally observed data. The combination of these concepts provides the possibility to simulate the whole crack propagation process in different stages from short crack growth until failure by means of a single model.

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