Two Parameter Fracture Mechanics: Fatigue Crack Behavior under Mixed Mode Condition

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ABSTRACT. *The fatigue crack path has been studied on tensile specimen with holes. The experimental crack path trajectories were compared with those calculated numerically. To incorporate the influence of constraint on the crack curving, we predicted the fatigue crack path by using the two-parameter modification of the maximum tensile stress (MTS) criterion. The values of the mixed-mode stress intensity factors K_I, and K_{II} as well as the corresponding T stress were calculated for the obtained curvilinear and reference crack path trajectories. The influence of constraint on the fatigue crack propagation rate under mixed-mode conditions is discussed.*

INTRODUCTION

When a crack propagates in a non-homogeneous stress field, its crack path is generally curved. The crack path in brittle isotropic homogeneous material has a tendency to be one for which the local stress field at its tip is of a normal mode I type and for which K_{II} tends to zero. This is consistent with the various proposed mixed-mode criteria such as e.g. the maximum tensile stress criterion [1], the maximum energy release rate criterion [2, 3], and the stationary Sih strain energy density factor [1, 4]. All these criteria have in common that if $K_{II}\neq 0$, the crack extends with non-zero change, θ_0, in the tangent direction to the crack path. Moreover, according to the above criteria, the direction of crack propagation θ_0 depends on the ratio of the stress intensity factors corresponding to mode II and mode I, i.e. $\theta_0=\theta_0(K_{II}/K_I)$. This approach, based on the assumptions of the standard fracture mechanics, does not account for the changing of the constraint level during the crack propagation.

In the contribution two-parameter constraint based fracture mechanics is used to account for these differences in constraint and the influence of the constraint level on the crack path under mixed mode conditions is described by using the T-stress. The elastic T-stress represents a constant tensile stress acting parallel to the crack flanks. It is related to the second term (the first non-singular term) in the Williams expansion of the stress field [5]:

$$\sigma_{ij}(r,\theta)=\frac{K_I}{\sqrt{2\pi r}}f_{ij}^{I}(\theta)+\frac{K_{II}}{\sqrt{2\pi r}}f_{ij}^{II}(\theta)+T\delta_{1i}\delta_{1j}\text{, as } r\to 0, \tag{1}$$

where σ_{ij} denotes the components of the stress tensor, K_I and K_{II} are the corresponding stress intensity factors and $f_{ij}(\theta)$ are known angular functions. The fracture parameters K_I, K_{II} and T depend on the geometry, size and external loading of the body and corresponding boundary conditions. The T-stress can be characterized by a non-dimensional biaxiality ratio B given by Leevers and Radon [6]:

$$B=\frac{T\sqrt{\pi a}}{K_I}, \tag{2}$$

where K_I is the stress intensity factor corresponding to the crack length a.

The facts in the literature confirm the dependence of the stability, under Mode I loading, of a straight crack path on the sign of the T-stress. The straight path is shown to be stable under Mode I for T<0 (low constraint) and unstable for T>0 (high constraint), see e.g. [7]. The aim of the present paper is to show how the T stress influences the crack propagation path under mixed mode conditions. At the same time the influence of constraint on the fatigue crack propagation rate under mixed-mode conditions is discussed.

THEORETICAL BACKGROUND

The Direction of crack growth

The growth of a fatigue crack is usually taken as a number of discrete incremental steps. After each increment of the crack growth the quantities K_I, K_{II}, T and the corresponding crack propagation direction θ_0 has to be calculated. Thus the estimation of the θ_0 is of paramount importance. In this paper a two-parameter modification of the maximum tensile stress (MTS) criterion is applied to determine the θ_0 value.

The MTS criterion has been introduced by Erdogan and Sih [1, 8, 9] for elastic material. It states that a crack propagates in the direction for that the tangential stress is maximum. It is a local approach since the direction of crack growth is directly determined by the local stress field within a small circle of radius r centered at the crack tip. The direction angle of the propagating crack is computed by solving the following equation:

$$K_I\sin(\theta_O)+K_{II}(3\cos(\theta_0)-1)=0 \text{ with } \begin{cases} K_{II}\sin(\theta_0/2)<0 \\ \theta_0\in(-\pi;\pi) \\ K_I>0 \end{cases}, \tag{3}$$

where K_I and K_{II} are the stress intensity factors corresponding to mode I and mode II loading respectively, and θ_0 is the direction angle.

The two parameter modification of the MTS criterion has the following form, see e.g. [10]:

$$K_I \sin(\theta_0) + K_{II}(3\cos(\theta_0) - 1) + \frac{16}{3} T \sin\left(\frac{\theta_0}{2}\right)\cos(\theta_0)\sqrt{2\pi r} = 0, \qquad (4)$$

where T is the corresponding value of the T-stress. Based on equation (4) it can be found that the negative T-stress decreases the crack initiation angle, but the positive T-stress increases. The crack growth direction θ_0 can be now expressed in the following form:

$$\theta_0 = \theta_0(K_{II}/K_I, T, r), \qquad (5)$$

where r is an additional length scale representing the fracture process zone size, see e.g. the article by Kim, et al [11]. Correspondingly, we have taken $r=a/100$. Notice also that if $r/a=0$ there is no effect of the T-stress on the crack propagation direction.

The Fatigue crack propagation rate

Under mixed-mode conditions the fatigue crack propagation rate generally depends on the values K_I, *and* K_{II}. A modified Paris law for mixed mode fatigue crack growth is usually used (e.g. see the work by Henn et al. [12]). The disadvantage of the approach is the fact that the corresponding material constants are not known in most cases.

The curvature of naturally growing cracks is usually slight. In the paper by Knésl [13, 14] the fatigue crack propagation under slightly changing mixed mode conditions have been studied experimentally and theoretically. It was shown that, under given conditions ($K_{II}<0.25K_I$), the shear mode of loading affects the crack path and consequently the values of the stress intensity factor K_I, but to estimate the fatigue crack propagation rate, the standard version of Paris-Erdogan law for mode I can be used.

In Knésl et al. [15] the modified form of the Paris-Erdogan law for two parameter fracture mechanics was introduced. It makes it possible to account for the effect of constraint on the fatigue propagation rate in the form:

$$da/dN = C[\lambda(T/\sigma_0)K_I]^m \qquad (6)$$

where C and m are the material constants obtained for the conditions corresponding to T=0 and σ_0 is the cyclic yield stress. The value of the T-stress in equation (6) represents the level of the constraint corresponding to the given specimen geometry and

$$\lambda(T/\sigma_0) = 1 - 0.33\left(\frac{T}{\sigma_0}\right) + 0.66\left(\frac{T}{\sigma_0}\right)^2 - 0.445\left(\frac{T}{\sigma_0}\right)^3 \qquad (7)$$

The approximation equation (7) holds for $-0.6 < T/\sigma_0 < 0.4$.

EXPERIMENT - A CRACK APPROACHING A HOLE

This section describes a testing procedure used for experimental verification of the modeling techniques proposed for predicting the curved crack fatigue problem. For this purpose, the experiments were performed on center cracked plate tension specimens with two holes (see Figure 1). The crack paths in the vicinity of the holes were measured optically with resolution 0.1 mm. The material used (steel ČSN 12010) has the following parameters: Young's modulus $E=2.1\times10^5$ MPa and Poisson's ratio $\nu=0.3$.

The materials constants in the Paris-Erdogan region are C=2.49x10^{-9} and m=2.97. The threshold value for mode I was found K_{Ith}=6.0 MPa m$^{1/2}$ and the cyclic yield stress is σ_0=202 MPa. Experiments were performed under load controlled conditions (the load amplitude was kept constant). The measured crack path (the average of the measurement on both sides of the specimens) is shown on Figure 2.

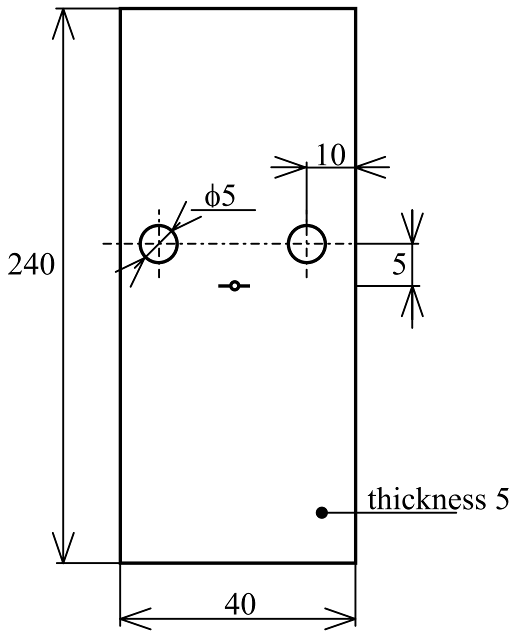

Figure 1. Modify center cracked tension (CCT) specimen with two holes

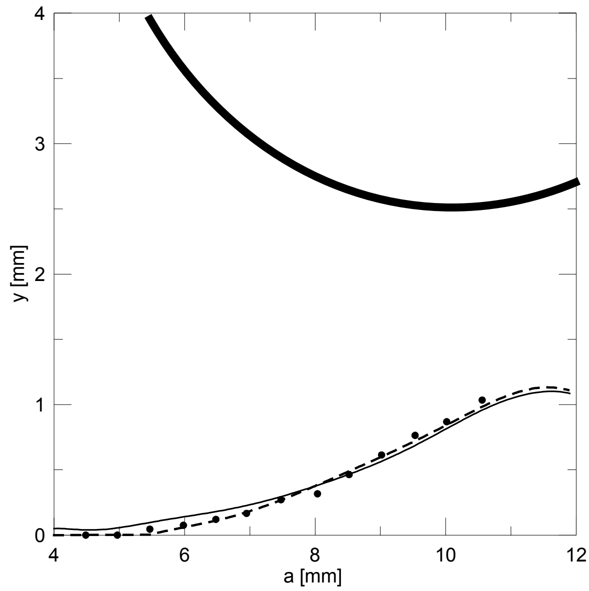

Figure 2. Experimental curves for the CCT specimen with holes The full line holds for classical and dash one for two parameter fracture mechanics.

NUMERICAL SIMULATION

A fatigue crack path in a center cracked tension specimen with two holes (Figure 1.) was simulated numerically. The stress intensity factors K_I and K_{II} and the T-stress values were computed by the commercial finite element (FE) code ANSYS (see Knésl et al. [16] and Seitl et al. [17], respectively).

In summary, the crack path simulation procedure was similar to those used by [10, 18] and its phase was: (i) the FE model of the holed specimen with the specified initial crack in the centre of the specimen was solved to obtain the initial values of K_I, K_{II} and T-stress; (ii) the corresponding crack propagation direction was calculated; (iii) the crack incremented in the growth direction by the (small) specified step; (iv) afterwards, the model was re-meshed to account for the new crack size; and (v) the process was repeated until the required final crack size was reached.

In (ii) the crack propagation direction was calculated by means of the equation (3) with no influence of the T stress, or by means of the equation (4) taking the T-stress into account. As a result two simulated crack paths were obtained, see Figure 2.

RESULTS AND DISCUSSION

First, the values of the stress intensity factor K_I and T-stress for a standard CCT-specimen without holes (i.e. the reference crack) were calculated, see Figure 3. The values correspond to the data from the literature, see e.g. [19].

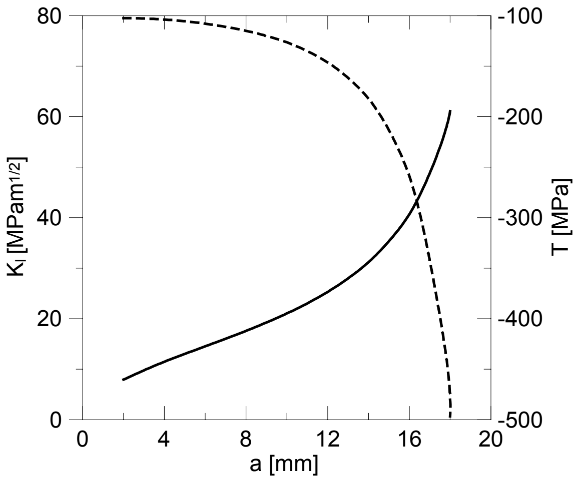

Figure 3. The curves of the stress intensity factor K_I (full line) and the T-stress (dash line) for a standard CCT-specimen (reference crack).

Further, for the simulated crack path, the values K_I , K_{II} and T-stress were calculated, see Figures 4, 5 and 6. The ratio K_{II}/K_I (Figure 5) and the T-stress (Figure 6) control the variables for the crack propagation direction according to the equations (3) and (4). It was shown in Knésl [13, 14] that under slightly changing mixed mode conditions corresponding to naturally growing cracks the fatigue crack propagation rate could be calculated by using the standard Paris-Erdogan law corresponding to the normal mode of loading only, but K_I had to be calculated for the curved crack path. In the same way, the fatigue crack propagation rate taking the T-stress into account may be estimated for mixed mode load conditions by means of the equation (7), where K_I and T correspond to the curved crack path. The results for the present case are given in Figure 7.

CONCLUSIONS

The modeling of a crack propagation path is of prime importance when estimating the fatigue life of engineering structures. Generally, under mixed mode conditions, the crack path and the fatigue crack propagation rate are influenced by the corresponding constraint level. With the aim to assess the influence of constraint on fatigue life under mixed-mode conditions two-parameter constraint based on fracture mechanics was applied. Two aspects of fatigue crack propagation, namely the fatigue crack propagation direction and the fatigue crack propagation rate, were analysed. The behaviour of the

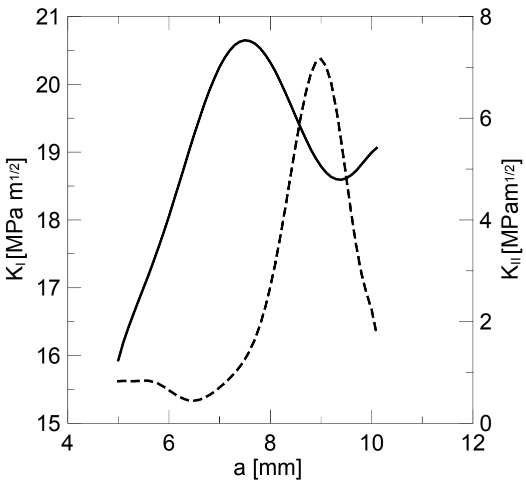

Figure 4. K-calibration for a CCT specimen with holes. Mode I loading – full line and mode II loading dash line

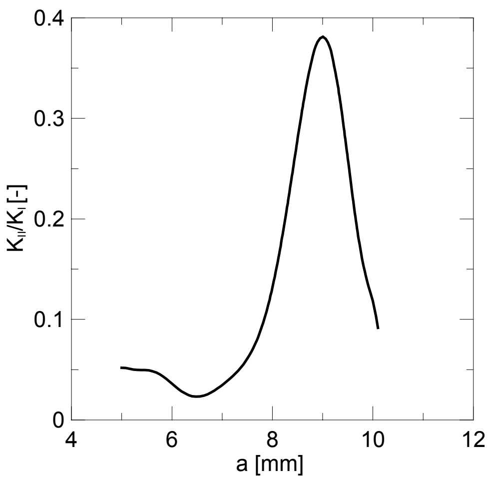

Figure 5. The ratio of the stress intensity factors K_{II}/K_I for a CCT specimen with holes. Values correspond to a numerically calculated crack path.

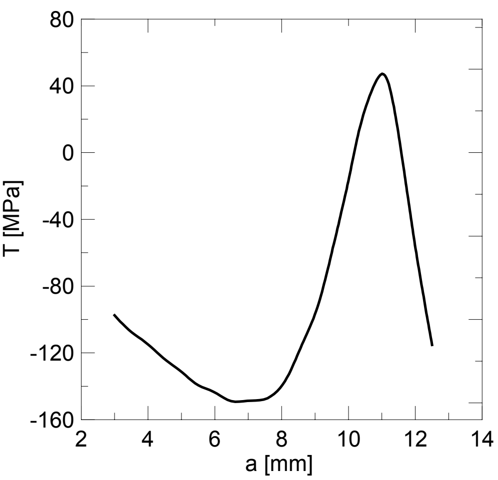

Figure 6. The dependence of the T-stress for a CCT specimen with holes.

fatigue crack near a hole in a tensile specimen was experimentally studied and the results obtained were compared with those of the corresponding numerical simulation. The fatigue crack path was predicted by using the modified maximum tensile stress criterion. The following results have been obtained:

(1) in the present case of a fatigue crack propagation in the tensile specimen with two holes the influence of constraint on the fatigue crack path is non-significant. Both simulated crack paths (with and without T-stress) are practicaly identical and correspond to the experimentaly determined curve well.

(2) the constraint level influences the fatigue crack propagation rate. For the studied configuration, the values of the T-stress are predominantly negative and contribute to the increase of the propagation rate.

(3) for naturally growing cracks where the crack curvature is slight the fatigue crack propagation rate can be calculated by using the Paris-Erdogan law corresponding to the normal load of loading only, but the corresponding values of K_I and T have to be calculated for the curved crack path.

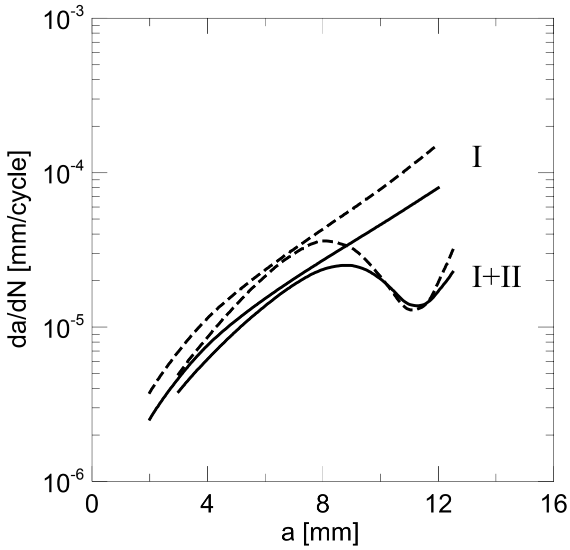

Figure 7. Fatigue crack propagation rate for the reference crack (pure mode I) and for CCT specimen with holes (mode I+II). The dash line represents the fatigue crack rate including the constraint.

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