

# A Fracture Mechanical Model and a Cohesive Zone Model of Interface Fracture

**Henrik Myhre Jensen**

Department of Civil Engineering  
Sohngaardsholmsvej 57  
Aalborg University  
DK 9000 Aalborg  
Denmark

***ABSTRACT.** A comparison between the prediction of crack propagation through an adhesive interface based on a fracture mechanics approach and a cohesive zone approach is presented. Attention is focussed on predicting the shape of the crack front and the critical stress required to propagate the crack under quasi-static conditions. The cohesive zone model has several advantages over the fracture mechanics based model. It is easier to generalise the cohesive zone model to take into account effects such as plastic deformation in the adherends, and to take into account effects of large local curvatures of the interface crack front. The comparison shows a convergence of the results based on the cohesive zone model towards the results based on a fracture mechanics approach in the limit where the size of the cohesive zone becomes smaller than other relevant geometrical lengths for the problem.*

## INTRODUCTION

A cohesive zone model is formulated to model the propagation of a crack through a plane shear-loaded adhesive bond region containing a flaw. In the model a cohesive zone in which non-linear springs are used to model the fracture process represents the adhesive bond region. The cohesive zone is embedded in a linear elastic finite element model of the adherends. Previously, cohesive zone models have been applied to model fracture in elastic-plastic solids as in Tvergaard and Hutchinson [1] and Wei and Hutchinson [2]. In Mohammed and Liechti [3] interface fracture and crack nucleation at bimaterial corners was modelled, using a cohesive zone representation of the bimaterial interface.

For the suggested cohesive zone model, results for the joint strength and crack front shape during crack propagation, for a circular bond region is obtained. These results are compared with similar results using an alternative fracture mechanical model suggested in Jensen [4,5]. The fracture mechanical model uses a mixed mode fracture criterion including modes 1, 2 and 3 coupled with a crack propagation criterion,

embedded in an outer finite element model. The purpose of the comparison is to verify the accuracy of the cohesive zone model, which has greater potential than the fracture mechanical model to be generalised to cases where plasticity plays a significant role or where the curvature of the crack front is large.

## ANALYSIS

In the cohesive zone model the relation between tractions,  $\sigma_s$ , and separations,  $\delta$ , in the adhesive is modelled as a tri-linear spring with the peak stress,  $\hat{\sigma}$ , and the toughness,  $G_{ss}$ , as the two main parameters characterising the adhesive. Here,

$$G_{ss} = \int_0^{\delta^c} \sigma_s(\delta) d\delta \quad (1)$$

where  $\delta^c$  is the separation at which the traction in the adhesive becomes zero, and thus the bond is locally broken. Two examples of traction separation relations are shown in Fig. 1 where the peak stress is the same but the toughness differs. Experimental methods for extracting the relationship have been discussed in Sørensen [6].

In the fracture mechanical approach the boundary between bonded and unbonded adherends is treated as an interface crack front and the following fracture criterion formulated in Jensen et al. [7] for non-oscillating singular crack tip fields is applied in the form

$$G_I + \lambda_2 G_{II} + \lambda_3 G_{III} = G_{Ic} \quad (2)$$

where  $\lambda_2$  and  $\lambda_3$  denote parameters between 0 and 1 adjusting the relative contributions of mode 2 and 3 to the fracture criterion, and  $G_{Ic}$  is the mode 1 fracture toughness of the bond. For  $\lambda_2 = \lambda_3 = 1$  the fracture criterion (2) is the Griffith criterion. In (2)  $G_I$ ,  $G_{II}$  and  $G_{III}$  denote the mode 1, 2 and 3 components of the energy release rate, respectively.

It is clear that there is no distinct crack front in the cohesive zone model but rather a fracture process zone, which presents a difficulty when comparing results of the fracture mechanical model with the cohesive zone model. In the comparisons below the position of the crack front is defined by  $\delta_2$ , where the traction drops below the strength  $\hat{\sigma}$ .

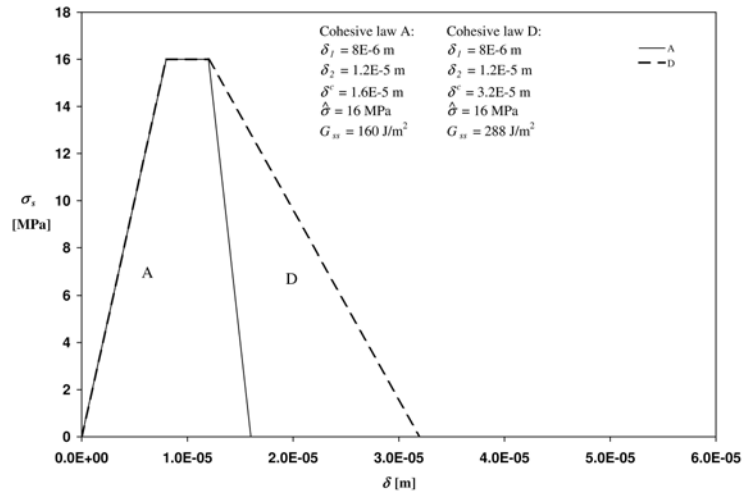


Figure 1. Two traction separation relations for the adhesive.

## RESULTS AND DISCUSSION

The adherends are assumed to behave linear elastic but the solution in both formulations must be followed by loading the system incrementally due to non-linearities associated with the crack growth process. In the fracture mechanical formulation a crack growth criterion is applied [4,5] and it is subsequently checked that the solution satisfies the fracture criterion (2). In the cohesive zone model (1) must be extended to three-dimensional load cases, for details see Feraren and Jensen [8]. The generalisation of (1) to three-dimensional loadings has been carried out without introducing mode dependence, and thus a comparison of the results for the cohesive zone model and the results of the fracture mechanical model is relevant only for  $\lambda_2 = \lambda_3 = 1$  in (2) in which case  $G_{ss} = G_{1c}$ .

The shape of the bond region is taken to be initially circular and a constant shear load is applied to the adherends. The amount of crack growth in the adhesive bond is characterised by the relative area change of the bond  $\Delta A/A$ . Fig. 2 shows a comparison of the predicted relationship between the externally applied stress  $\sigma$  and the relative area change of the bond. The calculations are performed based on the cohesive zone model for the two traction separation relations given in Fig. 1. Bond fracture is initiated at  $\Delta A/A = 0$  and the critical stress is seen to be nearly the same as this is mainly governed by the peak stress which by Fig. 1 is the same in the two cases. After crack initiation, the crack starts propagating through the interface bond and this may happen at increasing

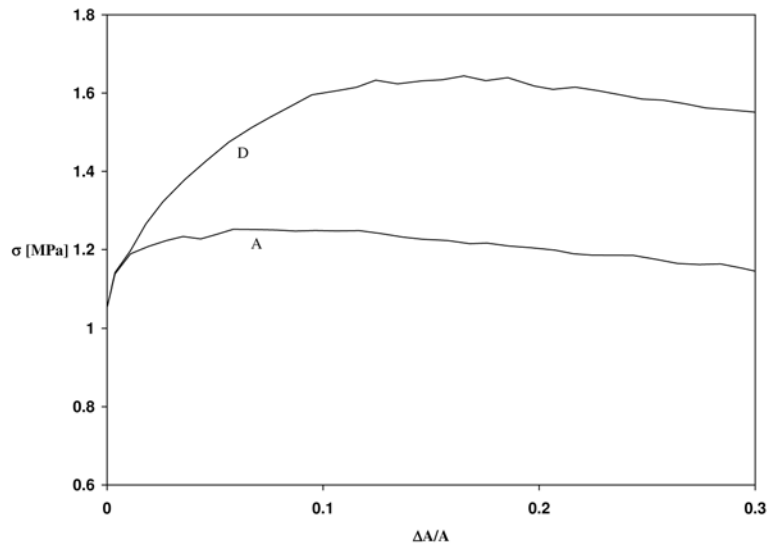


Figure 2. Relationship between applied stress and bond area change for two cohesive relations.

externally applied stress indicating a residual strength in the bond. This response is much more sensitive to the toughness,  $G_{ss}$ , of the bond as can be seen in Fig. 2.

In Fig. 3 a comparison between the cohesive zone model and the fracture mechanics based model is shown. The difference between the predicted bond strengths at which crack propagation is initiated is around 5% and the differences grow to around 10% during crack propagation. These minor differences are mainly caused by the finite size of the cohesive zone, which in the fracture mechanical approach is zero. In general the agreement between the formulations becomes better as the size of the cohesive zone is reduced.

The agreement between predicted shapes of the crack front during crack propagation is even better. In Fig. 4 a late stage of crack growth is shown corresponding to  $\Delta A/A = 0.1$ . In this figure the fracture mechanical prediction of the crack front shape is shown as a solid line. The dots are the places where the separation lies between  $\delta_1$  and  $\delta^c$  in Fig. 1. For the black dots  $\delta_1 < \delta < \delta_2$  and for the grey dots  $\delta_2 < \delta < \delta^c$ .

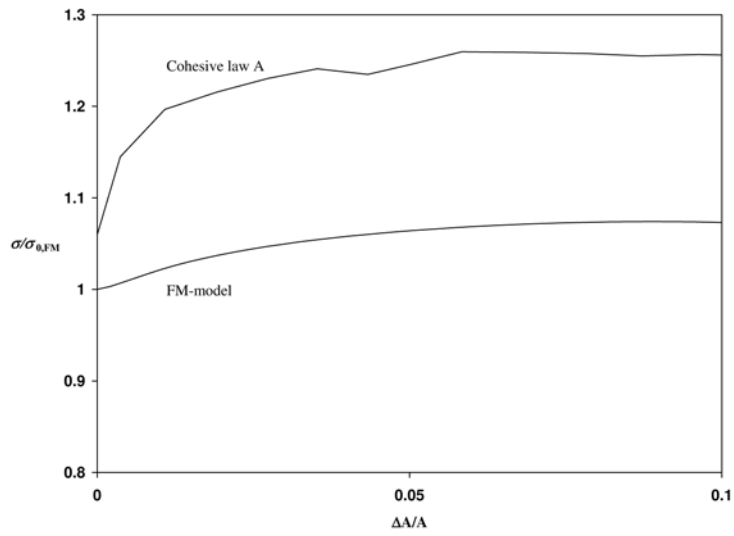


Figure 3. Relationship between applied stress and bond area change for fracture mechanical and cohesive zone model.

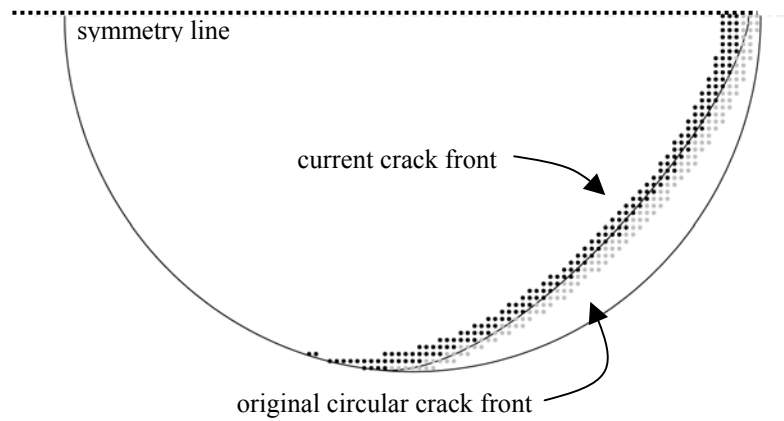


Figure 4. Predicted crack front shapes.

## CONCLUSION

A model developed to study the fracture of adhesive bonds based on the concept of cohesive zones was compared to model predictions based on a previously developed

fracture mechanical model. The initial strength of the bond and the shape of the crack front are in very good agreement, while some differences between the model predictions show up in the load-area change response after crack growth. These differences are mainly caused by the finite size of the cohesive zone.

## REFERENCES

1. V. Tvergaard and J.W. Hutchinson. The relation between crack growth resistance and fracture process parameters in elastic-plastic solids. *Journal of the Mechanics and Physics of Solids* **40**, 1377-1397, 1992.
2. Y. Wei and J.W. Hutchinson. Interface strength, work of adhesion and plasticity in the peel test, *International Journal of Fracture* **93**, 315-333, 1998.
3. I. Mohammed and K.M. Liechti. Cohesive zone modeling of crack nucleation at bimaterial corners, *Journal of the Mechanics and Physics of Solids* **48**, 735-764, 2000.
4. H.M. Jensen. Three-dimensional numerical investigation of brittle bond fracture, *International Journal of Fracture* **114**, 153--165, 2001.
5. H.M. Jensen. Crack initiation and growth in brittle bonds, *Engineering Fracture Mechanics* **70**, 1611-1621, 2003.
6. B.F. Sørensen. Cohesive law and notch sensitivity of adhesive joints. *Materials Research Department, Risø National Laboratory*, 2001.
7. H.M. Jensen, J.W. Hutchinson and K.-S. Kim. Decohesion of a cut prestressed film on a substrate, *International Journal of Solids and Structures* **26**, 1099-1114, 1990.
8. P.F. Hansen and H.M. Jensen, Cohesive Zone Modelling of Interface Fracture near Flaws in Adhesive Bonds. *Engineering Fracture Mechanics* **71**, 2125-2142, 2004.