Crack Growth Prediction in Aluminium Alloy at Different Environmental Temperatures

Andrea Carpinteri, Roberto Brighenti, Sabrina Vantadori, Danilo Viappiani

Department of Civil and Environmental Engineering & Architecture
University of Parma – Viale G.P. Usberti, 181/A – 43100 Parma – Italy
E-mail: andrea.carpinteri@unipr.it ; brigh@unipr.it

ABSTRACT. *The evaluation of the crack initiation and propagation under different environmental conditions is of great interest for many engineering applications. Several crack growth criteria have been proposed in the last decades, but each criterion can be employed only for particular loading situations, environmental conditions and materials. In the present paper, an extension of the criterion known as R-criterion (minimum extension of the core plastic zone) is proposed in order to take into account the environmental temperature dependence. The modified criterion is herein employed to predict the crack growth path in an edge-cracked finite plate under tension, by using a simplified procedure to determine the Stress-Intensity Factor. Then, some experimental mixed-mode fracture tests performed on edge-cracked aluminium alloy plates at different temperatures are described. Finally, the theoretical results are compared with the experimental data.*

INTRODUCTION

The evaluation of the crack initiation and propagation under different environmental conditions is of great interest for many engineering applications. Since several practical cases involve mixed-mode fracture, an appropriate theory to correctly describe the crack growth in such situations is needed. In the last decades, many criteria (e.g. [1-11]) have been proposed in order to determine the crack propagation paths under mixed mode loading.

When experimental crack growth data are compared with theoretical predictions, it is quite difficult to find a criterion which can cope with all the possible situations (loading types, environmental conditions, materials). In the present paper, an extension of the *R-criterion* [9, 11] is proposed in order to take into account the environmental temperature dependence. Some displacement-controlled fracture tests at different temperatures are described for aluminium alloy plates under mixed mode (I+II) loading. The extended *R-criterion* is assessed by comparing theoretical results with experimental data obtained.

REVIEW OF SOME CRACK PROPAGATION CRITERIA

Some crack propagation criteria are briefly reviewed in the present section.

According to the maximum principal stress criterion (*MTS-criterion*) proposed by Erdogan and Sih [1], crack grows perpendicular to the maximum principal stress direction. Analytically, it can be stated as follows:

$$\frac{\partial \sigma_\theta}{\partial \theta} = 0, \quad \frac{\partial^2 \sigma_\theta}{\partial \theta^2} < 0 \tag{1}$$

where the polar coordinate θ is shown in Fig.1.

According to the *M-criterion* proposed by Kong et al. [7], the crack propagation direction is defined by the maximum value of the stress triaxiality ratio $M = \sigma_H / \sigma_{eq}$ around the crack tip (σ_H is the hydrostatic stress, whereas σ_{eq} is the equivalent stress which can be assumed equal to the Von Mises stress). Analytically, the criterion can be stated as follows:

$$\frac{\partial M}{\partial \theta} = 0, \quad \frac{\partial^2 M}{\partial \theta^2} < 0 \quad \text{with} \quad \sigma_H = \frac{\sum_{i=1}^{3} \sigma_{ii}}{3} = \frac{2(1+\nu)}{3\sqrt{2\pi \cdot r}} \left[K_I \cos\frac{\theta}{2} - K_{II} \operatorname{sen}\frac{\theta}{2} \right] \quad \text{(plane strain)} \tag{2}$$

Following the minimum strain energy density criterion (*S-criterion*) proposed by Sih [2, 3], crack grows in the direction of minimum of the strain energy density S around the crack tip. Analytically, the criterion can be stated as follows:

$$\frac{\partial S}{\partial \theta} = 0, \quad \frac{\partial^2 S}{\partial \theta^2} > 0 \tag{3}$$

The angle θ between the initial crack and the crack growth direction can be determined by solving the following equation: $K_I \operatorname{sen}\theta + K_{II} \cdot (3 \cdot \cos\theta - 1) = 0$.

According to the maximum dilatational strain energy density criterion (*T-criterion*) proposed by Theocaris et al. [4-6], crack grows in the direction of maximum of the dilatational strain energy density T_V around the crack tip. The criterion can be written as follows:

$$\frac{\partial T_V}{\partial \theta} = 0, \quad \frac{\partial^2 T_V}{\partial \theta^2} < 0 \quad \text{or} \quad \frac{\partial I_p}{\partial \theta} = 0; \quad \frac{\partial^2 I_p}{\partial \theta^2} < 0 \tag{4}$$

where $I_p = I_1^2 - 2I_2$ with I_1, I_2 the first and the second stress tensor invariants, respectively. On the other hand, the distortional strain energy T_D along the boundary of the Von Mises plastic zone is constant and equal to the yield stress and, therefore, cannot be used to find a minimum value.

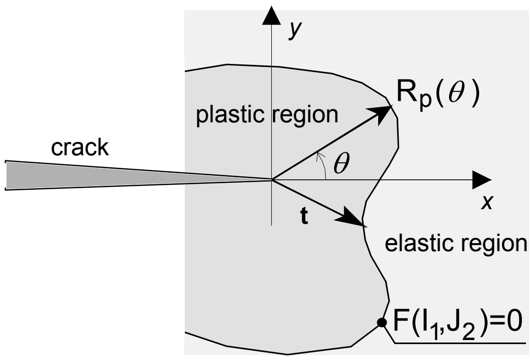

Figure 1. Graphical representation of the *R-criterion*.

From experimental results, it can be observed that the crack propagation direction tends to follow the local or global minimum extension of the plastic core region. From a physical point of view, it can be explained by considering that the plastic core region is a high-strain area, and the crack tends to reach the elastic region of the material outside the plastic zone, propagating through the plastic region which develops around the crack tip. Therefore, it is reasonable to assume that crack follows the "easiest" path to reach the elastic region. Such a path can be assumed to coincide with the shortest path from the crack tip to the elastic material, as is stated by the *R-criterion* proposed by Shafique et al. [9, 11] (Fig. 4). Mathematically, the *R-criterion* can be written as follows:

$$\frac{\partial R_p}{\partial \theta} = 0, \quad \frac{\partial^2 R_p}{\partial \theta^2} > 0 \tag{5}$$

where $R_p(\theta)$ is the function which defines the distance from the crack tip to a generic point of the plastic zone boundary $F(I_1, J_2) = 0$ (Fig. 1). When the conditions stated in eqns (5) are fulfilled, the crack propagation direction vector $\mathbf{t}$ is determined (Fig. 1).

Besides the above criteria, other crack growth criteria have been proposed (for example, see [8, 10]).

Extension of the R-criterion

The *R-criterion* has a well-defined physical meaning, and can be extended in order to take into account the environmental temperature effects on crack propagation. Since the yield stress and the yield function (necessary to identify the plastic core region) are strongly dependent on the environmental conditions, the crack growth criterion can be modified as is hereafter proposed.

As is well-known, materials usually show a sort of embrittlement by decreasing temperature, while large plastic deformations occur at high temperatures. A Drucker-Prager-like yield criterion can be considered in order to quantitatively describe such a behaviour. A generalisation of the yield function could be written as follows:

$$F(I_1, J_2) = \alpha(T) \cdot I_1 + \beta(T) \cdot \sqrt{J_2} - k(T) = 0 \tag{6}$$

being $I_1 = \sum_{i=1}^{3} \sigma_{ii}$, $J_2 = \sum_{i,j=1}^{3} \sigma'_{ij}\, \sigma'_{ij} / 2$ the first stress tensor invariant and the second deviatoric stress invariant, respectively, whereas $\alpha(T), \beta(T)$ and $k(T)$ are three temperature-dependent parameters of the material. The parameters $\alpha(T)$ and $\beta(T)$ respectively represent the hydrostatic and the deviatoric stress dependence on the temperature, and $k(T)$ defines the temperature-dependent yield stress.

The generalised yield function can be rewritten in the classical Drucker form:

$$F(I_1, J_2) = \gamma(T) \cdot I_1 + \sqrt{J_2} - \kappa(T) = 0 \tag{7}$$

where $\gamma(T) = \alpha(T) / \beta(T)$ and $\kappa(T) = k(T) / \beta(T)$ can be obtained through the following expressions (according to the Drucker formulation):

$$\gamma(T)=\frac{1}{\sqrt{3}}\frac{\sigma_c(T)-\sigma_t(T)}{(\sigma_c(T)+\sigma_t(T))}, \qquad \kappa(T)=\frac{2}{\sqrt{3}}\frac{\sigma_c(T)\cdot\sigma_t(T)}{(\sigma_c(T)+\sigma_t(T))} \tag{8}$$

with $\sigma_c(T), \sigma_t(T)$ equal to the compressive and the tensile yield temperature-dependent stress of the material, respectively.

If the tensile strength and the compressive strength are equal to each other and characterised by the same temperature dependence ($\sigma_c(T)=\sigma_t(T)=\sigma_y(T)$, typical of metal-like materials at *ordinary* or *high temperatures*), the yield condition becomes:

$$F(I_1,J_2)=+\sqrt{J_2}-\kappa(T)=0 \quad \text{with} \quad \gamma(T)=0,\ \kappa(T)=\frac{2}{\sqrt{3}}\frac{\sigma_c(T)\cdot\sigma_t(T)}{(\sigma_c(T)+\sigma_t(T))}=\frac{\sigma_y(T)}{\sqrt{3}} \tag{9}$$

that is, we obtain the classical Von Mises criterion.

On the other hand, a brittle behaviour can be observed at *very low temperatures*: in such a case, the tensile strength can be assumed to be much more lower than the compressive one (i.e. $\sigma_t(T) << \sigma_c(T)=\sigma_y(T)$), and the yield condition reduces to:

$$F(I_1,J_2)=I_1+\sqrt{3J_2}-\sqrt{3}\cdot\kappa(T)=0 \quad \text{with} \quad \gamma(T)\cong\frac{1}{\sqrt{3}} \tag{10}$$

which is a hydrostatic stress-dependent law (i.e. a particular case of the Drucker criterion).

For *intermediate values of the temperature*, a transition behaviour can be assumed between the situations described by eqns (9) and eqns (10) (Fig.2).

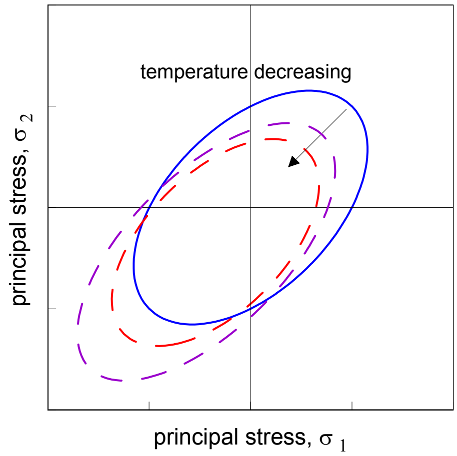

Figure 2. Generalised Drucker criterion.

EXPERIMENTAL TESTS

Thin edge-cracked aluminium alloy plates have been subjected to mixed-mode loading at different temperatures (below and above 0 ºC), by employing a universal test machine under uniaxial displacement control conditions (Fig. 3). The material is characterised by the following mechanical and physical parameters: $E=76GPa$, $\nu=0.34$, $\sigma_y=330MPa$, $\lambda=3.31\cdot10^{-5}\ {}^{o}C^{-1}$ (Young modulus, Poisson coefficient, yield stress, thermal expansion coefficient, respectively). The plate and crack sizes are: $b \times h \times t = 120 \times 240 \times 1.3$ mm, initial crack length $a_0 = 6\,\text{cm}$ with different values of the initial orientation ($\theta_0 = 30^\circ, 45^\circ, 90^\circ$), measured with respect to the loading direction (Fig. 3a).

From the experimental results, it can be deduced that the environmental temperature does not heavily influence the crack paths, independent of the initial crack orientation value, but the failure mode changes from brittle- (for T = -20 ºC) to ductile-type (for T = +80 ºC). As an example, crack paths for $\theta_0 = 30^\circ$ are shown in Fig. 4a to c.

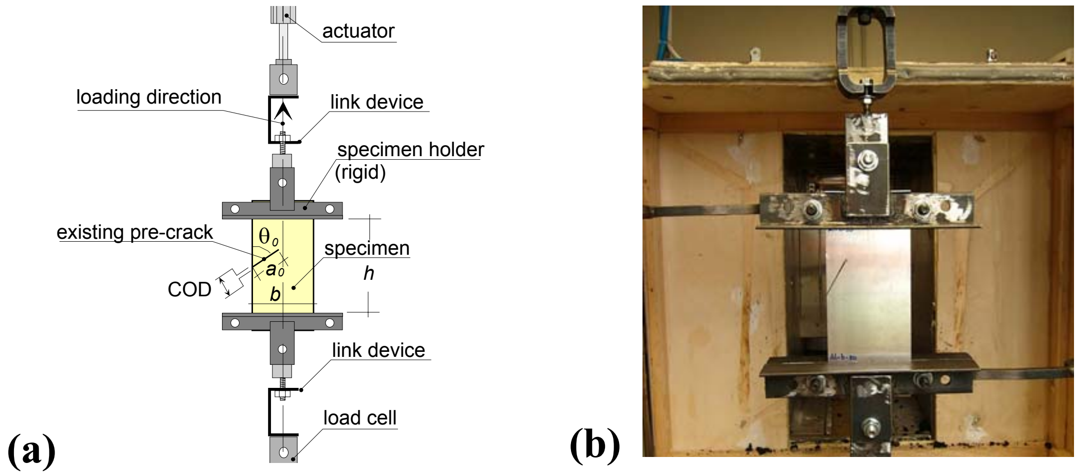

Figure 3. Experimental set-up.

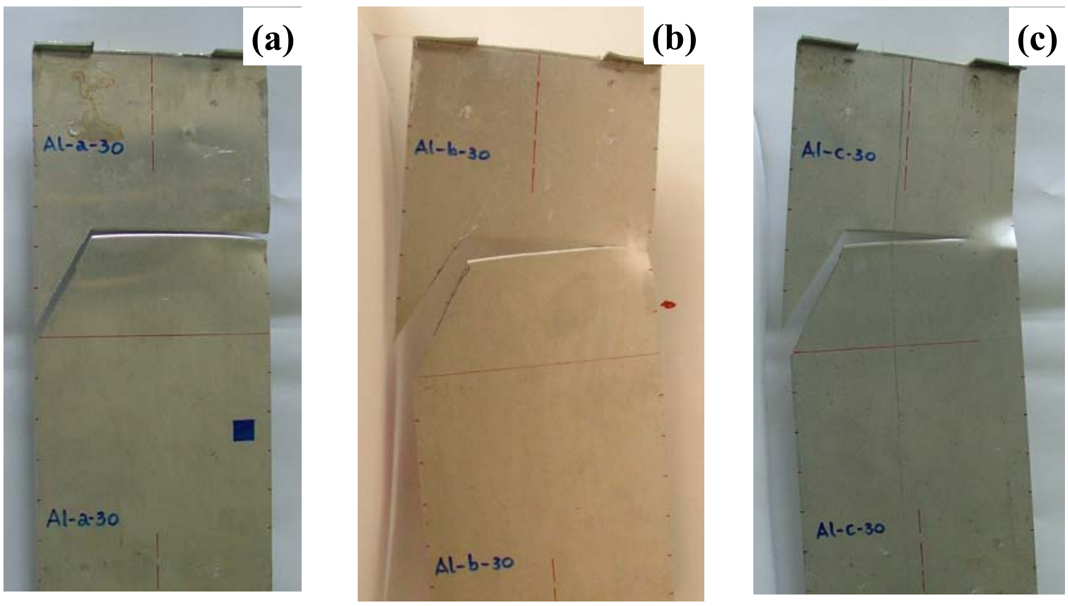

Figure 4. Crack paths and final collapsed configurations for aluminium plates with an initial crack having $\theta_0 = 30^\circ$, in the case of T equal to : (a) $-20^\circ C$, (b) $+20^\circ C$, (c) $+80^\circ C$.

NUMERICAL SIMULATIONS

The Mode I and Mode II SIFs under remote tension have been evaluated by Hasebe and Inohara [12] for a semi-infinite plate with an inclined edge crack, whereas SIFs results for the corresponding finite plate can be found in [13, 14]. Since cracks do not generally remain straight during crack growth [15], the following approximation is considered in the present study: at each stage i $(i > 2)$ of crack propagation (simulated by using a small straight crack increment with length da, Fig, 5a), the SIFs are assumed to be the same as those of an equivalent straight crack with length a_i and orientation $\beta_i = \theta_{i-1}$ (Fig. 5a). Mode I and Mode II SIFs determined through the above method and those obtained from a FE analysis are reported in Fig. 5b for cracks with a two-straight-segment shape ($\theta_0 = \theta_1 = 45^o$ and different angles θ_2). Plane stress condition is assumed. The approximate results seem to be appropriate and, therefore, are used in the numerical crack growth simulations below.

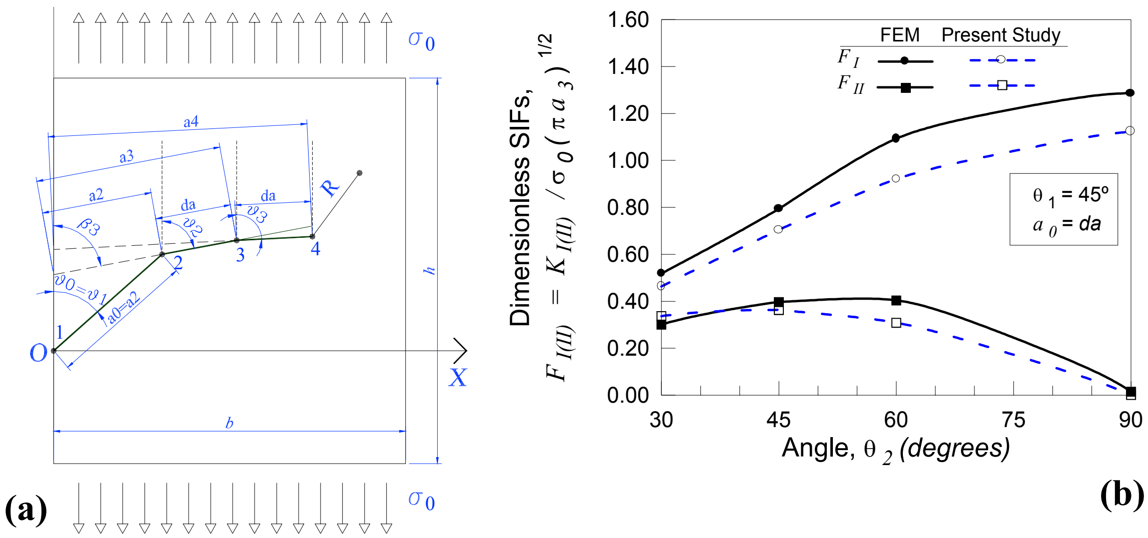

Figure 5. (a) Edge-cracked finite plate. (b) SIFs, obtained from a simplified method (present study) and a FE analysis, for cracks with a two-straight-segment shape.

In Fig. 6, numerical crack paths determined through the modified *R-criterion* are shown for three different initial angles ($\theta_0 = 30^o, 45^o, 90^o$), at (a) low ($-20^o C$), (b) room ($+20^o C$), (c) high temperature ($+80^o C$). The extensions of the plastic regions for the first steps of crack growth are also reported. For the temperature range considered in our tests, compressive yield stress (σ_c) and tensile yield stress (σ_t) have been experimentally observed to slightly increase with decreasing temperature, and the following relationship can describe such a behaviour:

$$\sigma_{c,t}(T) = \sigma_{c,t}(T_0) - 1.974 \cdot 10^5 \cdot (T - T_0) \tag{11}$$

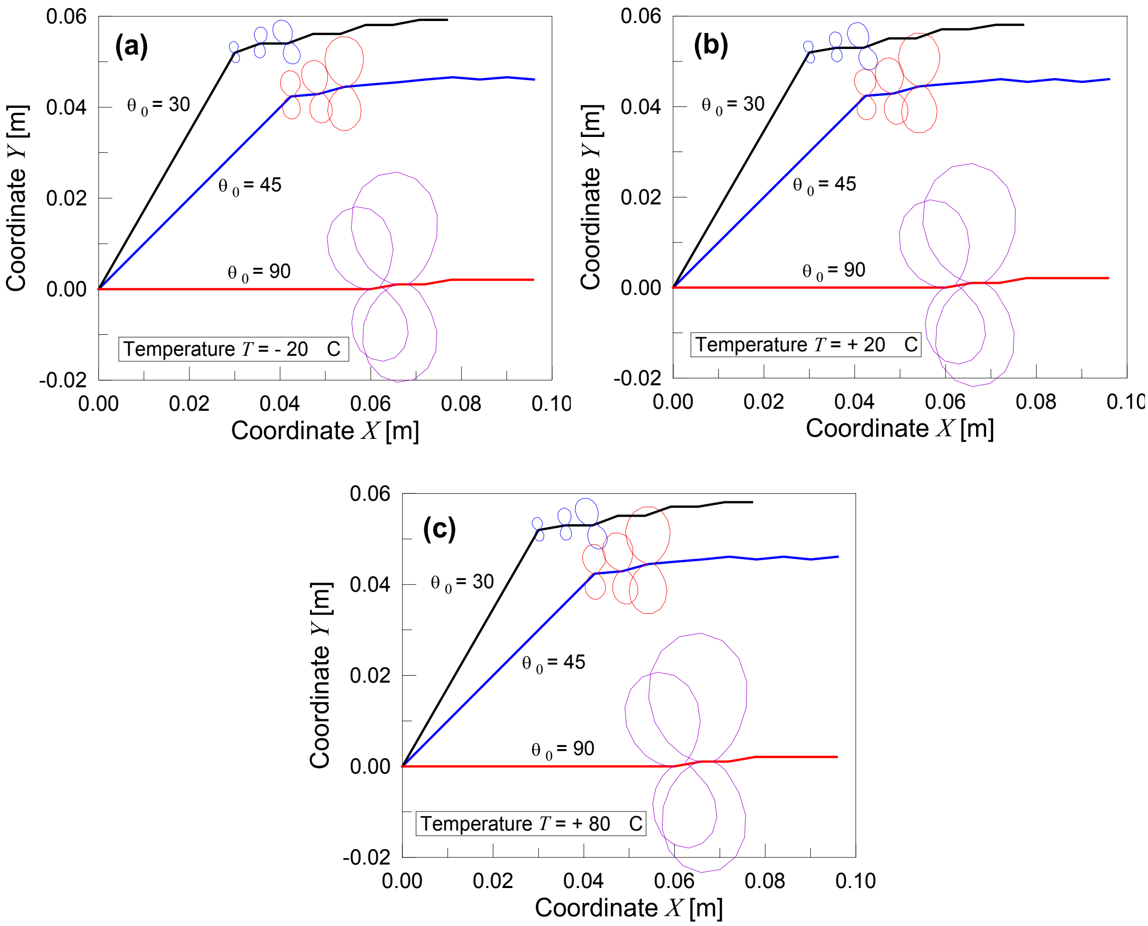

Figure 6. Numerical crack paths for initial cracks having $\theta_0 = 30^o, 45^o, 90^o$, in the case of T equal to : (a) $-20^o C$, (b) $+20^o C$, (c) $+80^o C$.

where $\sigma_{c,t}$ (i.e. σ_c or σ_t) is expressed in Pa, and the reference room temperature T_0 is expressed in °C.

It can be remarked that, for the temperature range examined in the present study, the numerical crack paths plotted in Fig. 6 are almost independent of the temperature (as has also occurred in the experimental tests) since the yield criterion practically reduces to that in eqn (9) (Mises criterion) and the yield surface shape is almost independent of temperature. In other words, for the temperature range considered, the ratio $\sigma_c(T)/\sigma_c(T)$ is equal to about 1 (by using the relationship of $\sigma_{c,t}(T)$ reported in eqn 11) and the hydrostatic stress dependence on the temperature is practically absent.

On the other hand, for temperatures lower than those considered, the yield condition could be assumed as a hydrostatic stress-dependent law (eqns 7 and 8), and the crack paths would show different patterns for different temperatures. As a matter of fact, by decreasing temperature, the ratio σ_c / σ_t increases, the parameter $\gamma(T)$ increases (see eqn 8), and the hydrostatic stress dependence automatically derives from eqn (7).

CONCLUSIONS

In the present paper, the influence of the environmental temperature on the crack growth in aluminium alloy specimens is analysed. Some experimental tests have been conducted at different temperatures in order to observe the related crack paths. It can be remarked that, in the temperature range considered, the crack paths are practically the same for all the cases examined.

An extension of the *R-criterion* is proposed in order to take into account the temperature effects. Some simulations are performed and the results obtained are compared with the experimental data to assess such a theoretical approach.

ACKNOWLEDGEMENTS

The authors gratefully acknowledge the research support for this work provided by the Italian Ministry for University and Technological and Scientific Research (MIUR).

REFERENCES

1. Erdogan, F., Sih, G.C. (1963) *J. Basic Engng* **85**, 519–527.
2. Sih, G.C. (1973) *Engng Fract. Mech.* **5**, 365-377.
3. Sih, G.C. (1974) *Int. J. Fracture* **10,** 305-321.
4. Theocaris, P.S., Adrianopoulos, N.P. (1982) *Engng Fract. Mech.* **16**, 425-432.
5. Theocaris, P.S., Kardomateas, G.A., Adrianopoulos N.P. (1982) *Engng Fract. Mech.* **17**, 439-447.
6. Theocaris, P.S., Adrianopoulos, N.P. (1982) *Int. J. Fract.* **20**, R125-130.
7. Kong, X.M., Schulter, M., Dahl, W. (1995) *Engng Fract. Mech.* **52**, 379-388.
8. Ukadgaonker, V.G., Awasare, P.J. (1995) *Engng Fract. Mech.* **51**, 265-274.
9. Shafique, M.A.K., Marwan, K.K. (2000) *Engng Fract. Mech.* **67**, 397-419.
10. Richard, H.A., Buchholz, F.G., Kullmer, G. (2003), *Key Engng Mat*. **251**, 251-260.
11. Shafique, M.A.K., Marwan, K.K. (2004) *Int. J. Plasticity* **20**, 55-84.
12. Hasebe, H., Inohara, S., (1980) *Ing. Arch*. **49**, 51-62.
13. Nisitani, H. (1975) *Trans. Japan Soc. Mech. Engrs*. **41**, 1103-1110.
14. Bowie, O.L. (1973) Solutions of plane crack problems by mapping technique*, Mechanics of Fracture* (Ed. G.C. Sih) **1**, 1-55, Noordhoff.
15. Pook, L.P. (2002) *Crack Paths*. WIT Press, Southampton.