Fatigue Strength Assessments of Welded Joints: from the Integration of Paris' Law to a Synthesis Based on the Notch Stress Intensity Factors of the Uncracked Geometries

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ABSTRACT. *Decreasing the notch root ρ the theoretical stress concentration factor K_t increases and the fatigue limit of a notched component decreases. Below a given critical value for ρ, the fatigue limit is no longer controlled by K_t and the notch behaves like a crack of equal depth. In the welded joints the conventional welding procedures result in a small value of the weld toe and the weld root radius. The influence of the radius can be considered negligible for as-welded joints and the highly stressed regions are modelled as sharp V-notches. Then fatigue life assessments should be performed on the basis of the Notch Stress Intensity factors (NSIFs), which quantify the intensity of the asymptotic distributions.*
A synthesis of fatigue strength data in terms of NSIF needs the constancy of the V-notch angle. Fatigue data from failures originated from weld roots or weld toes can be summarised in a single diagram by using the mean value of the strain energy density in a well defined volume (area) surrounding the fatigue crack initiation points. The strain energy density is a function of the relevant NSIFs.
In view of practical applications of the NSIF approach, a simplified calculation procedure based on finite element analyses can be defined. A mesh pattern characterised by a constant element size must be used close to the critical point and then the elastic peak stress can be adopted to assess the fatigue life of the joint. Despite its simplicity, such a method fully includes the scale effect, differently from other commonly adopted engineering methods.

INTRODUCTION

By re-analysing a large body of data from various measurement techniques, it was shown that in the majority of cases the minimum weld toe radius in fillet and butt welded joints ranges from 0.05 mm to 0.6 mm (Yakubovskii and Valteris, 1989). Considerable variability characterises the mean values, particularly for butt welded joints from manual welding. In the notch stress intensity approach to the fatigue assessment of welded joints, the weld toe is modelled as a sharp V-notch, ρ=0, and local stress distributions in plane configurations are given on the basis of the relevant mode I and mode II notch stress intensity factors (NSIFs). These factors quantify the magnitude of asymptotic stress distributions obeying Williams' solution (1952). As far as a constant weld toe angle can be assumed and this angle is large enough to make the mode II contribution non-singular (this happens for $2\alpha>102$ degrees), the mode I NSIF can directly be used to assess the fatigue strength of fillet welded joints of different geometry (Lazzarin and Tovo, 1998).

The NSIF approach overcomes some difficulties inherent in the fatigue life concept based on fracture mechanics and, in particular, the very complex problems related to short crack propagation life and the multiple crack interaction on different planes. These phenomena are influenced by loading parameters and statistical variations related to the irregularity of the toe profile (Verreman and Nie, 1996). The NSIF approach has another advantage: the scale effect is fully included in the NSIF values, since the local stress distributions depend on the absolute dimensions of the joints.

Fatigue damage is generally described as the nucleation and growth of cracks to final failure, although the differentiation of two stages is "qualitatively distinguishable but quantitatively ambiguous" (Jiang and Feng, 2004). Dealing with fatigue data from specimens (and not from real size structures where redundant load paths are present) the Mode I NSIF was used to summarise the total fatigue life data (Lazzarin and Tovo, 98, Atzori et al., 1999a, 1999b, 2002, Lazzarin and Livieri, 2001, Lazzarin et al., 2003, 2004). This is possible when most fatigue life is spent at short crack depth, within the zone governed by the V-notch singularity. No demarcation line being drawn between fatigue crack initiation and early propagation, both phases are thought of as strictly dependent on the stress distribution initially present on the uncracked specimen.

A synthesis of fatigue strength data in terms of NSIF needs the constancy of the V-notch angle. This problem has been overcome in some recent papers by using the mean value of the strain energy density range evaluated in a control volume surrounding the weld toe or the weld root (Lazzarin and Zambardi, 2001, Lazzarin et al., 2003, Livieri and Lazzarin, 2005). This strain energy density was given in closed form on the basis of the relevant NSIFs for modes I and II and the control radius R_C of the averaging zone was identified with reference to conventional arc welding processes. The approach, reminiscent of Neuber "elementary volume" concept, was also applied to welded joints under multiaxial load conditions (Lazzarin et al., 2004).

The aims of the present work are:

1. To summarise the analytical frame and the guidelines of the NSIF approach.
2. To make explicit the link between the NIFS of the initial, uncracked geometry, and the SIF of a crack initiated from the sharp V-notch tip; then there is a bridging between the NSIF approach and the conventional LEFM approach, based on the integration of Paris' law.
3. To use the NIFS to evaluate the averaged strain energy density W in a finite size volume surrounding the fatigue crack initiation points and present a ΔW-N scatter band for welded joints made of structural steels, with failures originated both from the weld toes and the weld roots. The synthesis will involve more than 600 fatigue data.

4. Finally, to give a simplified approach suitable for estimating the mode I NSIF by using the peak stress at the V-notch tip numerically evaluated by means of coarse meshes.

NOTCH STRESS INTENSITY FACTORS APPROACH (failure from weld toe)

The degree of the singularity of the stress fields due to re-entrant corners was established by Williams both for mode I and mode II loading (Williams, 1952). NSIFs quantify the intensity of the asymptotic stress distributions in the vicinity the notch tip. By using a polar coordinate system (r, θ) having its origin located at the sharp notch tip, the NSIFs related to mode I and mode II stress distributions are (Gross and Mendelson, 1972)

$$K_1^N = \sqrt{2\pi} \lim_{r\to 0^+} r^{1-\lambda_1} \sigma_{\theta\theta}(r, \theta = 0) \qquad K_2^N = \sqrt{2\pi} \lim_{r\to 0^+} r^{1-\lambda_2} \sigma_{r\theta}(r, \theta = 0) \tag{1}$$

where the stress components $\sigma_{\theta\theta}$ and $\sigma_{r\theta}$ have to be evaluated along the notch bisector (θ=0) see Figure 1.

The idea of estimating the fatigue strength of welded joints on the basis of the local stress fields on the surface of the welded plates was originally proposed by Atzori and Haibach (1979) and verified by comparing finite element evaluations and strain gauges measurements by Atzori et al. (1985). It was then extended to the evaluation of the fatigue strength of notched components in general by Atzori (1985). Stresses acting on the surface of the welded plate were plotted versus the distance from the point of singularity. Then the stress field intensities calculated for joints having different overall geometries but the same V-notch angle were compared so that the relative fatigue strengths could be estimated. Moreover the slope generated by the stress-distance data plotted in a double logarithmic diagram made it possible to quantify the scale effect due to different thicknesses. Stress distributions were directly obtained by means of the finite element method, without any explicit definition of the local stress parameters.

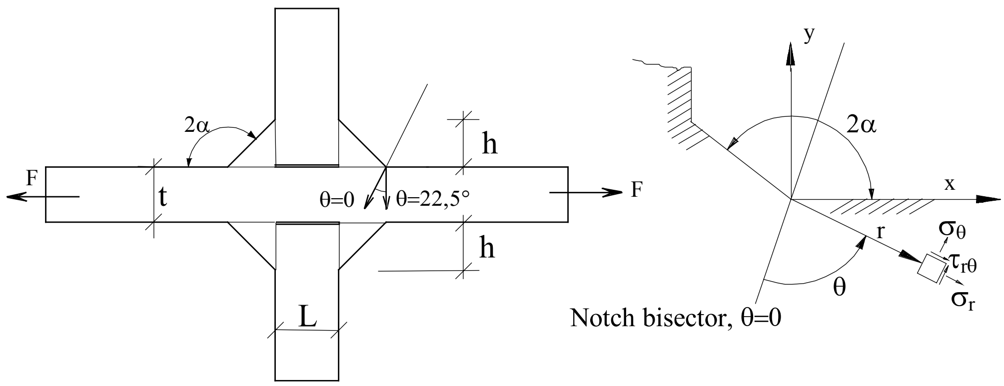

Figure 1. Typical welded joint geometry and local coordinate system.

More recently, NSIFs were proposed also by Boukarouba (1995) and Verreman and Nie (1996) as parameters useful to assess fatigue crack initiation life.

By using definitions (1), it is possible to present Williams' formulae for stress components as explicit functions of the NSIFs. Then, mode I stress distribution is (Lazzarin and Tovo, 1996)

$$\begin{Bmatrix}\sigma_\theta\\ \sigma_r\\ \tau_{r\theta}\end{Bmatrix}=\frac{1}{\sqrt{2\pi}}\frac{r^{\lambda_1-1}K_1^N}{(1+\lambda_1)+\chi_1(1-\lambda_1)}\left[\begin{Bmatrix}(1+\lambda_1)\cos(1-\lambda_1)\theta\\ (3-\lambda_1)\cos(1-\lambda_1)\theta\\ (1-\lambda_1)\sin(1-\lambda_1)\theta\end{Bmatrix}+\chi_1(1-\lambda_1)\begin{Bmatrix}\cos(1+\lambda_1)\theta\\ -\cos(1+\lambda_1)\theta\\ \sin(1+\lambda_1)\theta\end{Bmatrix}\right] \quad (2)$$

On the other hand, Mode II stress distribution results to be:

$$\begin{Bmatrix}\sigma_\theta\\ \sigma_r\\ \tau_{r\theta}\end{Bmatrix}=\frac{1}{\sqrt{2\pi}}\frac{r^{\lambda_2-1}K_2^N}{(1-\lambda_2)+\chi_2(1+\lambda_2)}\left[\begin{Bmatrix}-(1+\lambda_2)\sin(1-\lambda_2)\theta\\ -(3-\lambda_2)\sin(1-\lambda_2)\theta\\ (1-\lambda_2)\cos(1-\lambda_2)\theta\end{Bmatrix}+\chi_2(1+\lambda_2)\begin{Bmatrix}-\sin(1+\lambda_2)\theta\\ \sin(1+\lambda_2)\theta\\ \cos(1+\lambda_2)\theta\end{Bmatrix}\right] \quad (3)$$

Table 1 summarises the values of the parameters for some V-notch angles.

Table 1. Parameters λ and χ of Eqs (2,3) as a function of the V-notch angle 2α. Coefficients e_1 and e_2 for plane strain conditions and Poisson's ratio ν=0.3.

2α	Mode I			Mode II		
rad	λ_1	χ_1	e_1	λ_2	χ_2	e_2
0	0.500	1.000	0.133	0.500	1.000	0.340
$\pi/4$	0.505	1.166	0.150	0.660	0.814	0.244
$\pi/2$	0.544	1.841	0.145	0.909	0.219	0.168
$3\pi/4$	0.674	4.153	0.118	1.302	-0.569	0.111
$5\pi/6$	0.752	6.362	0.104	1.486	-0.787	0.096

Figure 2 shows the fatigue data related to some series of transverse non-load-carrying filled welded joints in structural steel, like those sketched in Figure 1. Original data were due to Maddox (1987) and Gurney (1991). In those 12 series of specimens, all under as-welded conditions, the main plate thickness t ranged from 6 to 100 mm and the variation of the transverse stiffeners was even more pronounced ($3 \le L \le 220$ mm). All fatigue failures originated from the weld toes, where the mean value of the weld angle was kept constant (2α =135 degrees). Due to large variations in the geometrical parameters, the scatter of the experimental data was obviously very pronounced when plotted in terms of nominal stress range (see Figure 2). However, the scatter greatly decreases as soon as the mode I NSIF is used as a meaningful parameter for summarising fatigue total life of all welded joints (and not simply the fatigue crack initiation life).

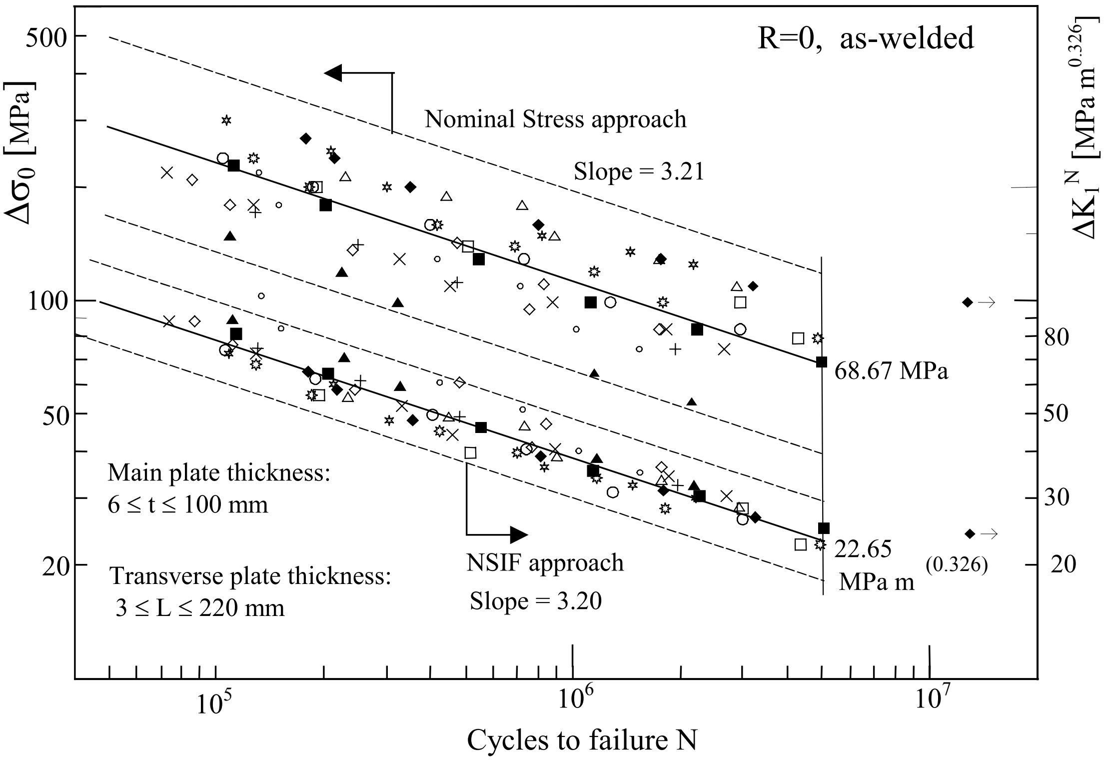

Figure 2. Fatigue strength data in terms of nominal stress range or the mode I NSIF range (from Lazzarin and Tovo, 1998. Original series from Maddox, 1987, and Gurney, 1991). All joints re-analysed here were as-welded, with a V-notch angle at the weld toe equal to 135 degrees. Scatter bands defined by mean values plus/minus 2 standard deviations.

In many cases of practical interest, it is possible to identify a nominal stress and correlate NSIFs to it. Two convenient expressions of NSIFs for welded joints are (Lazzarin and Tovo, 1998)

$$\Delta K_1^N = k_1 \, \Delta\sigma_0 \, t^{1-\lambda_1} \qquad \Delta K_2^N = k_2 \, \Delta\sigma_0 \, t^{1-\lambda_2} \tag{4}$$

where $\Delta\sigma_0$ is the range of the nominal stress, t is the main plate thickness and k_1 and k_2 are non-dimensional coefficients that depend on the welded joint geometry. Expressions for k_1 and k_2 have already been reported in the literature for transverse non-load carrying fillet welded joints subjected to tension or bending loadings (Lazzarin and Tovo, 1998, Atzori et al., 1999a).

Table 2 summarises geometrical parameters, NSIFs and fatigue strength data related to the 12 series of welded joints already shown in Figure 1.

Table 2. Geometric parameters, fatigue strength values and notch stress intensity factors of non-load carrying fillet welded joints. Nominal values of $\Delta\sigma_A$ obtained from the original data provided by Maddox (1987) and Gurney (1991); k_1 and k_2 coefficients determined according to Lazzarin and Tovo,1998.

Series	t mm	2h / t	L / t	k_1	k_2	$\Delta\sigma_A$ MPa $N=5\cdot10^6$	ΔK_1^N $MPa\cdot mm^{0.326}$	ΔK_2^N $MPa\cdot mm^{-0.302}$
1	13	1.231	0.769	1.141	0.813	79.52	209.37	29.80
2	50	0.640	1.0	1.097	0.894	59.64	234.21	16.36
3	100	0.320	0.5	0.883	1.375	55.47	219.80	18.98
4	13	0.769	0.231	0.968	1.290	91.70	204.83	54.52
5	13	1.538	0.769	1.154	0.769	76.68	204.19	27.18
6	25	0.4	0.120	0.787	1.727	93.92	211.09	61.36
7	25	0.720	1.28	1.153	0.766	66.02	217.39	19.13
8	25	1.2	8.80	1.359	0.433	59.72	231.78	9.78
9	38	0.421	0.342	0.873	1.462	68.69	196.30	33.48
10	38	0.789	5.789	1.408	0.351	45.46	209.53	5.52
11	100	0.1	0.030	0.551	2.230	95.70	236.63	53.11
12	100	0.3	2.200	1.271	0.423	40.09	228.66	4.22

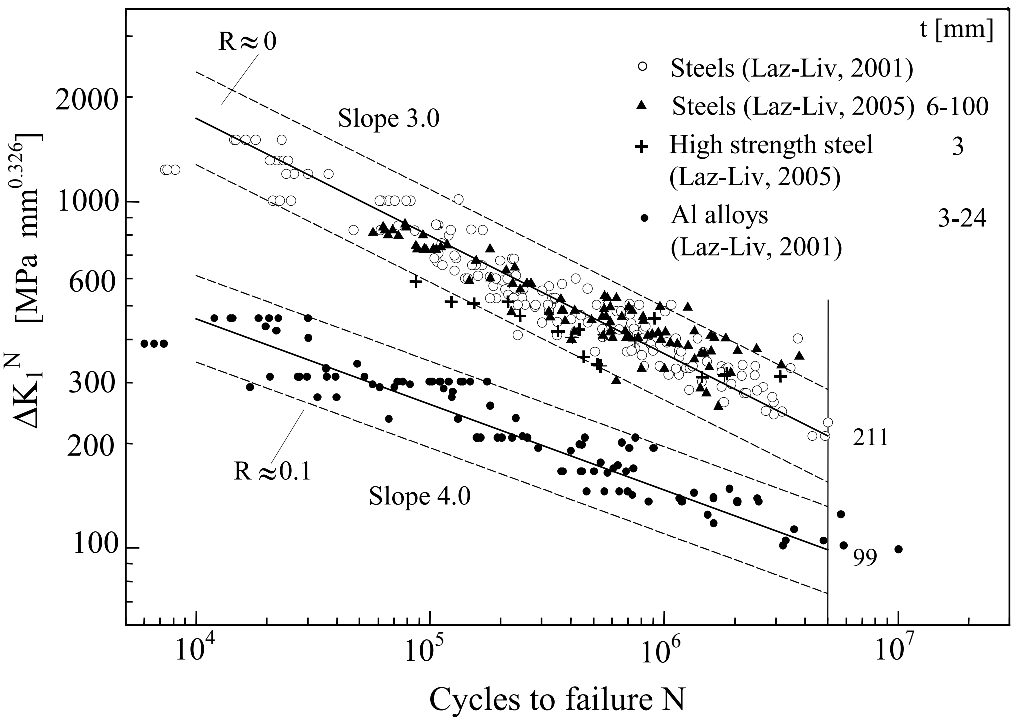

Figure 3. Fatigue strength of welded joints made of steel and aluminium as a function of mode I notch stress intensity factor. Scatter bands defined by mean values plus/minus 2 standard deviations. All failures from the weld toe, in the presence of 2α=135 degrees.

The NSIF approach was later applied to a large number of experimental data related to fillet welded joints made of structural steels and aluminium alloy, see Figure 3 (from Livieri and Lazzarin, 2005).

FRACTURE MECHANICS APPROACH (failure from weld toe)

The stress intensity factor K_I of a crack propagating in a zone affected by a stress gradient can be evaluated taking advantage of Bueckner superposition principle and, in particular, of Albrecht-Yamada's simplified method (1977), which makes it possible to determine the stress intensity factor as a function of the crack length a on the basis of a unique linear elastic analysis of the uncracked component. As soon as the direction of the propagating crack is known or simply established *a priori*, the SIF of a through-the-thickness crack is (see Figure 1):

$$K_I = Y\sqrt{\pi a}\,\frac{2}{\pi}\int_0^a \frac{\sigma_\theta}{\sqrt{a^2 - r^2}}\,dr = Y\sqrt{\pi a}\left[\sigma_\theta\big|_{r=a} - \frac{2}{\pi}\int_0^a \arcsin\left(\frac{r}{a}\right)\cdot\frac{d\sigma_\theta}{dr}\,dr\right] \tag{5}$$

The K_I value depends on the crack dimension a and needs the distribution $\sigma_\theta(r)$ to be known as well as its derivative with respect to radial distance r. Y is the shape factor, initially equal to 1.122 (lateral crack in an infinite plate).

If the propagation of a fatigue crack is believed as to be due to σ_θ, the crack will grow along the direction θ where such a component has its maximum value (according to k_1 and k_2 mutual influence). In order to use Eq. (1), let us simplify the crack path as a straight line. If such a direction coincides with the angle bisector ($\theta = 0$), the component σ_θ is independent of the sliding mode and turns out to be:

$$\sigma_\theta = \frac{1}{\sqrt{2\pi}}\,r^{-0.326}\cdot K_1^N \tag{6}$$

when the V-notch angle assumes its more typical value, *i.e.* 135 degrees. On the other hand, if the direction θ of propagation is perpendicular to the main plate surface (θ = 22.5° when 2α=135°) the two contributions due to Mode I and Mode II should be taken into account. Doing so, it is possible to write:

$$\sigma_\theta = 0.361\cdot r^{-0.326}\cdot K_1^N + 0.322\cdot r^{0.302}\cdot K_2^N \tag{7}$$

By introducing, alternatively, Eq. (6) or Eq.(7) into Eq.(5), one obtains:

$$\Delta K_I = Y\sqrt{\pi a}\cdot\left(0.53\cdot a^{-0.326}\cdot\Delta K_1^N\right) \qquad (\theta = 0°) \qquad (8)$$

$$\Delta K_I = Y\sqrt{\pi a}\cdot\left(0.479\cdot a^{-0.326}\cdot\Delta K_1^N + 0.269\cdot a^{0.302}\cdot\Delta K_2^N\right) \qquad (\theta = 22.5°) \qquad (9)$$

Equation (9) can be used to evaluate the residual life of welded joints by integration of Paris' law. In particular, referring to the data already shown in Figure 2, we assume here an initial crack length a_i=0.3 mm (Atzori et al., 1999a) whereas the coefficients in Paris' law are: m = 3.0 and C = $0.183\text{x}10^{-12}$ or, alternatively, m = 4.0 and C = $0.2046\text{x}10^{-15}$, both couples of values chosen according to Gurney (1991). In the y-axis it will appear the stress parameter $\Delta\sigma_{g,i}$, i.e. the initial value of the nominal stress range defined on the gross transverse section of the welded joints:

$$\Delta\sigma_{g,i} = \left(0.479\cdot a_i^{-0.326}\cdot\Delta K_1^N + 0.269\cdot a_i^{0.302}\cdot\Delta K_2^N\right) \qquad (\theta = 22.5°) \qquad (10)$$

or, alternatively,:

$$\Delta\sigma_{g,i} = \left[0.479\cdot k_1\cdot\left(\frac{t}{a_i}\right)^{0,326} + 0.269\cdot k_2\cdot\left(t\cdot a_i\right)^{0,302}\right]\cdot\Delta\sigma_0 \qquad (11)$$

The results are shown in Figure 4 and 5 where predicted values are compared with the experimental values reconverted in terms of $\Delta\sigma_{g,i}$.

The difference between experimental total life data and estimated residual life is evident in Figure 3: the mean lines, irrespective of joint thickness and geometry, are all translated in a way such that the experimental total life to residual life ratio is about 3:1. However, the most important result is not the position of the scatter band, the centre of which could easily be translated rightwards, maybe simply assuming a convenient elliptic crack front, but the substantially unmodified width of the two scatter bands. The inverse slope of the two curves is about 3, like the exponent m of the Paris' law.

By changing the parameter m and C in the Paris law, but obeying Gurney's equation $C\cdot(895.4)^m = 1.315\cdot10^{-4}$, the fatigue crack initiation time to total fatigue life ratio now depends on stress levels (Figure 5). This happens because m is different from the inverse slope of the Wöhler curves, k≈3, see Figure 2. The fact of major importance remains the capability of the fracture mechanics approach and the NSIF approach to unify the behaviour of very different geometries.

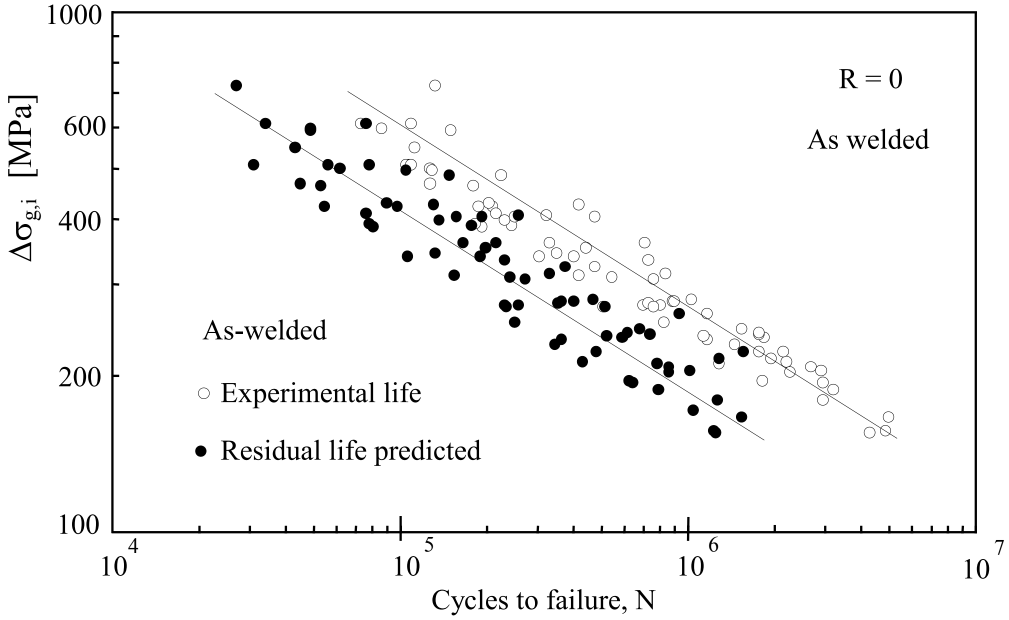

Figure 4: Fatigue strength data and residual life estimates. Residual life determined by assuming the initial crack length a_i = 0.3mm and the crack propagation along the direction (θ = 22.5°). Y=1.122√sec(πa/t); coefficients in the Paris law: m=3.0; C=0.183x 10^{-12}, (N and mm as units), (Gurney, 1991).

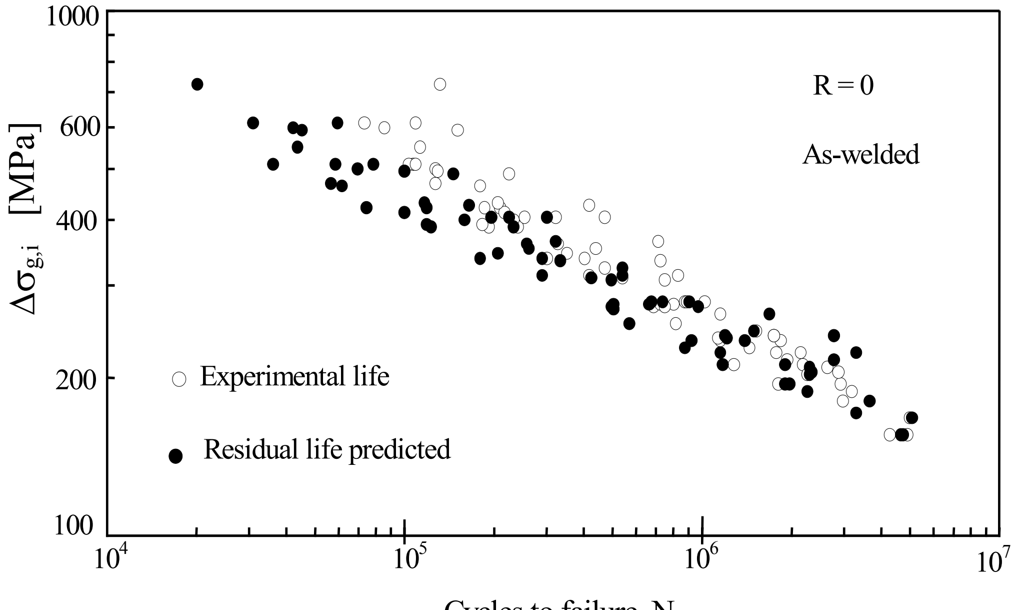

Figure 5: Fatigue strength data and residual life estimates. Residual life determined by assuming the initial crack length a_0 = 0.3mm and the crack propagation along the direction θ = 22.5°. Y=1.122√sec(πa/t). Coefficients in the Paris law: m=4.0; C=0.2046 10^{-15} (N and mm as units), (Gurney 1991).

LOCAL-STRAIN-ENERGY APPROACH (failure from both weld toe and root)

In a plane problem all stress and strain components in the highly stressed region are correlated to mode I and mode II NSIFs. Under a plane strain hypothesis, the strain energy included in a semicircular sector embracing the point of singularity is (Lazzarin and Zambardi, 2001):

$$\Delta\overline{W} = \frac{e_1}{E}\left[\frac{\Delta K_1^N}{R_C^{1-\lambda_1}}\right]^2 + \frac{e_2}{E}\left[\frac{\Delta K_2^N}{R_C^{1-\lambda_2}}\right]^2 \tag{12}$$

where R_C is the radius of the semicircular sector surrounding the weld toe or the weld root whereas e_1 and e_2 are two functions that depend on the opening angle 2α and the Poisson coefficient ν (see Table 1). A rapid calculation, with $\nu = 0.3$, can be made by using the following expressions (Lazzarin and Zambardi, 2001):

$$e_1 = -5.373 \cdot 10^{-6}\,(2\alpha)^2 + 6.151 \cdot 10^{-4}\,(2\alpha) + 0.1330 \tag{13}$$

$$e_2 = 4.809 \cdot 10^{-6}\,(2\alpha)^2 - 2.346 \cdot 10^{-3}\,(2\alpha) + 0.3400 \tag{14}$$

where 2α is in degrees.

Dealing with strain energy density, it is worth mentioning Sih's criterion based on the strain energy density factor S (Sih, 1974). The parameter S is the product of the strain energy density and a small distance from the point of singularity. Failure was thought of as controlled by a critical value of S, whereas the direction of crack propagation is determined by imposing a minimum condition on S. However, Sih's criterion is a point-related criterion. The minimum of S, correlated to a material-dependent parameter, is the failure criterion. Here we use an area- or volume-related averaged value of the strain energy density, which does not predict the direction of crack propagation, but only failure at a specific critical value, which is independent of the V-notch angle.

The radius R_C, which is thought of as a welded material property, can be estimated by using the fatigue strength $\Delta\sigma_A$ of the butt ground welded joints (in order to quantify the influence of the welding process, in the absence of any stress concentration effect) and the NSIF-based fatigue strength of welded joints having a V-notch angle at the weld toe constant and large enough to ensure the non singularity of mode II stress distributions. Under plane strain conditions and in the presence of a Mode II contribution non-singular, the expression for R_C becomes (Lazzarin and Zambardi, 2001):

$$R_C = \left(\frac{\sqrt{2e_1}\,\Delta K_{1A}^N}{\Delta\sigma_A}\right)^{\frac{1}{1-\lambda_1}} \tag{15}$$

where both λ_1 and e_1 depend on the V-notch angle (see Table 1).

At $N_A = 5 \cdot 10^6$ cycles and with a nominal load ratio R=0, a mean value of ΔK_{1A}^{N} is equal to 211 MPa $mm^{0.326}$ for welded joints with 2α=135°at the weld toe, as shown in Figure 3. At N_A= $5 \cdot 10^6$ cycles, with R=0, butt ground welds made of ferritic steels give $\Delta\sigma_A$ = 155 MPa (Atzori and Dattoma, 1983, Taylor et al., 2002). By introducing these two values into Eq. (15) one obtains R_C =0.28 mm. The choice of 5 million cycles as a reference value is due mainly to the fact that, according to Eurocode 3, nominal stress ranges corresponding to 5 million cycles can be considered as fatigue limits under constant amplitude load histories.

Finally, it is worth noting that at the weld root the V-notch becomes a crack-like notch (2α=0, λ_1 =0.5 and e_1 =0.133), and then Eq. (15) becomes:

$$R_C = \frac{0.85}{\pi}\left(\frac{\Delta K_{th}}{\Delta\sigma_A}\right)^2 = 0.85\, a_0 \qquad (16)$$

so that Eq. (15) establishes a bridging between the value of R_C and the well known material parameter a_0 (El Haddad et al., 1979). However, the coefficient 0.85 would be different if one had used different working hypotheses (plane stress conditions instead of plane strain conditions, for example, or deviatoric strain energy instead of total strain energy).

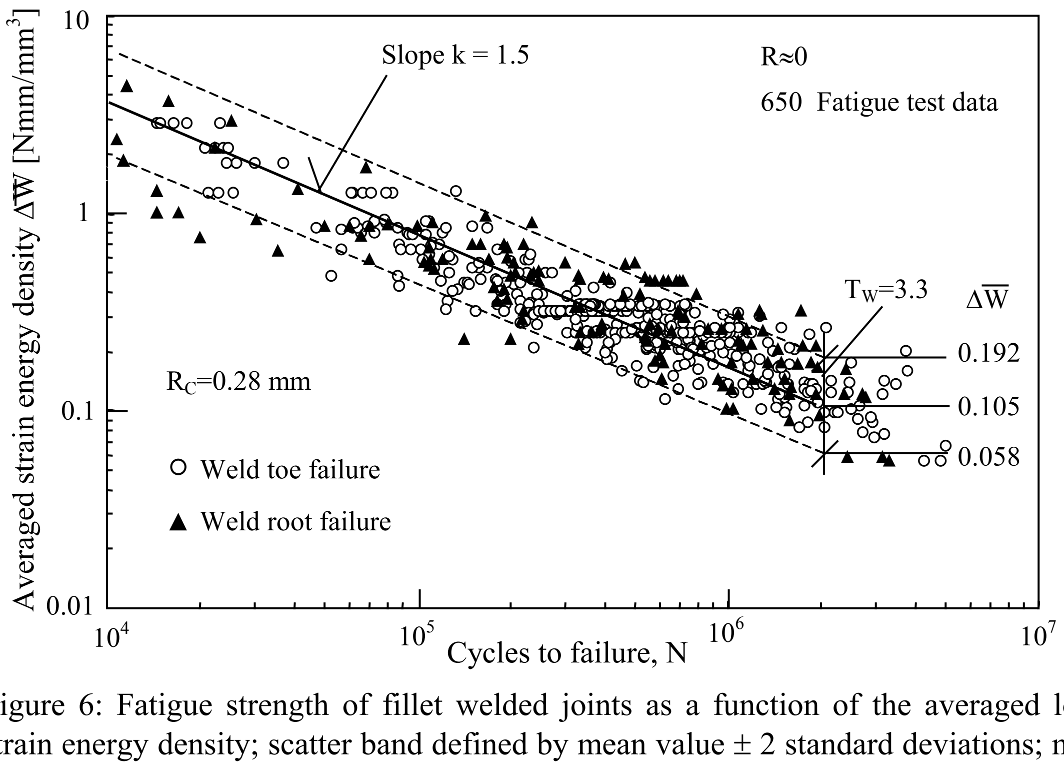

Figure 6: Fatigue strength of fillet welded joints as a function of the averaged local strain energy density; scatter band defined by mean value ± 2 standard deviations; main plate thickness ranging from 6 to 100 mm; crack initiation at weld toe or weld root.

A synthesis including about 650 fatigue data, mainly from transverse fillet welded joints with final fractures originating from the weld toe or weld root is shown in Figure 6. In all these cases, the weld toe (or the weld root) was always modelled as a sharp notch, ρ=0 (V-notch or crack). Detailed information on the adopted steels, welding technologies and all geometrical parameters is reported in (Livieri and Lazzarin, 2005). The scatter index T_W, related to probabilities of survival P_S=2.3% and 97.7 %, is found to be 3.3. It is worth noting that the scatter index becomes $\sqrt{3.3}/1.21 = 1.50$ if reconverted into an equivalent local stress range with probabilities of survival P_S=10% and 90%. The value of 1.50 is in agreement with Haibach's normalised S-N curve (Haibach, 1989).

PEAK STRESS METHOD (failure from the weld toe)

The three approaches summarised in the previous paragraph require that the NSIFs are known (see Eqs (4), (8,9), (12)). The NSIFs can be calculated by applying definitions given by expressions (1) provided that the local stress field $\sigma_{\theta\theta}(r,\theta)$ is accurately described. Typically, local stresses are calculated by means of dedicated finite element analyses by using very refined meshes and then time consuming numerical analyses in both two dimensional (Lazzarin and Tovo, 1998; Atzori and Meneghetti, 2001) and particularly in three dimensional cases (Meneghetti, 1998). As an example, the numerical analyses of the welded joints described in the previous sections often required the use of finite elements having edges as small as 1 μm or less.

Nisitani and co-workers recently proposed a simplified numerical method able to estimate the SIF of a crack (Nisitani et al., 2004). It is based on the usefulness of the elastic peak stress evaluated at the crack tip by the finite element method and obtained by means of a mesh pattern characterised by a given element size. The ratio between the SIF K_I and the elastic peak stress σ_{peak} was seen to be independent on the crack length: then the elastic peak stress can substitute the use of the SIF in fatigue analysis of cracked components.

From a practical point of view such a method is very convenient for at least two reasons: firstly a coarse mesh is sufficient in order to assure the constancy of the K_I/σ_{peak} ratio as compared with that needed to calculate the SIF; secondly just a nodal stress value is used for fatigue life calculations instead of a set of stress-distance data which are necessary in order to evaluate the SIF according to an expression similar to (1).

Such a method has been later extended to analyse the local stress state of components weakened by sharp V-notches characterised by an opening angle greater than zero, and, in particular, fillet-welded joints (Meneghetti, 2002). The joint geometries documented in Table 2 were re-analysed in terms of elastic peak stress evaluated by the finite element method, as well as the joints in aluminium alloy which have been re-analysed in terms of NSIF in Figure 3. A regular mesh pattern of PLANE 42 linear quadrilateral elements available in the Ansys 8.0 element library was adopted. During the automatic mesh generation a 'global element size' parameter equal to 0.5 mm was set such that a typical mesh pattern like that shown in Figure 5 was obtained. Figure 6 reports the results for welded joints in steel, while Figure 7 reports the results for the aluminium alloy case.

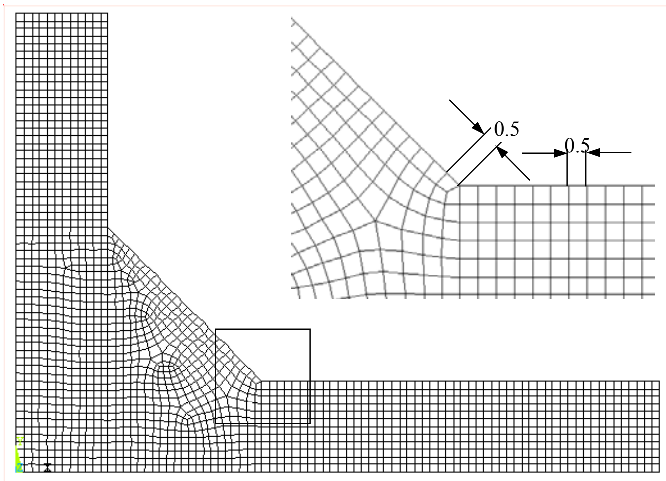

Figure 5: adopted mesh pattern for the stress analysis of cruciform joints. (Main plate thickness t= 12 mm, adopted element size: 0.5 mm)

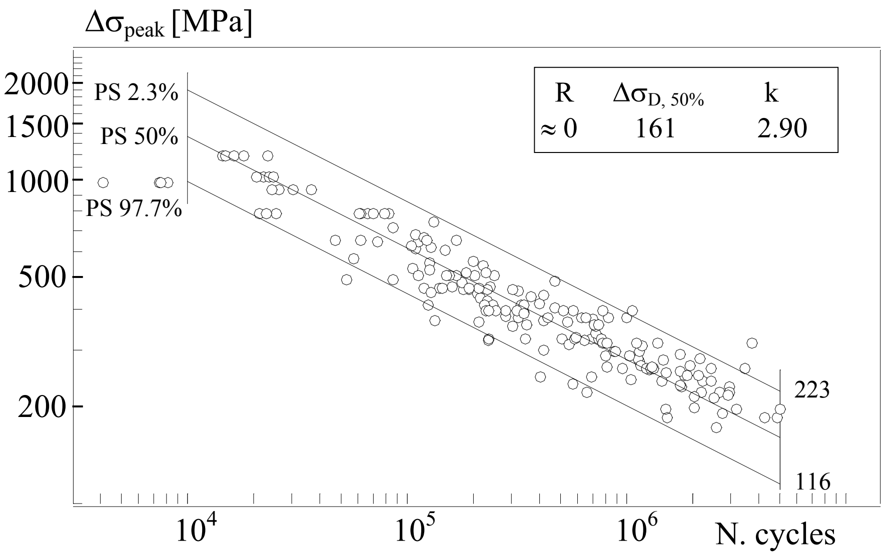

Figure 6: fatigue strength of fillet welded joints in structural steel in terms of elastic peak stress evaluated at the weld toe.

Figures 6 and 7 shows that the scatter band amplitudes are comparable with those reported in Figure 3. In fact the T_σ parameter, defined as the ratio between the fatigue strength for a survival probability of 2.3% and 97.7% at a given number of cycles, is equal to 1.92 in Figure 6, which is in good agreement with the value of 1.85 of the

NSIF-based scatter band; the slope of the fatigue curves is equal to 2.90, which substantially coincide with the value of 3.0 in Figure 3. Concerning the welded joints in aluminium alloys, the T_σ parameter is equal to 2.19, which is slightly higher than 1.8 in terms of NSIF, while the slope is 3.59, which is close to the value of 4.0 shown in Figure 3.

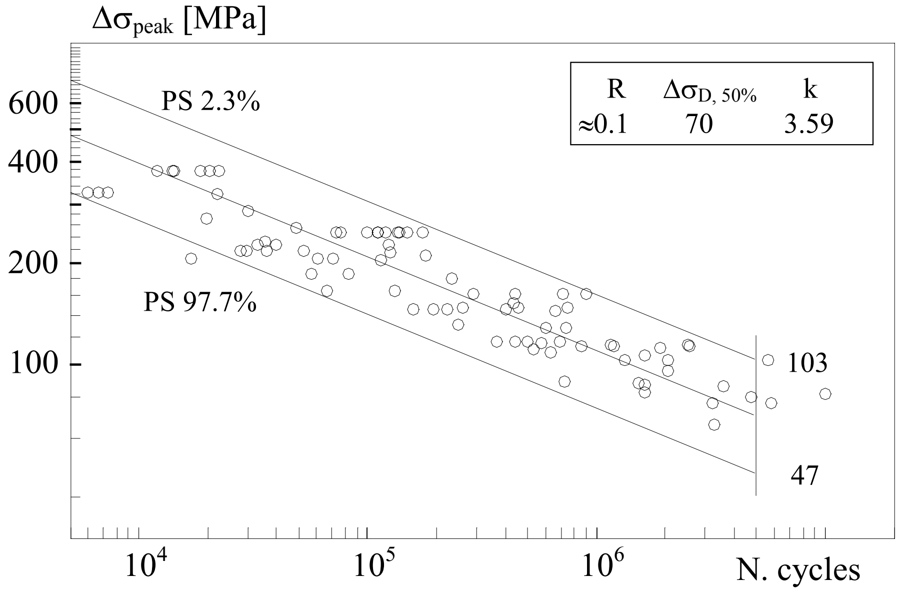

Figure 7: fatigue strength of fillet welded joints in aluminium alloys in terms of elastic peak stress evaluated at the weld toe.

It is believed that in view of practical applications the peak stress method can be adopted in an industrial context in order to efficiently assess the fatigue life of welded joints. In fact it combines the sound basis of the NSIF local approach with the simplicity of a point-like method. It should be noted that, differently from other engineering methods like the so-called hot-spot or structural stress approach, the peak stress method includes the scale effect 'by nature', because it represents a simplified way to estimate the local stress field parameter ΔK_1^N .

CONCLUSIONS

Fatigue life assessments are performed on the basis of the NSIFs, which are determined by setting the weld toe radius equal to zero and modelling the highly stressed regions as sharp V-notches.

Fatigue damage is generally described as the nucleation and growth of cracks to final failure, although the differentiation of two stages is quantitatively ambiguous. Therefore, the paper operates a second strong simplification. Since most of the fatigue life is spent in short crack propagation within the region of the virtual singularity due to the notch, the total fatigue life of specimens has been directly correlated to NSIF without any distinction between initiation and microcrack propagation. In the real size structures, where redundant load paths are generally present, a conventional demarcation line has to be drawn, and the NSIF should be used only to quantify the fatigue crack initiation life.

Units for NSIFs vary according to the V-notch angle. In order to collect fatigue data obtained from joints with different values of 2α, as well as cases of failures from weld root and weld toe, a simple scalar quantity was used as unifying parameter, i.e. the strain energy range included in a control volume being represented by a semicircular sector of radius R_C. The energy was evaluated under the plane strain hypothesis, assuming for the material a linear elastic law. As it happens for the El Haddad material parameter a_0, the evaluation of R_C needs the determination of an NSIF-based curve and the high cycle fatigue strength of butt ground welded joints. The radius R_C was found to be 0.28 mm for welded joints made of structural steels.

Finally a simplified application of the NSIF approach was used in order to assess the fatigue strength of fillet welded joints failing from the weld toe. Such a method takes as design stress the elastic peak stress evaluated at the weld toe by means of a finite element analysis performed with a mesh characterised by a constant element size. By so doing, it is possible to take advantage of both the simplicity of a point-like method and the robustness of the NSIF local approach.

REFERENCES

1. Albrecht, P., Yamada, K. (1977). *J. Struct. Division* ASCE 103, 377-389.
2. Atzori, B., Haibach. E. (1979). In: Procs VII Congress of Italian Society for Stress Analysis (AIAS ed.), Cagliari, Italy.
3. Atzori B., Dattoma V. (1983). IIW Document Doc XIII – 1089/83.
4. Atzori, B. (1985). In: Procs XIII Congress of Italian Society for Strain Analysis (AIAS ed.), Bergamo, Italy.
5. Atzori, B., Blasi, G., Pappalettere, C. (1985). *Exp. Mech.* **25** (2), 129-139.
6. Atzori, B., Lazzarin, P., Tovo R. (1999a). *Fatigue Fract. Engng Mater. Struct.* **22**, (5), 369-382.
7. Atzori, B., Lazzarin, P., Tovo, R. (1999b). *J. Strain Anal. Eng.* **34**, 437-453.
8. Atzori, B., Meneghetti, G. (2001). *Int. J. Fatigue* **23 (8)**, 713-721.
9. Atzori, B., Meneghetti, G., Susmel, L. (2002). *Int. J. Fatigue* **24**, 591-599.
10. Boukharouba, T., Tamine, T., Nui, L., Chehimi, C., and Pluvinage, G. (1995). *Eng. Fract. Mech.* **52**, 503-512.
11. El Haddad, M.H., Topper, T.H., Smith, K.N. (1979). *Eng. Fract. Mech.* **11**, 573-584.
12. Gross, R., Mendelson, A. (1972). *Int. J. Fract. Mech.* **8**, 267-276.
13. Gurney, TR. (1991). The fatigue strength of transverse fillet welded joints. Abington Publishing, Cambridge.
14. Haibach, E (1989). Service Fatigue-Strength – Methods and data for structural

analysis. Dusseldorf, VDI, 1992.
15. Jiang, Y., Feng, M. (2004). *J. Eng. Mater. Tech.* **126**, 77-86.
16. Lazzarin, P., Tovo R. (1996). *Int. J. Fracture* **78**, 3-19
17. Lazzarin, P., Tovo, R. (1998). *Fatigue Fract. Engng Mater. Struct.* **21**,1089-1104.
18. Lazzarin, P., Livieri, P. (2001). *Int. J. Fatigue* **23**, 225-232.
19. Lazzarin, P., Zambardi, R. (2001). *Int. J. Fracture* **112**, 275-298.
20. Lazzarin, P., Lassen, T., Livieri, P. (2003). *Fatigue Fract. Engng Mater. Struct.* **26**, 49-58.
21. Lazzarin P., Sonsino, C.M., Zambardi R. (2004). *Fatigue Fract. Engng Mater. Struct.* **27**, 127-141
22. Livieri, P., Lazzarin, P. (2005). *Int. J. Fracture* **133**, 247-278.
23. Maddox, S.J. (1987). The effect of plate thickness on the fatigue strength of fillet welded joints. Abington Publishing, Abington, Cambridge.
24. Meneghetti, G. (1998). PhD Thesis, University of Padova, Italy.
25. Meneghetti, G. (2002). *Rivista Italiana della Saldatura* **4**, 499-505 (in italian).
26. Nisitani, H., Teranishi, T. (2004). *Eng. Fract. Mech.* **71**, 579-585.
27. Sih, G.C. (1974). *Int. J. Fracture* **10**, 305-321.
28. Taylor, D., Barrett, N., Lucano, G. (2002). *Int. J. Fatigue* **24**, 509-518.
29. Verreman, Y., Nie, B. (1996). *Fatigue Fract. Engng Mater. Struct.* **19**, 669-681.
30. Williams, M.L. (1952). *J. Appl. Mech.* **19**, 526-528.
31. Yakubovskii, V.V., Valteris, I.I. (1989). IIW Document XIII-1326-89.