# Crack paths under mixed mode loading

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***ABSTRACT.*** *Long fatigue cracks that initially experience mixed mode displacements usually change direction in response to cyclic elastic stresses. Eventually the cracks tend to orient themselves into a pure mode I condition, but the path that they take can be complex and chaotic. In this paper we report on recent development in techniques for tracking the crack path as it grows and evaluating the strength of the mixed mode crack tip stress field.*

## INTRODUCTION

There are many opportunities for cracks and crack-like defects in engineering structures to exist in orientations that induce mixed mode crack tip displacements. Defects arising from fabrication processes such as welding or casting; cracks created under the action of residual tensile stresses; cracking of embrittled microstructures; and fatigue cracks that have grown under the action of some previously applied loading cycles that differ from the current load case can all create flaws with some arbitrary combination of mode I, II and III stress intensities.

Broberg [1] discussed aspects of the stability of the crack path under pure and mixed mode loads and concluded that crack paths remain straight under homogeneous remote stress fields. However, engineering structures in service rarely experience such well defined uniform stress fields. Stress and strain gradients arising from geometric features, multiple cracks and non-uniform, non-proportional remote loads commonly occur.

Applied mixed mode loading and interaction between multiple cracks are the principal causes of a major loss of directional stability. Highly anisotropic microstructures can also lead to significant changes in crack orientation but more often are responsible to local deviations, or ‘zigzags’, in the overall mode I crack trajectory.

Broberg also noted that the ideal mode I elastic crack tip stress field did not provided a sufficient condition for cracks to maintain a straight path. It was proposed, from the work of Rice *et al*. [2] and Anderson [3], that it is the plastic flow at the crack tip that dictates the crack path. The notion that the crack path is governed by the plastic behaviour of the crack tip is supported by the many workers. Under mode III loading, the propensity for flat, or shear, mode growth is strongly influenced by the plastic part of the crack tip displacement. Plumbridge [4] found, in experiments on aluminium plates cyclically loaded under Mode III loading conditions, that only when a fully plastic situation exists, does crack extension proceed by a valid Mode III mechanism. When plasticity is restricted to planes of maximum shear there is a strong component of Mode I cracking which results in delamination in the direction of macroscopic growth. In torsional fatigue, small axial loads, or prior residual strains, also play an important role in governing the flat or twisted path of cracks [5, 6]. The extent of crack tip plasticity, and hence the prevalence of flat mode growth, is also dependent upon the size of the cylindrical component [7,8]; small diameter shafts being more prone to flat crack growth than large shafts for the same stress, or strain, intensity factor. The shear *versus* branch crack competition is probably most apparent under sequential cyclic mode I and mode II loads, as experienced in cracked railway lines. The evidence for the role of the crack tip plasticity in governing the crack trajectory in this case is overwhelming [9].

The path of a fatigue crack under proportional loading from an initially mixed mode condition, as created by angled or inclined cracks in laboratory specimens, is surprisingly stable. One might expect major variations, as a function of mean stress for example, but there is little evidence to this effect. Nevertheless, there are subtle differences in the crack trajectory in specimens under identical test conditions. These small scale fluctuations in crack path are worthy of detailed investigation but, until recently, experimental techniques to evaluate the strength of mixed mode crack field have not been precise or reliable enough to yield useful information.

Understanding the behaviour of mixed mode cracks in general, and the path of such cracks in particular, requires a combination of high quality experimental data and observations as well as robust physically based models. Good data on the crack tip stress state, crack closure and contact, and the crack trajectory is hard to obtain and there has been much recent work in this area.

In this paper, we set out to report on some recent developments in gathering experimental data on mixed mode stress and displacement fields. We also consider how such techniques might provide an opportunity to investigate issues surrounding the stability of crack paths in varying stress fields.

## OVERVIEW OF FULL FIELD TECHNIQUES FOR CRACK ANALYSIS

Photoelasticity, moiré interferometry, electronic speckle pattern interferometry (ESPI), image correlation and thermoelasticity, or differential thermography, are all techniques which provide full field experimental data on crack tip displacements or strains. From these data, crack tip stresses can be inferred and hence stress intensity factors derived. With the advent of advanced computing power and digital image processing, techniques such as photoelasticity and moiré interferometry have moved from slow manual methods where fringe orders must be identified and located by an experienced operator, to those where stress intensity factors may be determined in a matter of minutes.

Fracture mechanics studies using transmission photoelasticity require fine slits to be introduced into epoxy models of engineering components [10, 11]. Several methods have been developed to determine $K_I$ and $K_{II}$ using the full field of data surrounding the slit tip [12, 13]. Nurse and Patterson [10] also developed a photoelastic method to predict the direction of crack growth using the theory that long cracks usually grow under mode I loading in direction perpendicular to maximum circumferential stress. They found that when $K_{II}/K_I$ is less than 0.7, this direction is approximately equivalent to the axis of symmetry observed in the isochromatic fringes loops and so one can predict the direction of crack growth. This method was further developed by Burguete and Patterson [14] to investigate the effect of friction on crack propagation in the dovetail fixings of gas turbine compressor discs.

Nurse and Patterson [15] used reflection photoelasticity to study a fatigue crack in an aluminium alloy using a stroboscopic light source over the complete load cycle. However the drawback to this method is the fact that the birefringent coating must not cover the crack and thus the crack growth direction must be predicted before applying the coating. Further investigations of fatigue crack closure were made by Pacey *et al.* [16], using transmission photoelasticity through a polycarbonate specimen, which is sufficiently ductile to allow fatigue crack growth. A method to evaluate mixed mode stress intensity factors was developed based on the Muskhelishvili stress field formulation together with a genetic algorithm and the downhill simplex method. This numerical optimisation procedure was found to offer a significant advance in the ability of characterise the behaviour of fatigue cracks with plasticity induced crack closure.

Similar studies on mixed mode fatigue crack propagation have been carried out using geometric moiré [17] and moiré interferometry [18]. Moiré methods are particularly useful when making high temperature measurements [19]. Moiré interferometry involves bonding a fine grating ahead of the crack tip which in the past risked debonding due to the high strain gradients in that area. Recent development of photoresist methods allow the production of well-adhered gratings of 0.75μm thickness [20]. Such developments mean that fatigue cracks can grow through the grating and allow detailed crack closure investigations to be carried out [21].

When studying fatigue crack propagation it is desirable to be able to evaluate the stress intensity factor range of the growing crack. To do so, the techniques based on photoelasticity and interferometry require data to be collected at maximum and minimum load. This can be done in several ways. Firstly, the cycling can be stopped at the required loads and data taken under static conditions. Alternatively, the component can be illuminated by a stroboscopic light synchronised with some part of the fatigue cycle. The development of modern high speed digital video cameras means that data can be collected at several points in the loading cycle and the changing stress field determined throughout the cycle.

Differential thermography, or Thermoelastic Stress Analysis (TSA), has proved to be an invaluable tool to explore the crack tip strain field during fatigue loading [22-25]. When a material is subject to cyclic strain under adiabatic conditions, an asynchronous cyclic temperature variation occurs on its surface which is directly proportional to the first strain invariant. In thermoelastic stress analysis, this temperature variation is measured using very sensitive infra-red detectors and processed to provide a map of the surface stress distribution. When the mixed mode stress field around a fatigue crack is examined, see Figure 1, the temperature data from the elastic field around the crack tip can be used to evaluate the range of both $\Delta K_I$ and $\Delta K_{II}$. A number of methodologies for calculating the stress intensity factor are available and have been reviewed by Tomlinson and Olden [26] in 1999. More recently, developments have focussed on more accurate determination of mixed-mode stress intensity factors [27-29].

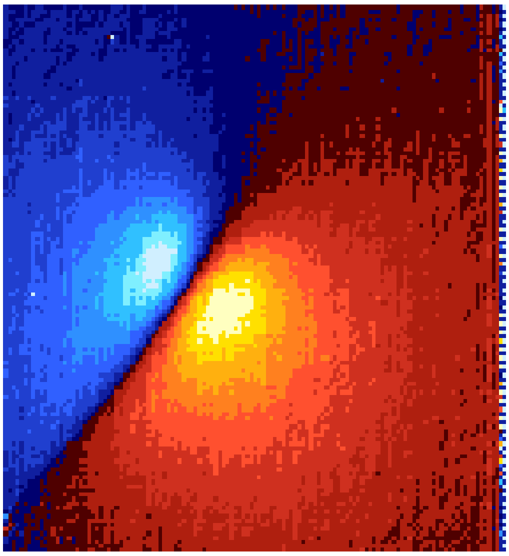


Figure 1. Mixed mode I+II crack tip stress field using TSA, from [37].

Historically, the analysis process required knowledge of the location of the crack tip and an initial estimate of the stress intensity factor. Further developments of the TSA technique [30, 31] provided both a means of tracking the location of the crack tip during propagation under cyclic loading and determining the stress intensity factor range *a priori*, see Figure 2.

Extracting the elastic stresses from around the growing crack tip provides a good estimate of, what is often called, the effective stress intensity factor range. In reality, this is the true stress field experienced by the crack, rather than the nominal, or applied $\Delta K$. Thermoelastic stress analysis, therefore, provides a method for estimating the crack closure levels directly.

In the last few years, Electronic Speckle Pattern Interferometry (ESPI) and image correlation have been used to measure crack tip displacements and strains. Shterenlikht *et al*. [32] developed the method used in photoelasticity by Nurse and Patterson [13] to accurately determine mixed-mode stress intensity factors using full field ESPI and image correlation data. An advantage of these techniques is that minimal specimen preparation is required, only using the painted or abraded surface of the component, unlike reflection photoelasticity and moiré where a coating or grating has to be bonded to the surface. The latest developments [33] in image correlation can provide information on the crack position and the crack tip displacement field.

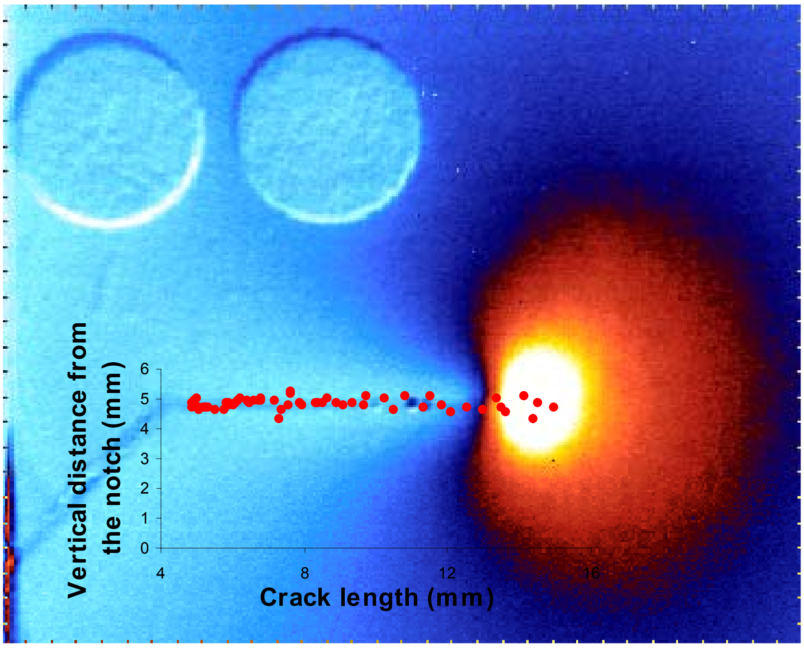



Figure 2. Tracking of a branched crack during fatigue cycling of a ferritic steel using differential thermography, from [37].

In this paper we make some observations on the paths of mixed I+III and interacting mode I cracks. We also compare the cyclic stress intensity factors obtained from differential thermography with numerical simulations using a popular finite element package.

## EXPERIMENTAL PROCEDURES

Two sets of experimental data have been examined. The first set is information on the path of mixed mode I+III cracks in a low alloy steel under different mean loads, previously reported by Yates and Mohammed [34-36]. Three point bend beams with electric discharged machined slits at angle of 45° and 60° were cyclically loaded in a Mayes servohydraulic test machine and the orientation of the crack monitored visually. Load ratios of 0.06, 0.17 and 0.5 were used.

The second set of information was obtained from offset double edge slit fatigue specimens. These were used to explore the trajectory and crack tip stress states of a pair of interacting fatigue cracks. Specimens 6 mm thick, 40 mm wide and 250 mm long were machined from a plate of 7010 T7651 aluminium alloy. Two slits, each 8 mm long, were electric discharged machined using 0.3 mm diameter wire on opposite sides of the specimens. The vertical offset between the two cracks was set at from 0, 8, 16, 32 and 64 mm for the series of tests conducted. One face of each specimen was painted with a thin coat of matt black paint (RS type 496-782) to provide a surface of uniform and known emissivity. A single rosette strain gauge (Tokyo Sokki Kenkyujo Co., 1 mm, 120 ± 0.5 Ω) was bonded to the specimen in a region of uniform and known elastic stress to provide a calibration for the thermoelastic data.

Specimens were loaded through two pins located 210 mm apart. Fatigue tests were conducted under load control at a frequency of 20 Hz, a range of 3.6 kN and a mean load of 14.4 kN for the 0 and 8 mm offset specimens and a range of 3.5 kN and a mean load of 8.5 kN for the remaining three specimens. The load range was reduced since considerable plasticity was observed in the first two tests. The frequency was chosen to be sufficiently high for adiabatic conditions to be attained in the material ahead of the crack tip. By doing so, we ensure that the thermoelastic signal contains information about the sum of the elastic principal stresses from which the mode I and mode II stress intensity factor ranges can be evaluated.

A Deltatherm 1550 instrument manufactured by Stress Photonics Inc. was used to gather thermoelastic data from the matt black surface. The crack tip position and the mode I and mode II stress intensity factor ranges occurring in the specimen were evaluated using the FATCAT software [37].

The FRANC2D finite element package [38] was used to predict the likely path of the cracks for each of the offset conditions. The predicted trajectory varies slightly according to the assumptions made in the calculation of stress intensity factors and the crack turning criterion chosen. Although there are no major discrepancies, there are small differences in the crack paths predicted, especially in the case where the cracks are initially only slightly offset. The paths are those found by using the MTS turning criterion and displacement correlation was used to evaluate the stress intensity factors.

## RESULTS AND DISCUSSION

The results of the mixed mode I+III crack path tests are shown in Figures 3 and 4 in terms of the angle of the crack front as a function of the extension of the crack from an initial 6 mm long slit. Since the cracks in the 45° slit specimens twist to about 60° after about 4 mm, the 60° data has been shifted by 4 mm along the X axis and superimposed on the 45° data in Figure 5 to compare the crack trajectories of the two configurations. It should be noted that the two sets of data align well and the forward trajectory of the twist crack appears to be independent of its previous path. The data suggest that the crack path is relatively insensitive to mean stress and hence, by inference, to crack closure or crack flank friction. There are local variations in crack trajectory, but these do not seem to be obviously associated with the global conditions at the crack tip. The insensitivity of the crack path to the cyclic stress intensity might be expected if the trajectory of the crack is governed by the directionality of the crack tip plasticity, as proposed by Broberg [1]. Crack face friction and other closure effects would only alter the magnitude of the plastic strains, and hence the growth rate, and not the shape of the plastic zone.

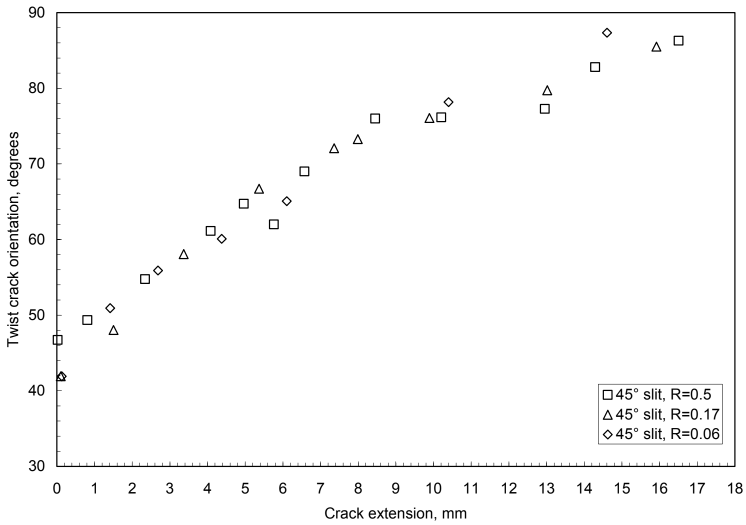



Figure 3. Mode I+III twist crack orientation in a three point bend fatigue specimen with an initial 45° slit

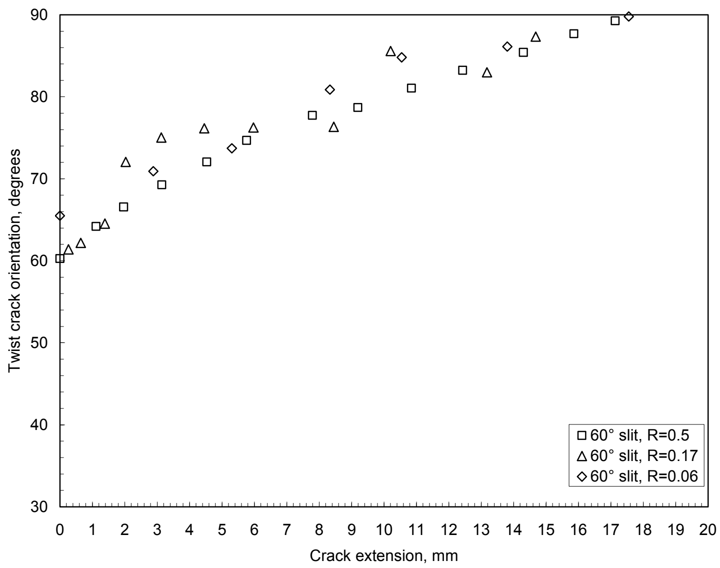



Figure 4. Mode I+III twist crack orientation in a three point bend fatigue specimen with an initial 60° slit

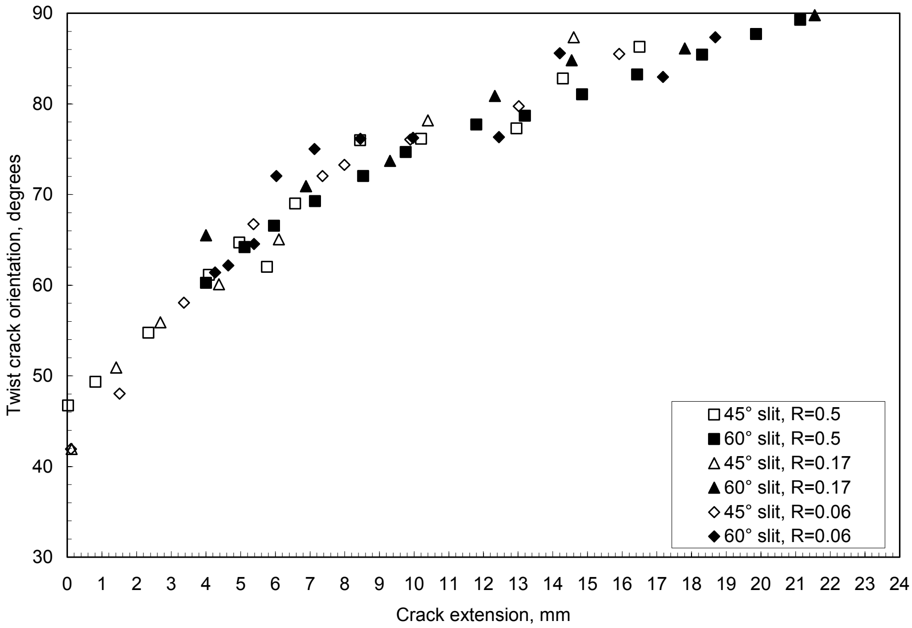



Figure 5. Mode I+III twist crack orientation in a three point bend fatigue specimen for 45° and 60° initial slit angles. The 60° data has been shifted along the X axis by 4 mm.

A qualitative comparison between the thermoelastic and finite element data is made in Figure 6. The experimental crack paths are very similar to those predicted by the finite element method. This is relatively surprising since the numerical simulations assume that both the left and right hand cracks start growing at the same time. In practice, the creation of a growing fatigue crack from the tip of the spark machine slit takes a different number of cycles in every case, and the cracks do not grow symmetrically.

Quantitative comparisons are made in Figures 7 and 8. The crack tip positions throughout the tests were located from the thermoelastic data and compared with the positions predicted by the FRANC2D finite element package for offsets of 0, 16 and 64 mm respectively. Whilst the paths, Figure 7, are very similar, the mode I stress intensity factor ranges, Figure 8, are quite different. It should be noted, as an aside, that the mode II stress intensity factors are approximately zero, as expected and as predicted by the numerical simulations.

The stress intensity factor ranges found using the thermoelastic data have been established [30] to be the true, or effective, conditions at the crack tip, and therefore incorporate the effects of crack closure and crack face friction. One might expect, therefore, that the experimental values of $\Delta K_I$ to be slightly smaller that those predicted by the finite element technique. However, the asymmetry of the crack growth completely swamps any subtle closure effects that may occur. In the zero offset case, for example, the $\Delta K_I$ of the right hand crack is much larger than that of the left hand crack since it started growing sooner and grew much longer than the left hand crack.

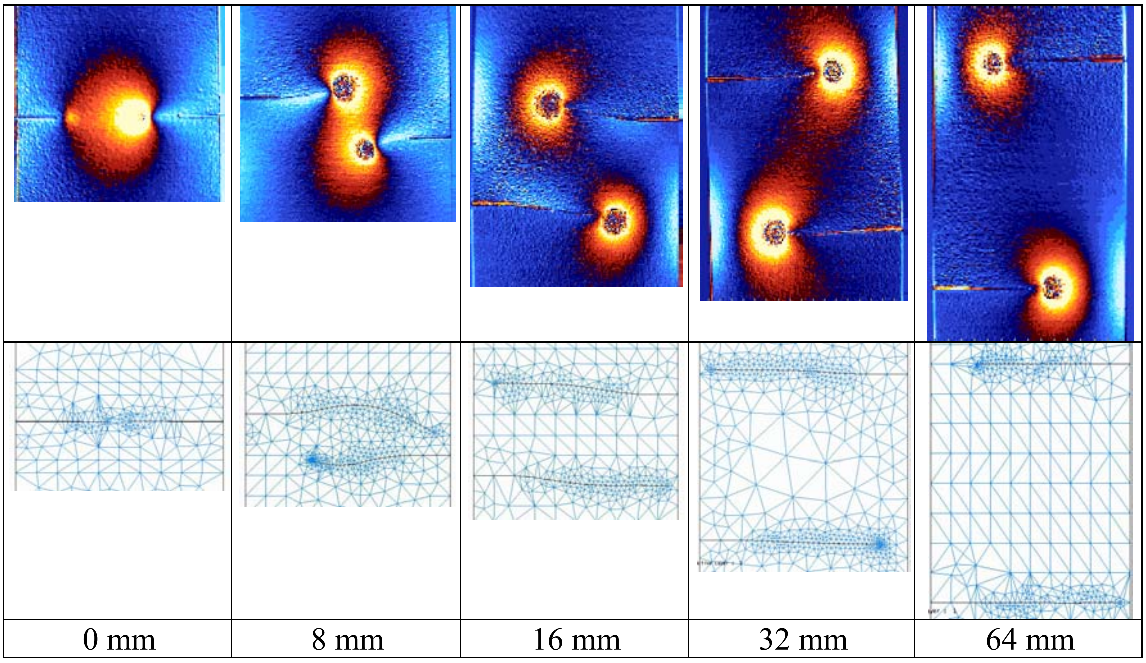
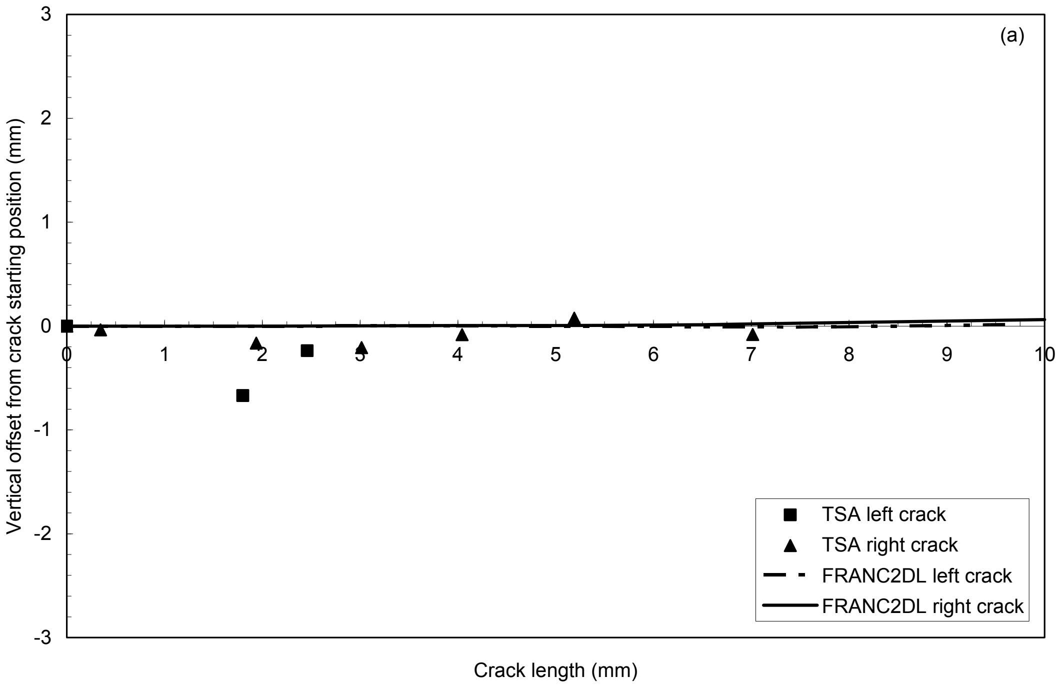


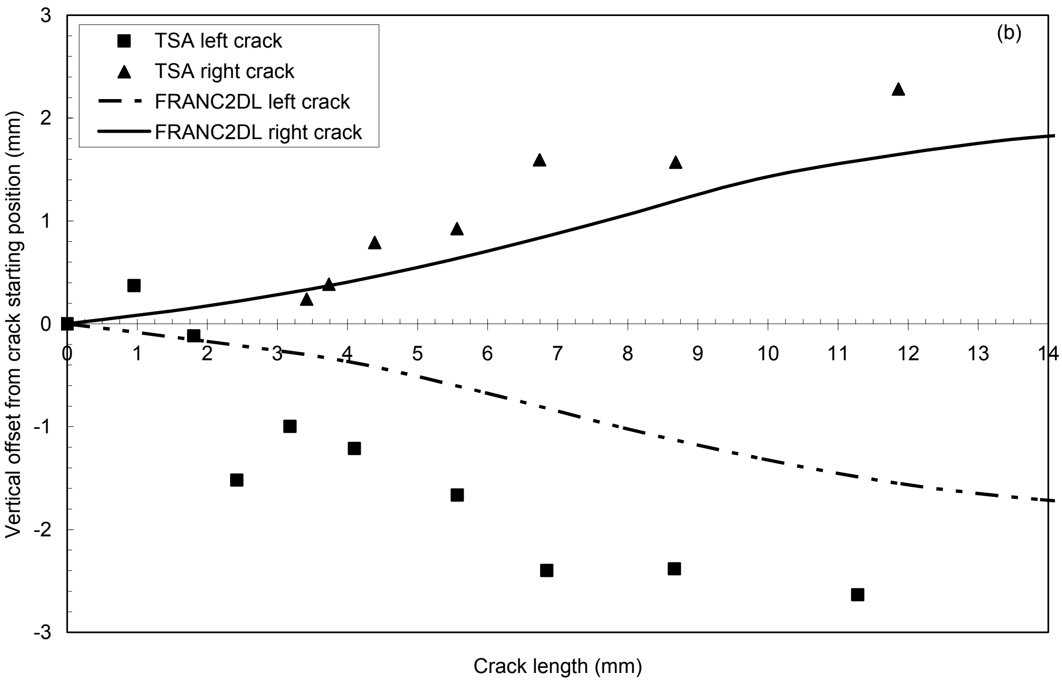

Figure 6 (top) Crack tip stress field using thermoelasticity, (bottom) predicted paths of interacting cracks using the FRANC2D finite element package for five different vertical offsets of 0, 8, 16, 32, and 64 mm.

It appears that the elastic stress field, as characterised by the stress intensity factor, is only indirectly related to the crack path. If Broberg's assertion is correct, and it is the directionality of the plastic strain field that governs the crack path, then we should be seeking ways of measuring plastic strains directly. It is suggested that the latest developments in image correlation techniques [33] and differential thermography [39] may provide a route to quantitative evaluation of the non-linear strains fields around a crack tip and hence offer some further insight into the trajectory of fatigue cracks.

## CONCLUSIONS

Recent developments in experimental mechanics offer an opportunity to explore the hypothesis that the direction of fatigue cracks may be governed more strongly by directionality of crack tip plasticity rather than by the magnitude of the elastic stress field.


(a)
3
2
1
0
-1
-2
-3
0
1
2
3
4
5
6
7
8
9
10
Vertical offset from crack starting position (mm)
Crack length (mm)
TSA left crack
TSA right crack
FRANC2DL left crack
FRANC2DL right crack


(b)
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12
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14
Vertical offset from crack starting position (mm)
Crack length (mm)
TSA left crack
TSA right crack
FRANC2DL left crack
FRANC2DL right crack

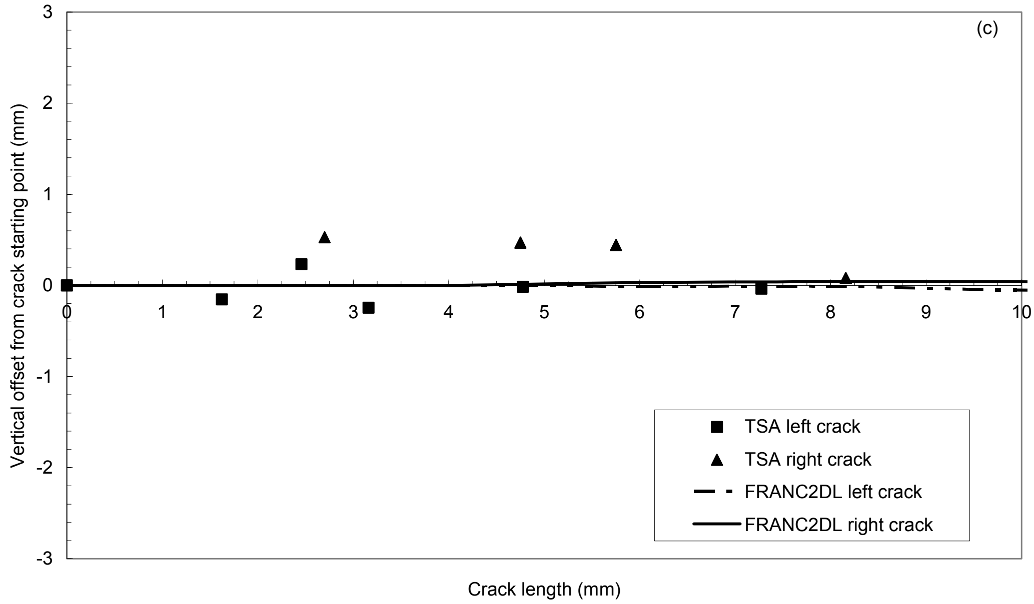
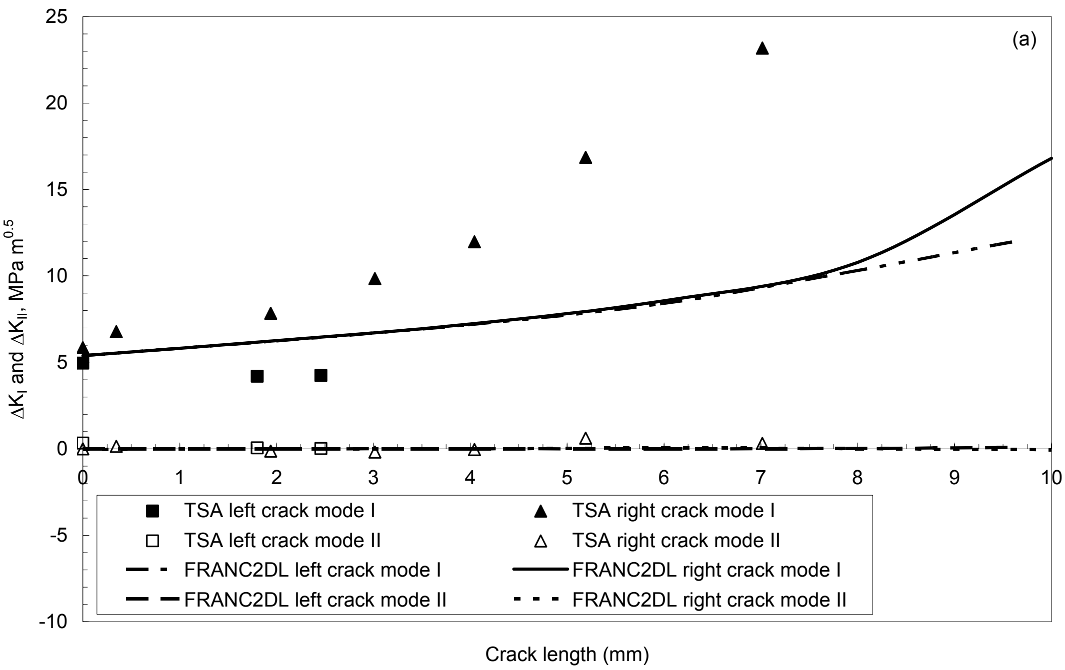



Figure 7. Left and right crack path comparison using thermoelastic stress analysis and finite element analysis (FRANC2DL). (a) 0 mm offset, (b) 16 mm offset, (c) 64 mm offset.



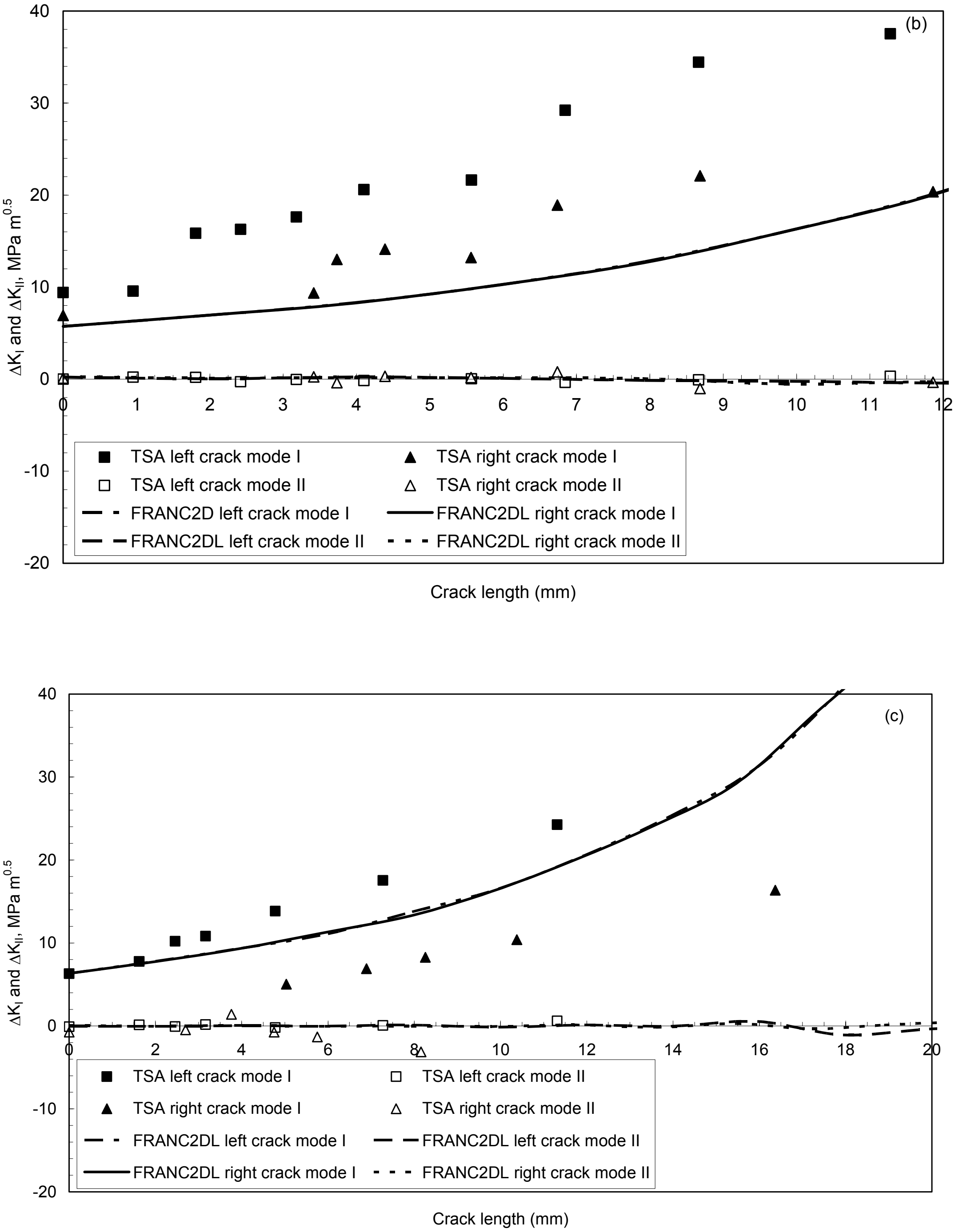



Figure 8. Left and right stress intensity factors ($\Delta K_I$ and $\Delta K_{II}$) using thermoelastic stress analysis and finite element analysis (FRANC2DL). (a) 0 mm offset, (b) 16 mm offset, (c) 64 mm offset.

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