Progress in Identifying the Real $\Delta K_{effective}$ in the Threshold Region and Beyond

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ABSTRACT *The use of the crack tip stress intensity factor, K, has survived almost 50 years as the key parameter correlating fatigue crack growth. As time past the range of the stress intensity,* ΔK*, was recognized as causing alternating plasticity at the crack tip. The threshold level for* ΔK *was discovered. Further the occurrence of crack closure was noted which effected the* ΔK *for different load ratios, R, of cyclic loading. The ASTM method of counting the linear part of the load displacement for determining* ΔK_{open} *was found to understate the* $\Delta K_{effective}$*, which correlates data for different load ratios. One approach to adjust for this problem is the "Partial Closure Model", where the closure only occurs away from the crack tip. Here it will be discussed that such a model leads to a universal growth law. Moreover, this law shows application in estimating the order of magnitude of crack growth life (*$>10^7$ *cycles) for example with very high cycle fatigue (*$>10^9$ *cycles). Some advances in this application will also be cited.*

INTRODUCTION

The use of the elastic crack tip stress intensity factor, *K,* was submitted for publication in 1959 [1] and was promptly rejected by 3 major journals (ASME, AIAA and a UK journal). In all three cases the reviewers argued that an elastic parameter could not correlate fatigue crack growth data because plasticity must be involved. Figure 1 shows the original plots of data from three independent sources on 2 aluminum alloys showing the correlation of data ignored by those reviewers. Further discussion appears in a subsequent paper [2], comparing earlier suggested parameters based on more limited data. The wide range of data provided by McEvily [3] settled this search for *K* as the leading parameter of interest. It is acknowledged that McEvily introduced a stress concentration type parameter, which was a less popular but correct approach.

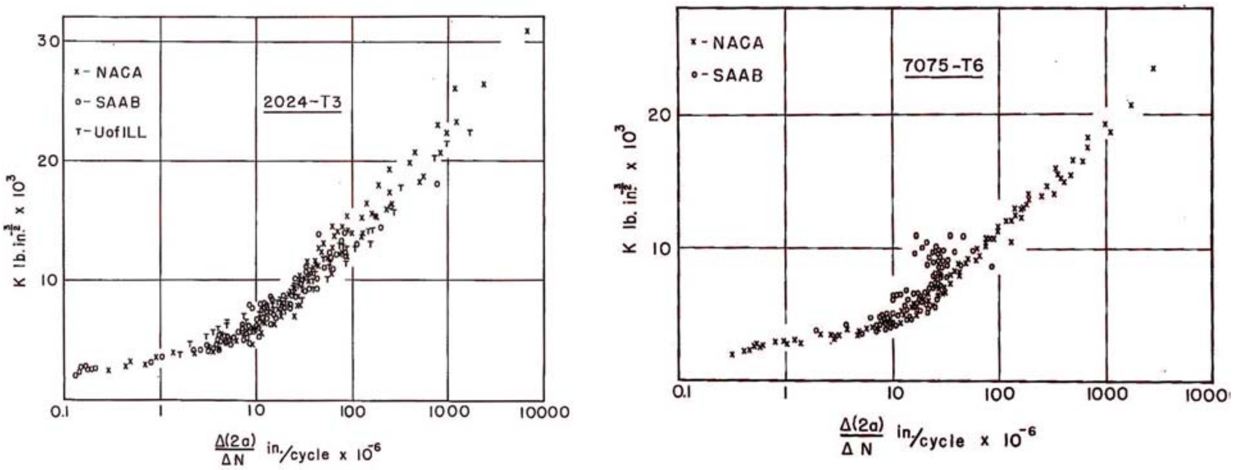

Figure 1 The original 1959 correlation of data on 2024 and 7075 aluminum alloys [1].

In this later paper [3] the power law of crack growth was presented in terms of the range of the stress intensity, ΔK, with a constant, C, dependant on the load ratio, *R,* to express the growth rate as:

$$\frac{da}{dN} = C(\Delta K)^n \qquad \text{where} \qquad C=C(R)$$

This form was merely an empirical fit of McEvily's data over a wide range of growth rates (5+ log cycles). It was observed by Hertzberg that this law failed at rates below one Burger's vector, *b,* per cycle by leveling to a threshold ΔK (private communication 1964). Even earlier Anderson [4] noted that growth rates were similar for all metal alloys if the stress intensity range was normalized by dividing by elastic modulus, *E.*

It was later in the 1960's that Elber [5] drew attention to crack closure in fatigue, although closure was noted by Christensen [6] much earlier. Thereafter, [7] Hertzberg noticed that for load ratios, *R,* above 0.7, where no closure occurs, that the preceding law herein can be made universal for all metal alloys as:

$$\frac{da}{dN} = b\left(\frac{\Delta K}{E\sqrt{b}}\right)^n \text{ where n = 3 and threshold occurs for } \frac{\Delta K}{E\sqrt{b}} = 1$$

Indeed this empirical law works for a wide variety of steels; aluminum, titanium, magnesium, and copper-beryllium alloys [7]. It remains to develop this law to an even more universal form by finding a $\Delta K_{effective}$ so that it may be applied to all load ratios, *R,* by including the effects of crack closure.

THE SEARCH FOR $\Delta K_{effective}$ WITH CRACK CLOSURE PRESENT

There is no analytical method of calculating the crack closure (or opening) level during cyclic loading. For variable amplitude there is also no method. The ASTM has tried to develop a method (see ASTM E 647-00) of measuring the opening load by determining the load level for which the load displacement record becomes linear as the crack peals open. Data in terms of load vs. displacement is analyzed to obtain the point at which the deviation from subsequent linearity is a certain small % of that slope. This load is used to compute $K_{opening}$, which along with the maximum load for $K_{\max}$, is used to compute a stress intensity range as:

$$\Delta K_{open} = K_{\max} - K_{open}$$

This was at one time regarded as the relevant ΔK causing fatigue crack growth. However, precise computer controlled load-displacement data from Donald [8] covers a wide range of load ratios, *R*. It shows that the ASTM method does not well correlate the data of widely differing load ratios. It improves correlation at high stress intensities but worsens correlation near threshold. This effect is shown on Figures 2 and 3. Donald [9]

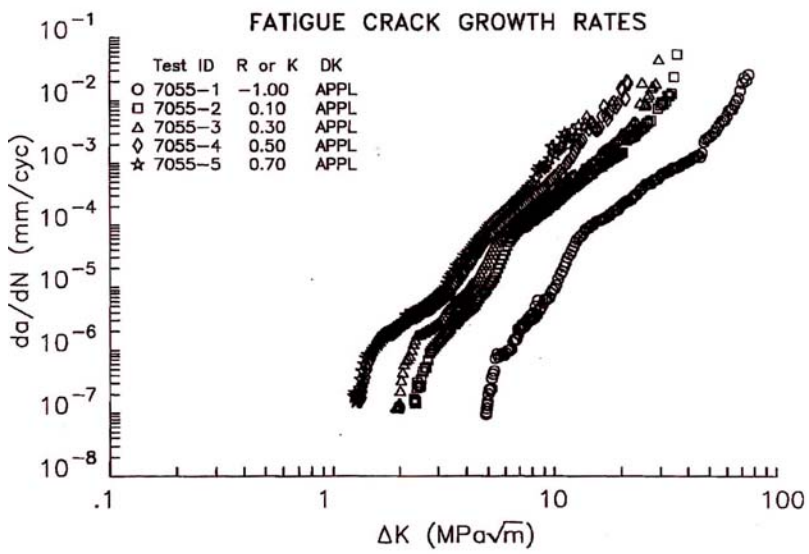

Figure 2 Data on 7055 aluminum alloy using applied stress intensity range, ΔK, [10].

proposed the "Adjusted Compliance Ratio Method" and also noted [10] a minor effect of $K_{\max}$ in the data. See Figures 4 and 5. After several years of consideration there is no known model or theory to justify this ACR method. On the other hand the "Partial Closure Model" [11] will be revisited here, which does have a physical and analytical basis. With it we shall show that the preceding normalized power law can be made universal for all load ratios.

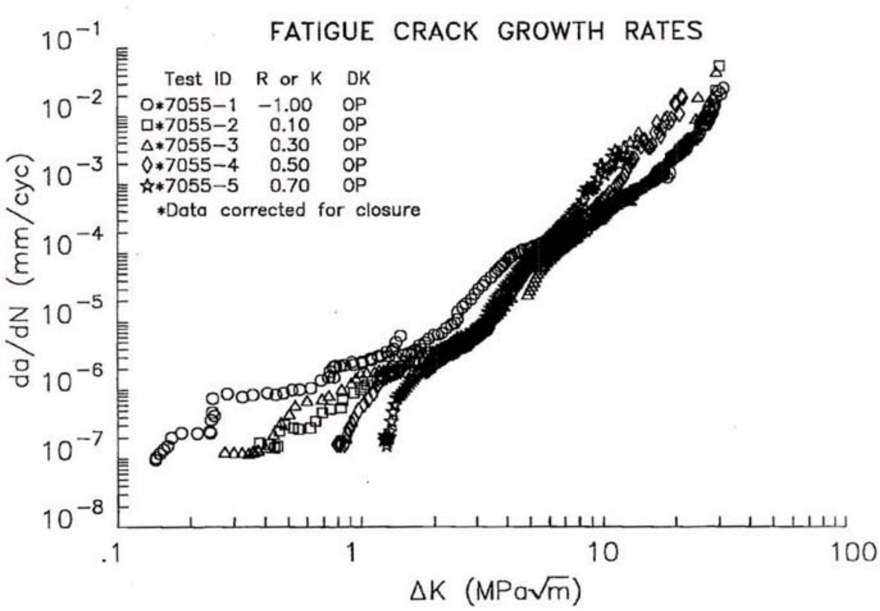

Figure 3 Data on 7055 using the ASTM $\Delta K_{opening}$ method, [10].

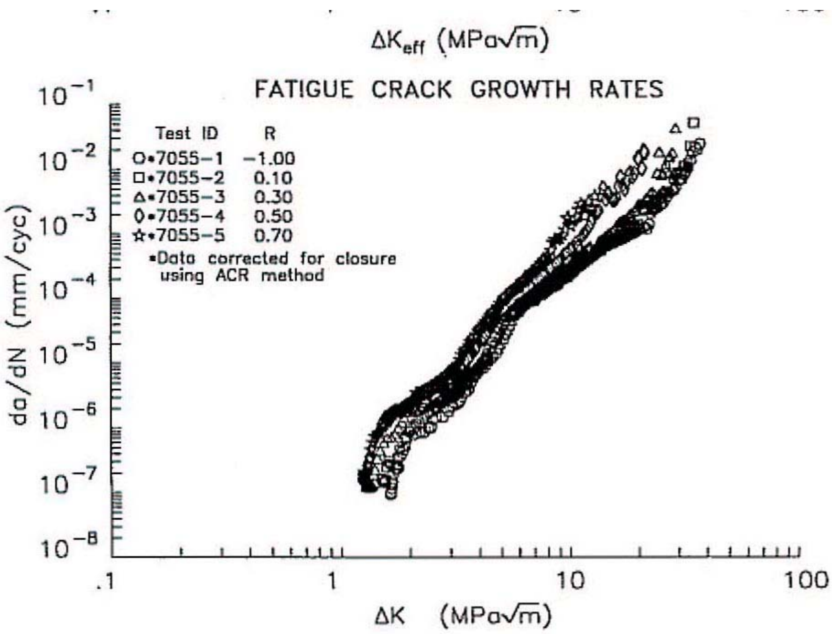

Figure 4 Data on 7055 using Donald's Adjusted Compliance ΔK_{ACR} method, [10].

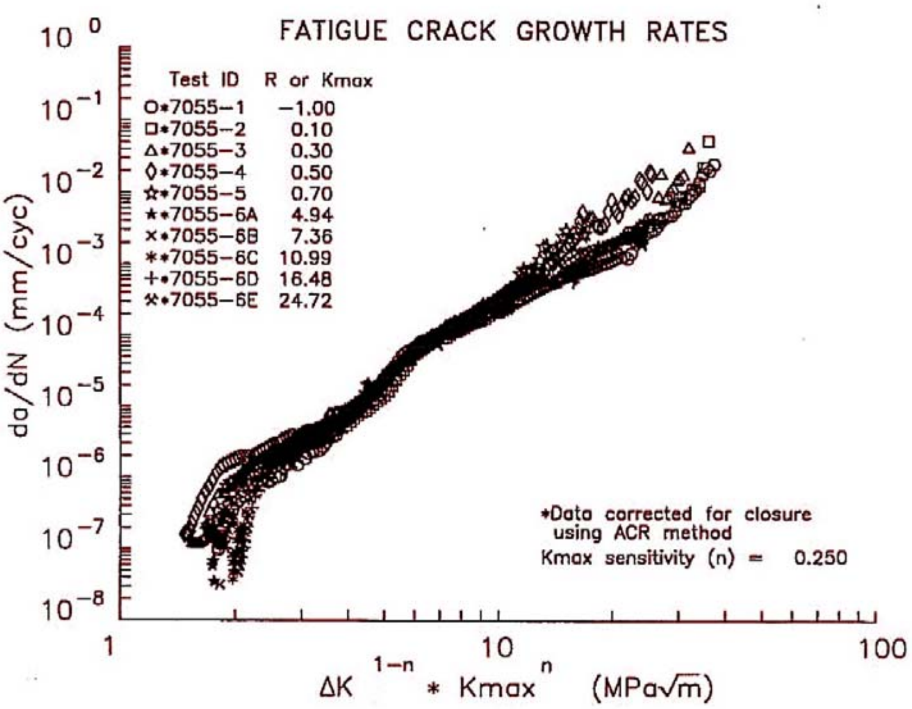

Figure 5 Data on 7055 using ΔK_{ACR} and with Donald's adjustment for $K_{\max}$, [10].

THE PARTIAL CLOSURE MODEL FOR $\Delta K_{effective}$

The doctoral dissertation of Bowles [12] noticed that with cyclic fatigue crack closure a region near the crack tip stays open at minimum load. Whether closure is due to plasticity, asperities on the surface, or fragments etc it can be modeled as a rigid layer of height, 2*h*, extending into the crack a distance, *d*, from the tip. Figure 6 shows the

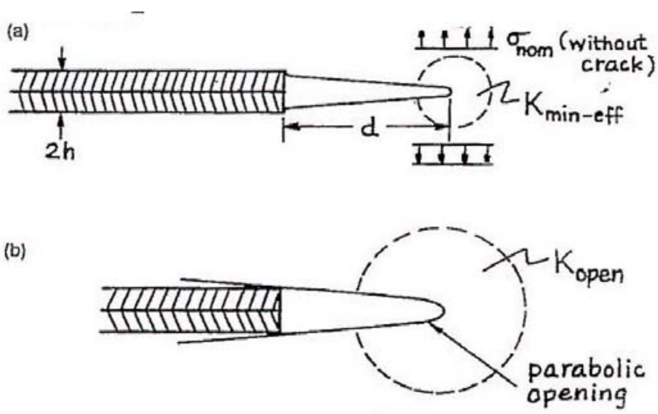

Figure 6 The computational model for the partial closure method [11].

model for (a) minimum load and (b) at opening load when crack closure occurs. For the condition at full unloading, (a), the crack tip stress intensity is found to be:

$$K_{eff-\min} = \frac{Eh}{\sqrt{2\pi d}} + \sigma_{nom-\min}\sqrt{\frac{\pi d}{2}}$$

For (b) at opening load the stress intensity is:

$$K_{open} = \frac{Eh}{2}\sqrt{\frac{\pi}{2d}}$$

Combining these gives:

$$K_{eff-\min} = \frac{2}{\pi}K_{open} + \sigma_{nom-\min}\sqrt{\frac{\pi d}{2}}$$

Where $\sigma_{nom-\min}$ is the nominal tensile stress perpendicular to the crack with the crack absent. Since the term is quite small because *d* is also small, it can be neglected. Consequently it is seen that the minimum effective stress intensity is very nearly:

$$K_{eff-\min} \cong \frac{2}{\pi}K_{open}$$

As follows from this we have called this the "Partial Closure Model" or 2/Pi0 – method where the effective stress intensity range is:

$$\Delta K_{effective} = K_{\max} - K_{eff-\min} \cong K_{\max} - \frac{2}{\pi}K_{open}$$

This implies that the ASTM opening stress intensity should be reduced by approximately $2/\pi$ to correctly compute the real stress intensity range. Figure 7 shows the same preceding data of Donald from Figures 2, 3, 4, and 5 where the data is correlated quite closely into a single curve. Though this is data on a single material the reader will find many other materials with comparative correlations in the references cited herein.

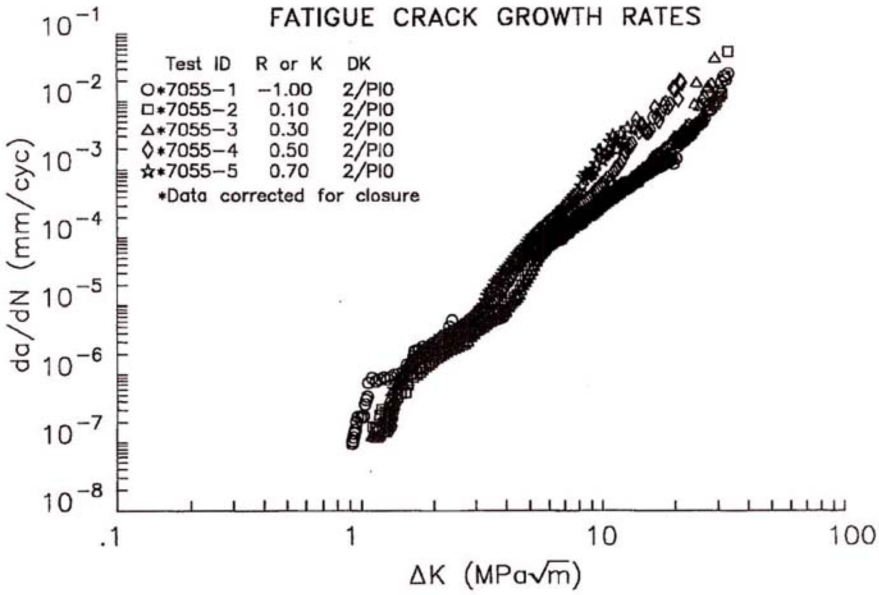

Figure 7 Data on 7055 using the partial closure model (2/Pi0) ΔK_{eff}.

The Partial Closure Model is emphasized here with some reservation. All physical models are crude approximations of reality and this one is no exception. However it happens to helpfully correlate data for considerations of whether the data is well founded and whether the material is not an oddity. The ACR method of Donald serves this same purpose in general. At least for one material he has tested, Donald has acknowledged (private communication) that the Partial Closure Model provides tighter correlation. The disadvantage of both of these models is that closure load levels must be measured experimentally, which make the data difficult to use in practical applications to life prediction. In any case these correlations do help to show that ΔK as modified for closure is the primary and dominant variable causing fatigue cracking.

It is of further interest to also revisit the preceding cubic power law using the effective stress intensity range developed here.

THE UNIVERSAL LAW OF MECHANICAL FATIGUE CRACK GROWTH

In order to make the previous third power law herein into a universal law for all load ratios, *R,* it is only necessary to substitute the effective stress intensity factor. It is acknowledge that a small effect of the maximum stress intensity factor is present, as illustrated in Figure 5. Since this effect is minor it shall be ignored in further discussion.

Consequently, the “Universal Law” is stated as:

$$\frac{da}{dN} \cong b\left(\frac{\Delta K_{eff}}{E\sqrt{b}}\right)^3 \text{ where for threshold } \frac{da}{dN} \leq b \text{ and } \frac{\Delta K_{eff}}{E\sqrt{b}} \cong 1$$

This Universal Law is a good approximation for all data on metal alloys known to these authors but is only an approximation. Figure 8 shows the results of the plotted lines of the law as compared to data 7055 aluminum (a very good fit) and for 2324 aluminum (a good fit except this alloy exhibits a superior threshold or larger Burger’s vector). These are extremes in the precision of fit and again the reader will find further supporting evidence in the references herein, especially [7]. The Universal Law is suggested to provide a maximum growth rate limit for data not influenced by aggressive environments. It applies equally well to “small cracks” as a maximum growth rate. As such it can be used in estimates of minimum and order of magnitude estimates of crack growth lives for many applications.

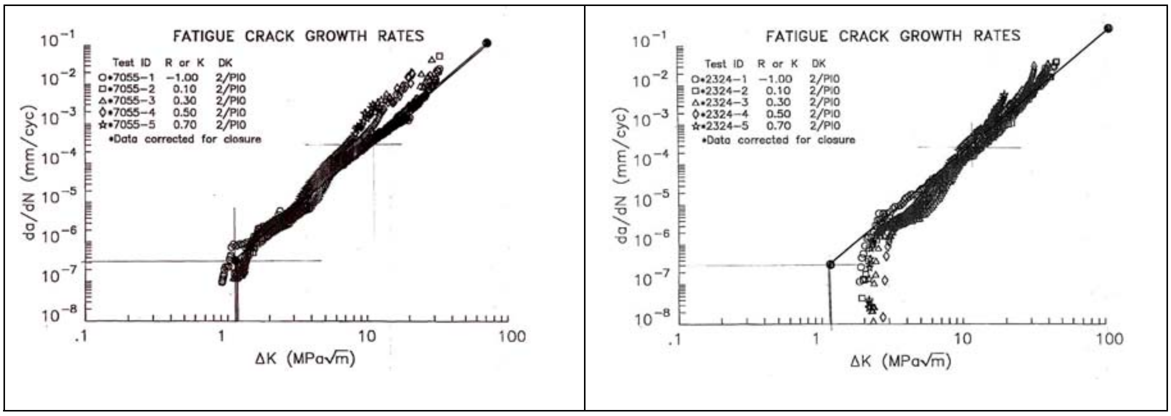

Figure 8 Data on both 7055 and 2324 with predicted lines from the Universal Law.

For example in a series of applications to Very High Cycle Fatigue, $>10^8$ cycles, exhibiting failure initiation from internal metallurgical discontinuities, this law can be used to show that the accompanying crack growth life is much smaller, $<10^6$ cycles. Therefore, VHCF life is dominated by initiation of cracking, see [13-17].

Dimensional Considerations of the Universal Law

The immediately above power law is noted to be dimensionally correct. If only the effective stress intensity range, the maximum stress intensity, the elastic modulus, and the Burger’s vector are present in the growth rate law, then the non-dimensional parameters involve are: dN, $\frac{da}{b}$, $\frac{\Delta K_{eff}}{E\sqrt{b}}$. and $\frac{K_{\max}}{E\sqrt{b}}$. Restricting the parameters to these items is strongly supported by the preceding data. A general form of the law can then be written as:

$$\frac{da}{dN} = b \cdot F\left(\frac{\Delta K_{eff}}{E\sqrt{b}}, \frac{K_{\max}}{E\sqrt{b}}\right)$$

It is acknowledged that b could be a micro-structural characteristic of the material of the order of the Burger's vector (such as micro-constituent phase size, etc.). However the Universal Law applied to data in all cases strongly supports the third power effect, i.e. a growth rate proportional to $\left(\frac{\Delta K_{eff}}{E\sqrt{b}}\right)^3$. As a consequence the law becomes:

$$\frac{da}{dN} = b\left(\frac{\Delta K_{eff}}{E\sqrt{b}}\right)^3 \cdot F_1\left(\frac{K_{\max}}{E\sqrt{b}}\right)$$

Donald [10] in his work chooses: $F_1\left(\frac{K_{\max}}{E\sqrt{b}}\right) = A \cdot \left(\frac{K_{\max}}{E\sqrt{b}}\right)^m$, (with $m = 1$) in an attempt to fit the data even better and where A is a non-dimensional constant. This choice might be subject to further investigation. However, with that choice the law becomes:

$$\frac{da}{dN} = A \cdot b\left(\frac{\Delta K_{eff}}{E\sqrt{b}}\right)^3 \cdot \left(\frac{K_{\max}}{E\sqrt{b}}\right)^m$$

where threshold occurs at:

$$F\left(\frac{\Delta K_{eff}}{E\sqrt{b}}, \frac{K_{\max}}{E\sqrt{b}}\right) = B$$

and where B is also a dimensionless constant.

It is noted that the Universal Law as previously stated above is within the restrictions of these dimensional considerations. Other attempts to formulate laws of mechanical fatigue crack growth incorporating other factors (such as yield stress, etc.) are contrary to the broad trends of data used in implying and developing the Universal Law through the analysis here.

It remains for someone to give a full physical explanation of the fact that stress intensity divided by elastic modulus times square root of Burger's vector is show by all the data on metal alloys to be the universal normalizing factor. Further, the influence of environment remains another effect requiring attention as well.

CONCLUSIONS

(1) The power law of stress intensity factor range, ΔK, has withstood almost 50 years of exploration and remains the most dominant parameter causing fatigue crack growth.
(2) Crack closure effects the stress intensity range.
(3) The ASTM method of determining open load and thereby ΔK_{open} does not adequately express the full stress intensity range with closure.
(4) Following the work of Bowles, the Partial Closure Model shows a ΔK_{eff} greater than the ASTM method. Donald's ACR method also correlates data better but lacks an analytical model's justification.
(5) All fatigue crack growth data strongly show that dividing the stress intensity by elastic modulus times square root of Burger's vector normalizes that data.
(6) From the previous conclusions a Universal Power Law of mechanical fatigue crack growth for all metal alloys has been reviewed and presented herein.
(7) This Universal Law may be affected in a minor way by the maximum applied stress intensity and sometimes in major ways by environmental influences.
(8) Applications of this Universal Law are only good for order of magnitude estimates of minimum crack growth lives (for example for very high cycle fatigue $>10^8$ applications).

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