IGF6 - VI Convegno Nazionale GRUPPO ITALIANO FRATTURA Ancona Giugno 1990

PREVISIONE TEORICA E VALIDAZIONE SPERIMENTALE DEI MECCANISMI DI ACCUMULO DEL DANNO IN LAMINATI REALIZZATI CON FIBRA DI VETRO.

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Sommario

I meccanismi di danno, su laminati compositi in fibra di vetro e resina epossidica, sono stati l'oggetto del presente lavoro. Utilizzando controlli non distruttivi, quali radiografia industriale e liquidi penetranti, sono state osservate cinque diverse modalità di danneggiamento: cricche trasversali nella matrice, rottura delle fibre, debonding delle fibre, delaminazione intralaminare e cricche longitudinali. Gli effetti di questi diversi meccanismi di danno si sono manifestati attraverso la globale riduzione delle proprietà meccaniche. E' stata realizzata un'indagine sperimentale su due laminati con differente stratificazione. Le variazioni dei moduli elastici e del modulo di Poisson sono stati misurati in funzione del numero di cicli a diversi livelli di tensione. L'approccio di base sviluppato da Talreja ed altri ha permesso di prevedere la riduzione dei moduli per un dato stato di danno. Tali riduzioni sono inoltre state correlate ai parametri di fatica (numero di cicli e livello di tensione). E' stato infine proposto un modello di danneggiamento cumulativo, basato sull'energia di deformazione assorbita.

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THEORETICAL FORECASTING AND EXPERIMENTAL VALIDATION OF DAMAGE TOLERANCE AND ACCUMULATION IN GLASS/EPOXY LAMINATES.

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ABSTRACT

Damage mechanism and accumulation under cyclic loads were studied in orthotropic cross-ply composite laminates realized in S-glass fibers and epoxy resin. Five different basic mechanisms of damage were observed making use in particular, of x-ray and dye-penetrant non destructive techniques. They are: transverse matrix cracks, fiber failure, fiber debonding, intralaminar delamination and random disperse longitudinal cracks. The effects of these different damage mechanisms are revealed through the reduction of the mechanical properties of the composites material. A comprehensive experimental analysis on two laminates with different lay-ups was made. Variation of the elastic moduli in two different direction and Poisson's ratio were measured as function of the number of cycles, for different stress levels, under alternate loads. The basic relations ,developed by Talreja et alt., which permit to predict the reduction of the elastic moduli for a given damage state, were directly related to the number of cycles N and to the stress levels employed in the fatigue tests. A cumulative damage model based on residual normalized strain energy ratio is also proposed.

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1 INTRODUCTION.

The relatively recent introduction and wide spread application of composites as structural materials has stressed the need for a better understanding of their actual capability to sustain loads particularly cyclic loads, without premature failure. This , in turn, has focused the attention on the basic mechanisms of damage, so different from those occurring on metals, and their cumulative effect on the final strength of the material.

Basically ,when a composite material is subjected to repeated load applications it undergoes a series of internal failures, more or less severe, that can be grouped into five fundamental types:

- Occurrence of cracking in the matrix transverse to the external loads.
- 2) Fiber fracture.
- 3) Fiber debonding.
- 4) Dispersed cracking parallel to the fibers.
- 5) Intralaminar delamination.

Of course of the five mechanism of damage presented, types (1) and (2) are by far the most important yet it must be recognized that the remaining three are very effective in demolishing the bulk structure of the composite drastically reducing its overall capability to withstand loads even though they do not directly act on fibers, which are resisting element. As to the time damage seems to take place in two stages. Under cyclic loads, transverse cracking occurs first and saturates relatively soon. In the second stage cracks parallel to the Fiber start to appear and grow at the intersection with transverse cracks. The growth is accompanied by a process of Fiber debonding and internal delamination. The fundamental approach to model the distributed damage can be of two types: a micromechanical approach and a continuum mechanics approach. The

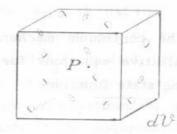
former aims at determining the overall response of a damage body from a local analysis of damage which requires an advanced capability of microscopic modelling with tremendous numerical analysis, the latter instead, asses the degradation occurred in the material on the base of the engineering properties such as Young's moduli Poisson's ratio yield and ultimate strength which are macroscopic quantities averaged on a scale much larger then the microscopic one. In the present work the continuum mechanics approach has been adopted. An empirical trend has been found which relates the damage state to the number of cycles N and to the stress levels used. Well before the final fracture of the specimen, its capability to substain external loads is reduced to such a point as to loose any structural resistance.

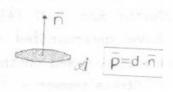
A damage tolerance shall be introduced since damage is produced already in the early application of the cyclic loads. Since this damage results in a reduction of the capability of the material to absorb strain energy ,the tolerance can be expressed as an energy ratio ,i.e., as a fraction of the initial strain energy that can still be absorbed by the material. The strain energy ratio is related to the number of cycles and to the stress level.

2 BASIC APPROACH.

2.1 CONTINUUM MECHANICS APPROACH.

According to the Talreja [1], Coleman and Gurtin [2] model ,the damage introduced in a composite material can be expressed in terms of number of cracks per elementary volume element ,dv, around a material point P ,as depicted in figure 1.





A vector p is assigned to any of the m existing cracks whose magnitude ,d ,is proportional to the dimensions of the flaw and whose direction ,n ,is that of the normal to the crack plane:

$$\vec{p}^{(r)} = d^{(r)} \vec{n}^{(r)}$$

and d is expressed as:

(2)
$$q^{(r)} = f(A^{(r)}, a^{(r)})$$

where Ar and ar are the area and length of the crack ,respectively.

Supposing n different orientations, damage can be represented by a vector field V:

(3)
$$\vec{V}^{(\alpha)} = D^{(\alpha)} \vec{n}^{(\alpha)}$$

where $\alpha = 1...n$.

D is the average of dr within the elementary volume dv for cracks having the same orientationa.

Using the scalar formulation of Kachanov [3]:

(4)
$$D^{2} = \frac{1}{dv} \sum_{i} \left(\overrightarrow{p}^{(r)} \overrightarrow{p}^{(r)} \right)$$

and

(5)
$$f\left(A^{(r)},a^{(r)}\right) = \left(cA^{r}a^{r}\right)^{1/2}$$
 where c is an empirical costant, equation (4) becomes:

$$(6) D^2 = \eta_{i,j} \langle c A a \rangle$$

with η_i indicating the crack density.

Coleman ,Gurtin and Noll [4] using the continuum mechanics approach have determinated the constitutive equations for a composite material based on the following state function:

Stress tensor o

Heat flux vector q

Helmhotz free energy ₩

Specific entropy η

Considering the second law of thermodynamics ,according to the Clausius' formulation, two important results can be obtained :

 The stress tensor ,the Helmhotz free energy and specific entropy turn out to be independent of temperature gradient,

(7)
$$\sigma = \sigma(\overline{F}, \tau, \overline{V})$$

$$\Psi = \Psi(\overline{F}, \tau, \overline{V})$$

$$\eta = \eta(\overline{F}, \tau, \overline{V})$$

and

$$q = q(\overline{F}, g, \tau, \overline{V})$$

where F is the strain gradient, g is the temperature gradient, r and V the damage vector ,already defined by equation (3).

II) the stress tensor and the specific entropy are related to each other by the Helmhotz free energy according to:

(8)
$$\sigma = \rho \frac{\partial \Psi}{\partial \vec{F}} (\vec{F}, \tau, \vec{V})$$

$$\eta = -\rho \frac{\partial \Psi}{\partial \tau} (\vec{F}, \tau, \vec{V})$$

where rho is the density.

Finally assuming to be dealing with an isothermal problem and small strains it is possible to write [5]:

(9)
$$\sigma = \sigma(\vec{e}, \vec{V})$$

$$\Psi = \Psi(\vec{e}, \vec{V})$$

where e is the strain vector.

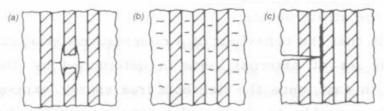
The result is that the behaviour of a composite material subjected to a known set of external loads is determined by the state function Ψ . In fact, once the Helmhotz free energy expression is known it is possible to derive the remaining state functions through equations (8) and (9) and therefore the thermodynamic state of the material. The function Ψ is then given the name of response function. The structure of the response function is such that if shall consider either geometrical and symmetrical properties as well. Rivlin and alt. [6] have determinated the effects of different symmetries on response function represented by polynomials, Adkins, Smith et alt [7] have determinated the irreducible integral bases for vector and tensors.

The composite material used in this investigation presents an orthotropic symmetry. Smith [8] has determined the integral bases for the orthotropic case where the group of transformations defining the class of crystals of interest is given by:

(10)
$$I = (1,1,1), C = (-1,-1,-1) R_1 = (-1,1,1), R_2 = (1,-1,1), R_3 = (1,1,-1) D_1 = (1,-1,-1), D_2 = (-1,1,-1) and D_3 = (-1,-1,1) (3.26)$$

In this case the response function ψ is:

$$(11) \begin{array}{l} \varrho\psi = a_1 I_1^2 + a_2 I_1 I_2 + a_3 I_1 I_3 + a_4 I_1^2 I_8 \\ + a_5 I_1^2 I_9 + a_6 I_1^2 I_{10} + a_7 I_1 I_{11} + a_8 I_1 I_{12} + a_9 I_1 I_{13} \\ + a_{10} I_1 I_2 I_8 + a_{11} I_1 I_2 I_9 + a_{12} I_1 I_2 I_{10} \\ + a_{13} I_1 I_3 I_8 + a_{14} I_1 I_3 I_9 + a_{15} I_1 I_3 I_{10} + b_1 I_2^2 \\ + b_2 I_2 I_3 + b_3 I_2^2 I_8 + b_4 I_2^2 I_9 + b_5 I_2^2 I_{10} + b_6 I_2 I_{11} \\ + b_7 I_2 I_{12} + b_8 I_2 I_{13} + b_9 I_2 I_3 I_8 + b_{10} I_2 I_3 I_9 \\ + b_{11} I_2 I_3 I_{10} + c_1 I_3^2 + c_2 I_3^2 I_8 + c_3 I_3^2 I_9 + c_4 I_3^2 I_{10} \\ + c_5 I_3 I_{11} + c_6 I_3 I_{12} + c_7 I_3 I_{13} + d_1 I_4 + d_2 I_4 I_8 \\ + d_3 I_4 I_9 + d_4 I_4 I_{10} + e_1 I_5 + e_2 I_5 I_8 + e_3 I_5 I_9 \\ + e_4 I_5 I_{10} + f_1 I_6 + f_2 I_6 I_8 + f_3 I_6 I_9 + f_4 I_6 I_{10} \\ + g_1 I_{14} + g_2 I_{15} + g_3 I_{16} + P_0 + P_1 (e_{ij}, V_k) + P_2 (V_i) \end{array}$$



Fatigue damage mechanisms in unidirectional composites under loading parallel to fibers; a) fiber breakage, interfacial debonding; b) matrix cracking; c) interfacial shear failure.

and the invariants are:

(12)
$$I_{1} = e_{11}$$

$$I_{2} = e_{22}$$

$$I_{3} = e_{33}$$

$$I_{4} = e_{23}^{2}$$

$$I_{5} = e_{13}^{2}$$

$$I_{6} = e_{12}^{2}$$

$$I_{7} = e_{23}e_{13}e_{12}$$

$$I_{8} = V_{1}^{2}$$

$$I_{10} = V_{3}^{2}$$

$$I_{11} = V_{2}V_{3}e_{23}$$

$$I_{11} = V_{2}V_{3}e_{23}$$

If one considers that the first eight terms of equation (12) are independent of the damage vector \underline{V} ,it is possible to write as:

$$\rho \Psi = \rho \Psi_0 + \rho \Psi_1$$

where the first terms on the right refers to the virgin material, the second to the damaged one. Using the Voigt notation it yields:

$$\sigma_{ij} = \varrho \frac{\partial \psi^0}{\partial e_{ij}} + \varrho \frac{\partial \psi^1}{\partial e_{ij}} \qquad \sigma_{\theta} = (C_{\rho q}^0 + C_{\rho q}^1) e_q$$

where:

$$C_{pq}^{0} = \begin{bmatrix} 2a_{1} & a_{2} & a_{3} & 0 & 0 & 0 \\ & 2b_{1} & b_{2} & 0 & 0 & 0 \\ & & 2c_{1} & 0 & 0 & 0 \\ & & & \frac{d_{1}}{2} & 0 & 0 \\ & & & \frac{f_{1}}{2} \end{bmatrix}$$

and

where

Supposing to be dealing with an orthotropic symmetry ,it is obtained:

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(17)
$$E_1 = \frac{C_{11}C_{22} - C_{12}^2}{C_{22}}, E_2 = \frac{C_{11}C_{22} - C_{12}^2}{C_{11}}$$

$$\nu_{12} = \frac{C_{12}}{C_{22}} \text{ and } C_{12} = C_{66}$$

With the additional hypothesis of a unique damage vector, the in-plane elastic moduli are given by:

(18)
$$E_{1} = E_{1}^{0} + 2D^{2}[k_{3} + k_{7}(\nu_{12}^{0})^{2} - k_{13}\nu_{12}^{0}]$$

$$E_{2} = E_{2}^{0} + 2D^{2}[k_{7} + k_{3}(\nu_{21}^{0})^{2} - k_{13}\nu_{21}^{0}]$$

$$\nu_{12} = \nu_{12}^{0} + D^{2}\left[\frac{1 - \nu_{12}^{0}\nu_{21}^{0}}{E_{2}^{0}}\right](k_{13} - 2k_{7}\nu_{12}^{0}) \text{ and } G_{12} = G_{12}^{0} + 2D^{2}K_{11}$$

The result is a set of equations that relates the elastic moduli to a given damage state. The numerical value of the damage function necessary to solve equation (18) is of experimental derivation. For laminates, Talreja [9] suggests the following expression to fit the experimental data:

$$(19) D^2 = \eta_c \overline{l_c w_c f_c}$$

where η_c is the crack density defined as number of cracks per unit surface ,1 and \underline{w} are the average length and width of the defect ,respectively and f_c is an adimensional constraint factor which takes into account the action exerted by the present layers on the crack plane.

If t is the overall thickness of the laminate and to that of the

plies containing cracks, than:

(20)
$$f_{c} = \frac{n \quad t_{c}}{t}$$

$$\overline{w} = n \quad t_{c}$$

$$\overline{l} = specimen \quad width$$

2.2 FATIGUE DAMAGE CORRELATION.

Damage nucleation and growth in composite materials subjected to cyclic loads depends on the stress amplitude and number of cycles. Cracking transverse to the matrix appears to be the most effective cause of reduction of elastic moduli. The results of the study presents in this paper have shown that the number $N_{\rm c}$ of transverse cracks developed in the composite materials under investigation saturates after a certain number of cycles which depends on the applied stress level. A relationship can be written to relate $N_{\rm c}$ to the number of cycles ,N ,and the stress amplitude , σ :

(21)
$$N_{c} = \frac{A \operatorname{Log}_{10}(N)}{1 + \rho(\sigma) \operatorname{Log}_{10}(N)}$$

where A is the maximum number of cracks at saturation for the material considered and ρ is a function of the stress level:

. As being each and
$$\rho = f(\sigma)$$
 which is the same of σ

that can be represented by a polynomial of the third order. In our case ,S-glass/epoxy laminate [90,0]s

$$A = 67$$

and

$$\rho = (.1294E - 3)\sigma^{3} - (0.02018)\sigma^{2} + 1.04409\sigma - 17.7611$$

It is worthwhile to that the same relationship described by equation (21) applies to other material such as graphite/epoxy. Experimental data obtained by Charewicz and Daniel on graphite/epoxy [9] have been analysed in this study yielding a value of 260 for the coefficient A and

$$(8.463E - 5)\sigma^3 - 0.01305\sigma^2 + 0.6723\sigma - 11.423$$

Results are presented in figures 4.1, 4.2, 4.3

Equation (21) is of paramount importance because it provides a practical tool to asses the reduction of the elastic moduli. Continuum mechanics ,in fact ,does not define the experimental trend of the Young's modulus observed on glass/epoxy and theoretical one assessed in this study for the data obtained by Talreja. Figures 4.5 and 4.6 show our predicted results and Talreja's data [10].

3 CUMULATIVE DAMAGE.

In a composite material ,damage free, subjected to a quasi-static tensile load ,the external work is converted into strain energy supposing that applied stress rate has not produced any damage. Upon unloading ,if the material undergoes a cyclic load application which produces a certain damage D and than is reloaded quasi-statically to the previous strain level it will be observed a reduction in the strain energy absorption. In the general case of tri-axial stress state, the strain energy is given by:

(23)
$$\mathfrak{D} = \frac{1}{2} \sum_{j=1}^{3} \sigma_{j} \epsilon_{j}$$

which reduces to:

$$\mathfrak{I} = \frac{1}{2}\sigma_1 \epsilon_1$$

for the single stress condition, or:

$$\mathfrak{I} = \frac{1}{2} E_{\perp} \epsilon_{\perp}^{2}$$

If \mathfrak{I}_{0} is the energy absorbed before a damage D is produced by the cyclic load and \mathfrak{I}_{r} is the corresponding value after fatiguing ,the ratio:

(26)
$$\frac{\Delta S}{S} = \frac{S_0 - S_f}{S_0}$$

is adimensional parameter representing the reduction of strain energy due to the damage introduced which can be introduced as:

(27)
$$\frac{\Delta \mathfrak{D}}{\mathfrak{D}} = -\frac{\eta_{\mathfrak{c}} \overline{lw} f_{\mathfrak{c}} k_{1}}{E_{1}}$$

and recalling the dependence of the damage D on the number of cycles ,N, and stress , $\boldsymbol{\sigma}$:

(28)
$$\frac{\Delta S}{S} = f(N, \sigma)$$

It may be particularly interesting to study how the ratio given by equation (28) ,which can be consider as an index of the damage ,is actually varying as a function of the number of cycles and stress level.

4 EXPERIMENTAL PROCEDURES AND RESULTS.

The material used in this investigation was 3M-SP250 S2-glass/epoxy obtained from 3M Corp. in prepreg form. Two layups $,[90_4,0]_s$ and $[0_4,90]_s$, were fabricated by ITALCOMPOSITI (Anagni-Italy). From this laminates were obtained specimens according to ASTM D 3479 and UNI in form of coupons of dimensions:

End tabs 50 mm long of glass/polyester by pultrusion ,glued with epoxy resin, were employed. Specimens were cut using a diamond disk saw.

Fatigue testing was conducted with an electro-hydraulic MTS load

cell, using a sinusoidal tension-tension cyclic load with a stress ratio R = 0.1.

The frequency was 1 Hz for the first 1000 cycles and 6 Hz for the following. These frequencies were selected to study low cycle fatigue damage, and also to reproduce the natural frequencies of some ae: onautical structures as wings. Six different stress levels were investigated. For each stress level a minimum of three tests was conducted. Young's modulus in the longitudinal direction, E_1 , and Poisson's modulus, nu_{12} , were measured on every specimen under quasi-static condition prior and after fatiguing.

Control of residual moduli and damage entity was done at selected number of cycles N:

N = 0, 100, 1000, 10E4, 10E5, 2.5*10E5, 5*10E5, 7.5*10E5 and 10E6. Damage was monitored using non-toxic dye-penetrant (Spootcheck FP/v6)and X-radiography. Specimens that did not fail at 10E6 cycles were loaded to failure under tensile test and residual strength were measured. Larger specimens were used to measure the elastic moduli in two normal directions. Temperature was also monitored during fatigue tests. Two thermocouples were employed. Temperature rises of 1.5 - 2 °C on the surface of specimen were recorded.

Finally, some tests of cumulative damage were made using a load spectrum. Result are reported in fig 5.1, 5.2 ,5.3 ,5.4 ,5.5 ,5.6.

5 FINITE ELEMENT ANALYSIS.

A Intralaminar stress analysis on composite material was made using the finite element code MARC. A comparison was also made with an isotropic case as aluminium 7075. Model in figure 6.1 show a quarter of specimen geometry used in experimental investigation. Load condition was a simple tension uniform distributed on the edge. An bilinear thick shell element with six degree of freedom ,for each node, was employed. Boundary conditions permit plane

displacements only. In figure 6.2, 6.3 are shown different nodal displacements for different cases with the same load condition and thickness. In figure 6.4 are reported the components of average membrane stress. In figure 6.5 ,finally, is reported the comparison of results obtained with f.e.m. MARC and program GENLAM of Think Composite.

6 CONCLUSION AND RECOMMENDATION.

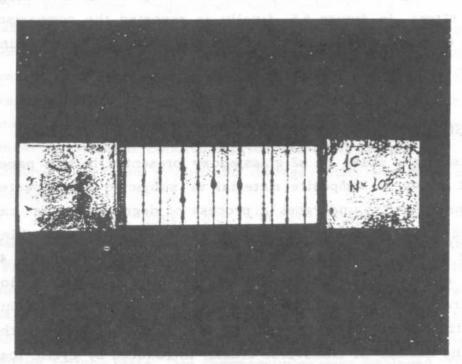
The results of the study on s2-glass/epoxy presented in this paper show that when a composite material is subjected to repeated applications of load it undergoes a process of degradation that can be ascribe to five fundamental modes of failure: 1) cracking transverse to the matrix ,2) fiber failure ,3) fiber debonding., 4) dispersed cracking parallel to fibers., 5) internal delamination. Cracking transverse to the matrix appears to be the most dangerous and effective cause of reduction of the elastic moduli of the material. The number of transverse cracks introduced by cyclic load saturates after a certain number of cycles which depends on the stress amplitude.

From that moment on, the applied load cycles produce dispersed parallel cracking ,fiber debonding. and internal delamination. This mechanism of degradation suggests that a composite material can be used for structural purposes in as long as number of applied cycles remains below that limit value which saturates the transverse cracking and a safety factor can be applied on that number.

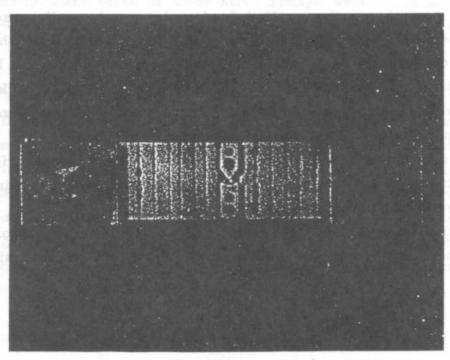
it may be interesting to check if and how the energy index varies with the stress level at the maximum number of transverse cracking and whether it can be used as a design parameter.

The investigation must be extended to asses the effects of load reversal and mean stress on the fatigue resistance of a composite materials.

Non destructive examination techniques , such as X-ray, can be us to check the damage introduced in the material, some , as acous emission , can be utilized to perform continuous monitoring of t structural material.



Dye-penetrant investigation on (90, , 0)s Lam.



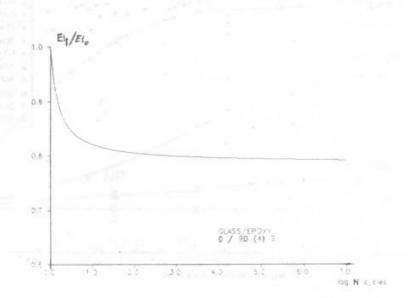
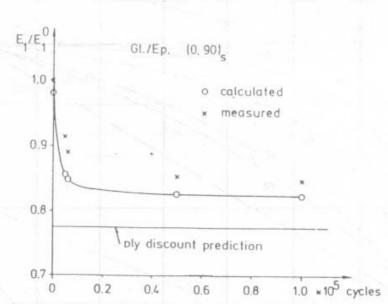


Fig. 4.5



Variation of the longitudinal Young's modulus of (0,90), laminate with the applied tension-tension cycles.

Fig. 4.6

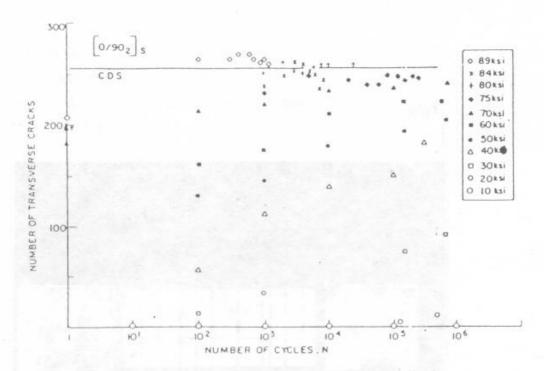


Fig. 4.1

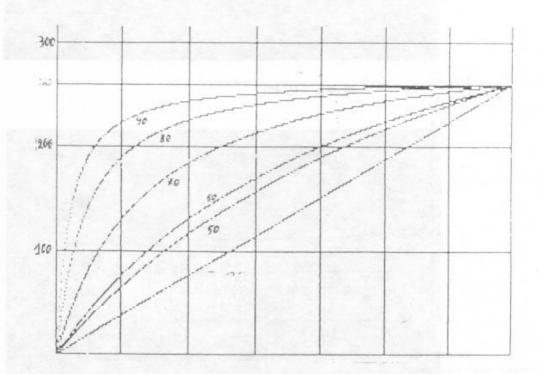
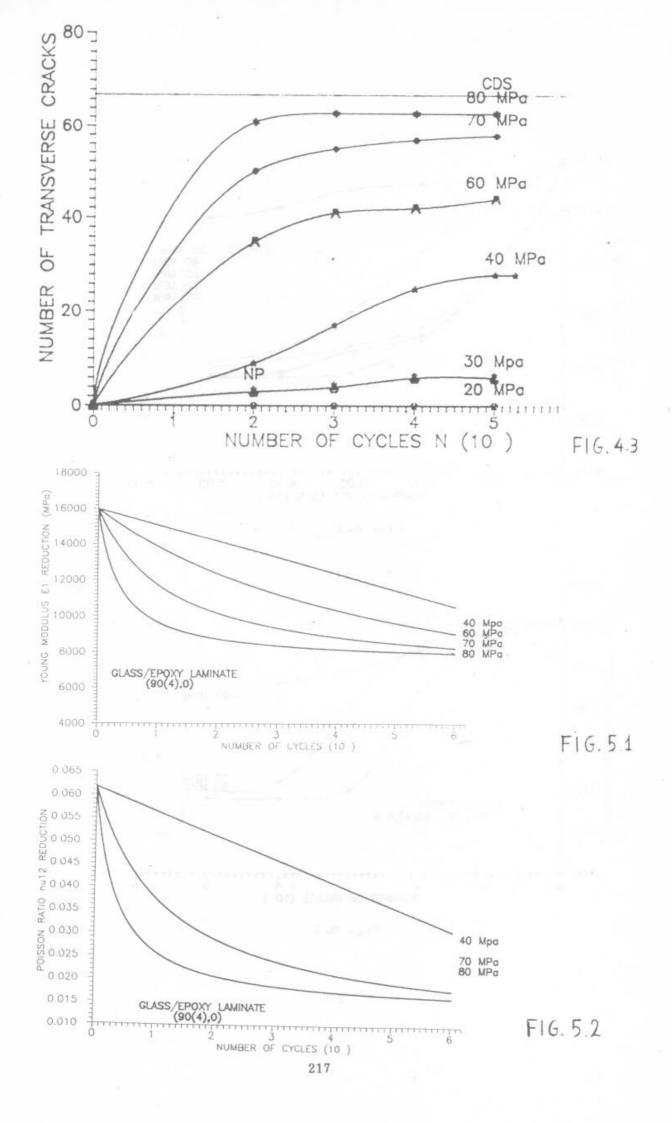


Fig. 4.2 Predicted trend.



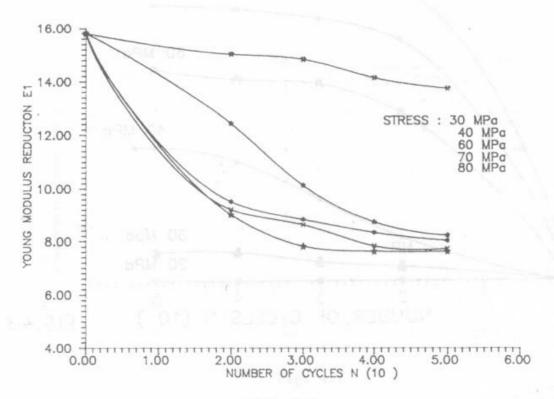


FIg. 5.3

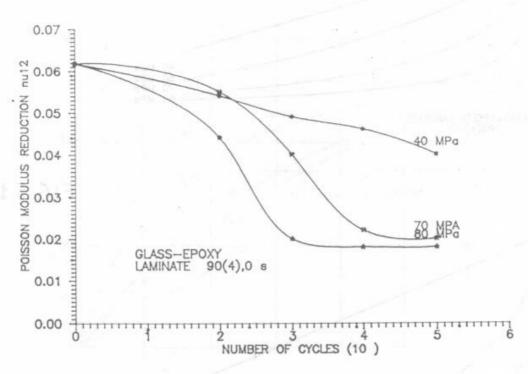


Fig. 5.4

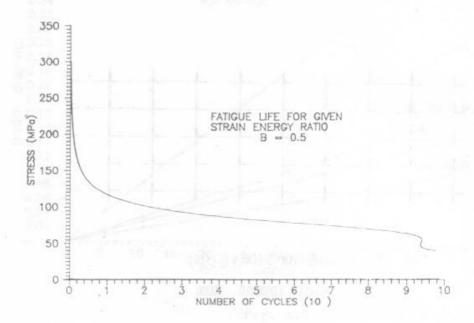


Fig. 5.5

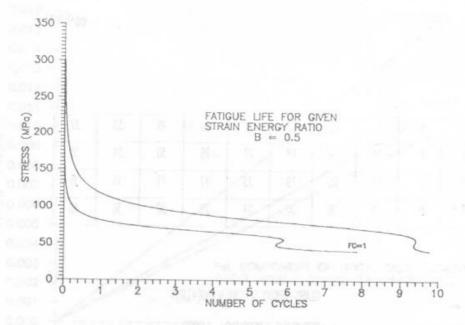
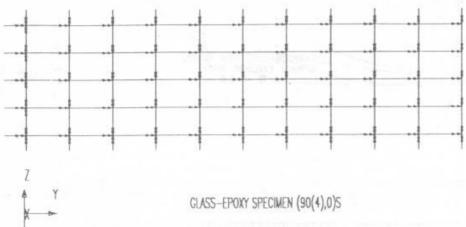


Fig. 5.6

COMENTAT



ORTHO V.P. 1.0000 0.0 1.0000

CO MENTAT

1	5	9	13	17	21	25	29	33	37
2	6	10	14	18	22	26	30	34	38
3	7	11	15	19	23	27	31	35	39
4 -	8.	12	16	20	24	28	32	36	40

Z X Y

GLASS-EPOXY SPECIMEN (90(4),0)S

ORTHO V.P. 1.0000 0.0 1.0000

Fig. 6.1

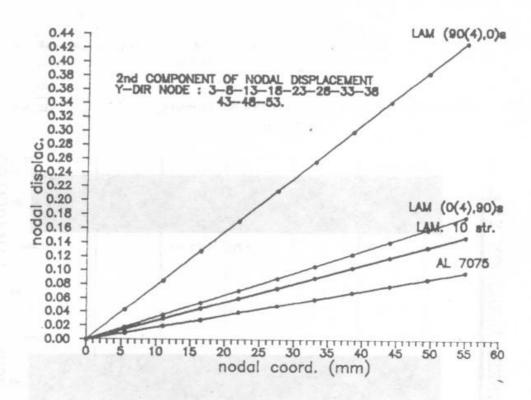


Fig. 6.2

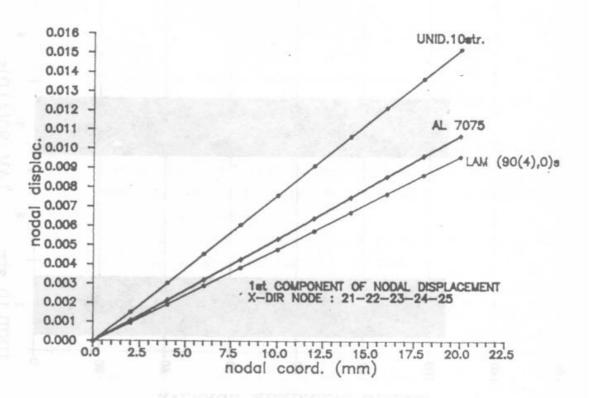
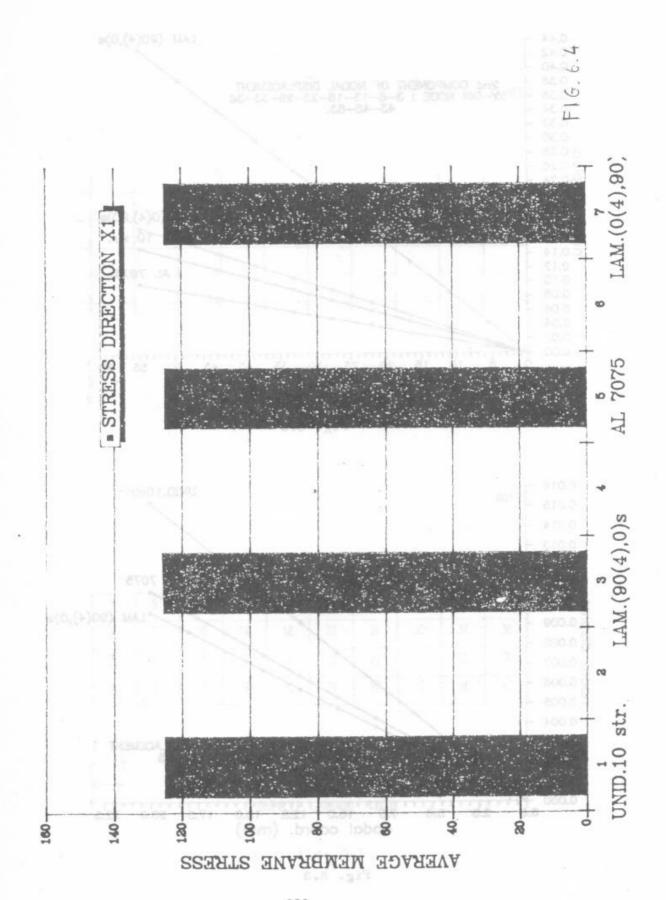


Fig. 6.3



CASE 1 :

Trazione uniforme Ni : .180E+06 [MN/m]

sigma1: 120 [MPa]

	Ply No.	sigma1 (MARC)	sigma1 (GENLAM)	[MPa]
1	to 4	65.86	63.23	
5	to 6	361.6	347.09	
7	to 10	65.86	63.23	
	Ply No.	sigma2 (MARC)	sigma2 (GENLAM)	[MPa]
1	to 4	-3.656	-3.51	
5	to 6	14.62	14.4	
7	to 10	-3.656	-3.51	

F16.65

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