Numerical and experimental assessment of the hodograph cone method (HCM) for the prediction of the COD of off-centered circumferential cracks in pipes

D. Gentile, G. Iannitti
University of Cassino, Dept. of Mechanics, Structures and Environment (IT)

ABSTRACT. In this work the applicability and the accuracy of the hodograph cone method (HCM) for estimating the COD distribution of off-centered cracks in pipe under bending has been verified by means of extensive finite element analysis and experimental investigation. The need to develop appropriated numerical model and the guideline to follows in order to get accurate results is discussed. Experimental measures performed by means of digital image correlation confirmed both the accuracy of the FEM model adopted as well as the robustness of the HCM. A correction for further improvement of the HCM accuracy and extension to large crack length is also proposed.

SOMMARIO. In questo lavoro è stata verificata l'accuratezza e l'applicabilità del metodo del cono odografo (HCM) per la stima della distribuzione del COD, di cricche poste fuori asse presenti nei tubi soggetti a momento flettente, mediante un'estensiva analisi agli elementi finite e attraverso prove sperimentali. È stata discussa la necessità di sviluppare un accurato modello numerico e le linee guide da seguire per ottenere un risultato accurato. Le misure sperimentali, eseguite attraverso la tecnica del Digital Image Correlation (DIC), hanno confermano sia l'accuratezza del modello FEM sviluppato che la robustezza del metodo HCM. Infine è stata proposta una correzione per migliorare l'accuratezza del metodo HCM per cricche molto lunghe.

KEYWORDS. LBB; Hodograph Cone Method; COD.

INTRODUCTION

In the leak-before-break (LBB) approach the shape and size of the leak area (COA), or alternatively the crack opening displacement (COD) distribution, is the necessary information in order to estimate the flaw size. For piping systems, a through-thickness circumferential crack under bending and/or internal pressure load is the reference configuration for structural integrity evaluations. The crack is always assumed to be symmetrically placed with respect to the bending plane since the maximum COD/COA, for given applied moment, is obtained in this configuration. The COD distribution along the crack length is assumed elliptical and several reference solutions have been determined, by means of finite element simulation, and made available in design code such as GE-EPRI, [1].
However, during service the bending plane can change due to different operative conditions (load transients, vibrations, ram load, etc.), and therefore the crack may result to be off-center with respect to the acting load. Under these circumstances, the reference solution available for the symmetric case is no longer conservative since, as a result of the reduction of the crack opening caused by a lower opening stress, also the leak area is reduced leading to a shorter flaw size estimation.
In order to derive the effective leak area for a generic off-centered crack configuration, finite element analysis becomes necessary. Rahman et al. [2] firstly investigated the effects of off-centered crack configuration and restraint on the crack

opening area. They found that the COA is considerably reduced and the shape of the COD is no longer elliptical as soon as the crack is no longer symmetric with respect to the bending plane.
The same year Bonora proposed the Hodograph Cone Method (HCM) [3]. In the HCM, the COD distribution and the leak area are predicted, for varying crack sizes and off-center angles, only by means of geometrical considerations. Later Rahman et al., [4] performed further numerical investigations on off-centered crack configurations, and Firmature and Rahman [5] investigated the behavior of off-center cracks in elastic-plastic regime.
Today, the HCM method is the only solution available that does not require extensive use of finite element simulation and allows quick estimation of the COD distribution and associated COA, given the crack length and the off-axis angle. For this reason, it is attractive for its potential use in design-by-analysis procedures.
In [3], Bonora validated the HCM by means of finite element simulation. Although conceptually simple, the numerical simulation of off-centered crack configurations presents a number of operative problems that need to be solved in order to come up with a reliable representative model and accurate results. Firstly, half of the pipe needs to be modeled using brick elements with enough mesh refinement along the crack front (thickness), the crack length (for COD evaluation) and near the crack tip (for blunting). Secondly, crack closure may occur for large off-center angles. In order to account for this, contact analysis becomes necessary. The use of simple gap elements or fixed boundary conditions lead to poor estimation of the effective closure underestimating the effective opening crack length and modifying the COD distribution. Similarly, the simulation of the boundary conditions (supports and applied load) would also require the use of contact surfaces in order to avoid unrealistic peak stress at the nodes, when loaded with concentrated forces, or over-stiffening the model by using fixed boundary conditions in order to simulate the pipe supports.
Because of the limited computational capabilities available in the past, most of these features have been neglected in previous computational analyses and most of the numerical results have been obtained with simplified models.
In this study, the HCM has been investigated in order to establish its accuracy and applicability range.
To this purposes an extensive 3D FEM investigation over a wide number of geometry configurations (different flaw size and off-axis angles) has been performed. Particular attention has been given to the FEM model preparation in order to avoid any of the numerical problems previously discussed.
In order to validate both the HCM solution and the FEM models, the COD for off-centered cracks has been measured experimentally by means of 3D Digital Image Correlation technique (DIC).
The results indicates that the HCM is a robust method for estimating the effective COD/COA for an off-centered configuration but also showed some discrepancies with FEM solutions proposed in the literature.

THE HODOGRAPH CONE METHOD

The COD distribution for a centered crack configuration is assumed to be elliptical. This assumption has been verified and found very accurate in the elastic deformation range [1-2]. From a geometrical point of view the COD distribution on the cylindrical pipe can be seen has the locus resulting from the intersection of a cone and a cylinder. The cone represents an hypothetical 3D COD surface and the cylinder is representative of the pipe. The cone has an elliptical base, with semi-axes equal to the half maximum COD, for given bending load, and half of the projected crack length. The vertex is at 180° on the pipe circumference with respect to the maximum COD (that is the height of the cone is equal to the pipe diameter), Fig. 1.
The intersection between the cone and the cylinder that gives the COD distribution for a circumferential crack length $2\alpha R$ and a generic off-center angle ϕ is given by the following equations:

$$\begin{cases} y = \dfrac{b}{2}\Big\{\left[\cos(\beta)+\cos(\phi)\right]^2 - \\ \left[\sin(\beta)-\sin(\phi)\right]^2 \tan^{-2}(\alpha/2)\Big\}^{1/2} \\ x = R\sin(\beta) \\ z = R\cos(\beta) \end{cases} \tag{1}$$

where $\beta \in [-\alpha; +\alpha]$.
When the crack is off-center, the COD cone vertex rotate along the cylinder circumference in the opposite direction, by an angle equal to that of off-center angle, Fig. 2.

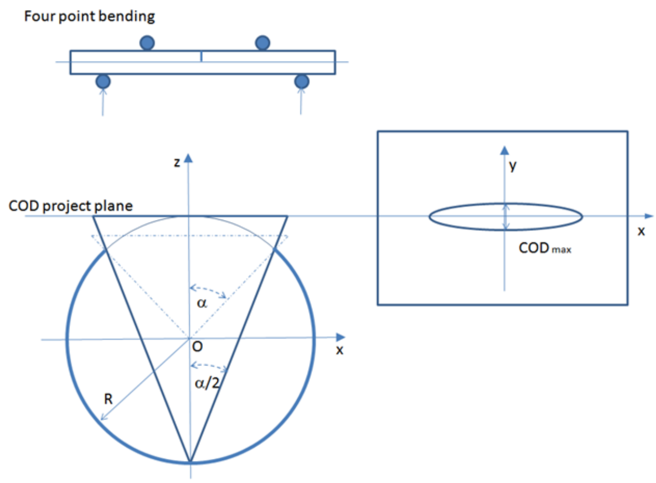

Figure 1: Geometric representation of the hodograph cone for the reference in-axis configuration.

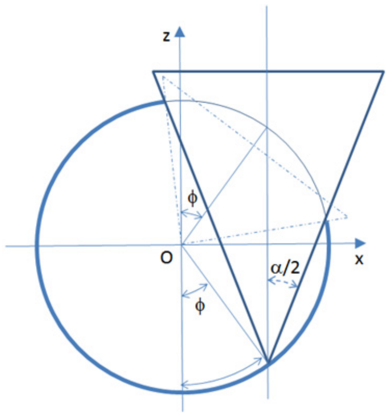

Figure 2: HCM for the generic off-center crack configuration.

According to this, crack closure stars when the crack goes in the compression region that is predicted to occurs at an angle that depends on the crack length such as,

$$\phi = \frac{1}{2}(\pi - \alpha) \tag{2}$$

Details on the derivation of these equations are given elsewhere, [3].

FINITE ELEMENT SIMULATION

The validity of the HCM has been verified by numerical simulation using the same reference pipe configuration used in [3]. The pipe dimensions are: 400 mm outer radius, 100 mm thickness, the outer span distance 10545.5 mm and inner span distance 3781,8 mm. Several crack sizes have been investigated: α=25°, 45°, 60° and 90°.
For each crack length the centered (ϕ=0°) configuration has been analyzed in order to determine the maximum COD under given bending load. This case has also been used to validate the present FEM model by comparison with the experimental measures performed in this work and with other reference solutions such as those given in [1,6,7]. Therefore, for each crack length three off-axis angle values, namely 30°, 60° and 90°, have been investigated.
Due to the symmetry condition only half pipe has been simulated. Several FEM models have been tested in order to avoid mesh size effect. The element numbers for the model used to obtain the results presented in this work was 4896 eight

node elements. Loading and supports condition were simulated by means of rigid surfaces. In order to simulate the crack closure and to prevent axial displacement of the crack face across the symmetry plane for large off-center angles, again a rigid surface has been used as shown in Fig. 3.

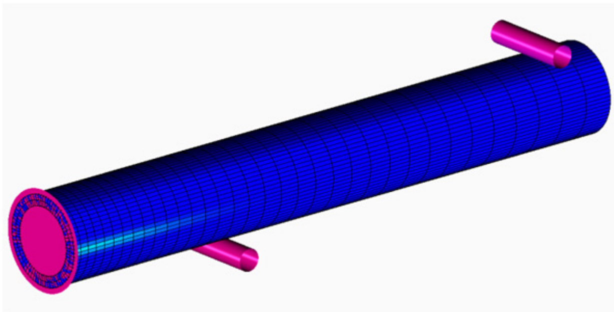

Figure 3: FEM model showing the details of the boundary conditions (supports, symmetry plane and pin load).

Numerical simulations have been performed with the commercial FE code MSC.MARC 2010r1. The analyses have been performed using large displacement and lagrangian updating formulation in order to account for large rotations occurring with extensive cracks. Since the displacement field varies linearly between nodes in eight node brick elements, even if a fine mesh is used near the crack tip, the curvature of the COD profile may not be accurately described. In order to have more accurate description of the COD profile automatic remeshing has also been used. In Fig. 4 a detail of the FE mesh at the crack tip is given.

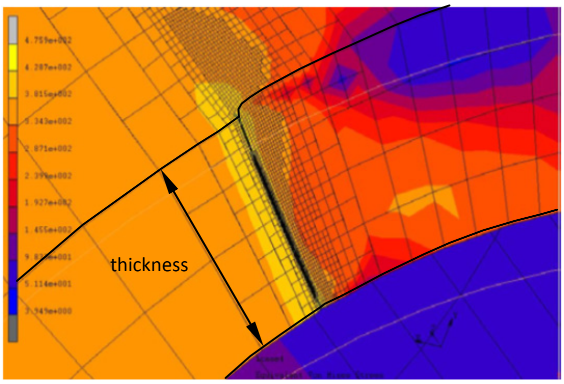

Figure 4: FEM model showing the details of the boundary conditions (supports, symmetry plane and pin load)

EXPERIMENTAL MEASURE OF COD DISTRIBUTION

The COD of off-center circumferential cracks in pipe under bending load has been measured. To this purpose a pipe with 60.2 mm diameter, 3.4 mm thick, has been used. The pipe is obtained by drawing ensuring very limited ovalization and constant thickness. The crack has been machined using a circular saw. Therefore, the resulting tip shape was square with half-average distance between the crack faces of 0.5 mm. No fatigue pre-cracking has been performed to sharp the tip radius since, at this stage, the interest was only on the COD distributions. It has been verified by FEM that the tip shape does not influence the measure of the COD distribution.
The pipe material was FE360 steel with 360 MPa minimum nominal yield stress and 190GPa Young modulus.
The pipe has been loaded under four point bending using a Instron 300kN electromechanical testing machine.
The measure of the COD distribution along the crack length has been determined by means of the digital image correlation (DIC) system ARAMIS by GOM which ensure an accuracy on strain measurements of 0.01%.

The pipe surface has been previously prepared in order to allow speckle analysis, an example is given in Fig. 5. Static measures have been performed for three reference load levels: 500,1000 and 1500N. A pre-load of 40N has been used in order to compensate the loading fixture compliance.

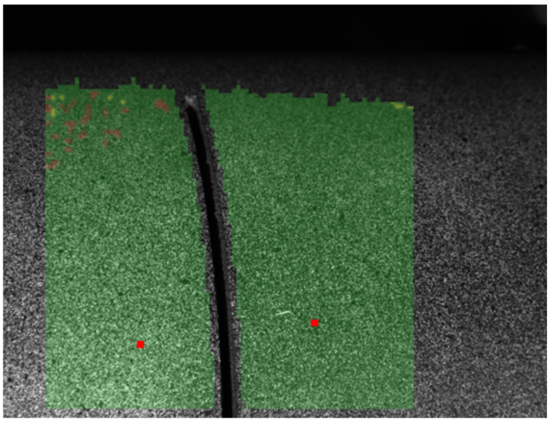

Figure 5: Detail of the pipe prepared for speckle analysis: the lighter zone is the portion of the pipe used by the DIC chosen for strain/displacement measurements.

RESULTS AND DISCUSSION

The HCM assumes that, given the maximum COD for the center crack δ_0, the COD distribution for the off-center crack configuration is given by,

$$\delta(\phi) = \delta_0 \left\{ \left[\cos(\beta) + \cos(\phi)\right]^2 - \left[\sin(\beta) - \sin(\phi)\right]^2 \tan^{-2}(\alpha/2) \right\}^{1/2} \quad (3)$$

This solution overestimates the effective maximum opening for given off-axis angle.
It has been found that applying the following correction usually leads to much better agreement between the HCM and FEM results,

$$\delta_0^* = \delta_0 \cos(\phi / \phi_0) \quad (4)$$

where ϕ_0 is a factor that depends on the amplitude of the applied bending and crack length and it is equal to the ratio of half crack angle α and the off center angle τ for which there is half closure of the crack.

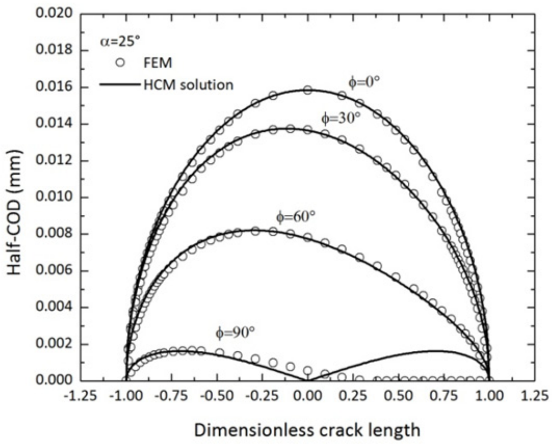

Figure 6: Comparison of HCM solution and FEM for α=25°.

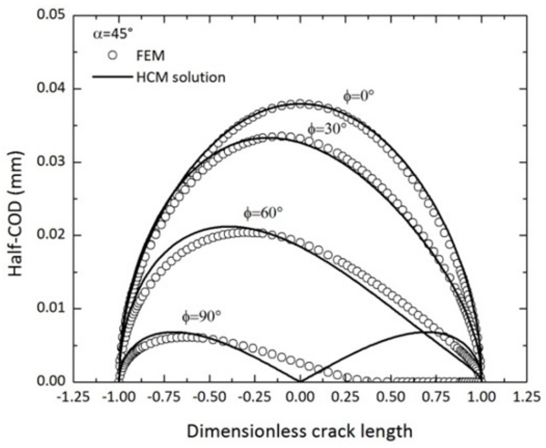

Figure 7: Comparison of HCM solution and FEM for α=45°.

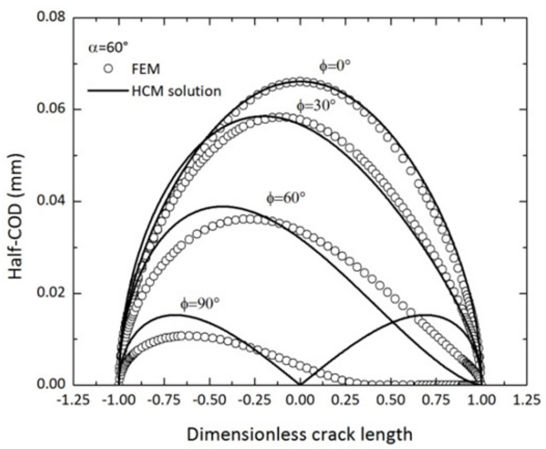

Figure 8: Comparison of HCM solution and FEM for α=60°.

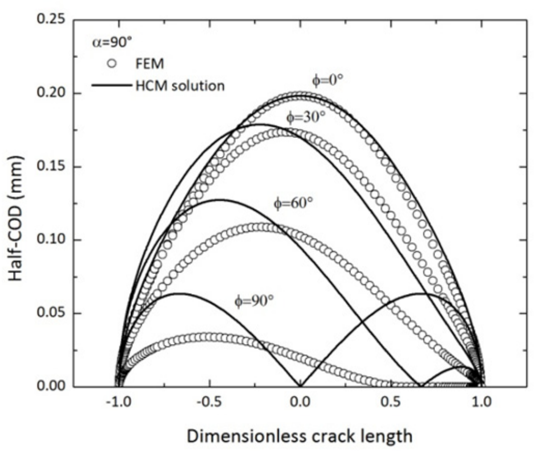

Figure 9: Comparison of HCM solution and FEM for α=90°.

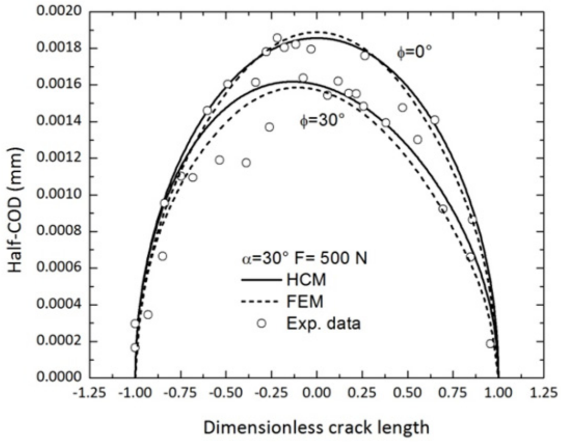

Figure 10: Comparison of HCM, FEM and experimental measures for α=30°, ϕ=0° and 30°, and 500 N load case.

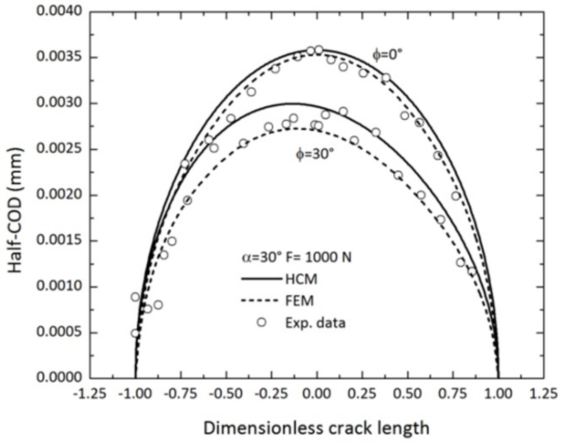

Figure 11: Comparison of HCM, FEM and experimental measures for α=30°, ϕ=0° and 30°, and 1000 N load case.

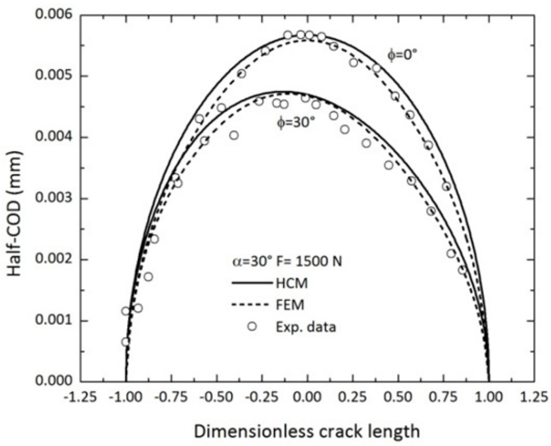

Figure 12: Comparison of HCM, FEM and experimental measures for α=30°, ϕ=0° and 30°, and 1500 N load case.

Although a precise determination of ϕ_0 would requires finite element simulation, in a fist approximation it can be taken equal to 4.0.

The comparison of the corrected HCM solution with the finite element results relative to the original pipe geometry used by Bonora [3] are given in the Figs. 6-9, showing a very good agreement of the predicted COD both in shape and amplitude for smaller crack sizes (α<45°) while for larger cracks the solution becomes less accurate for large off-center angles.
In Figs. 10-12 the comparison of the COD distributions for centered and off-centered crack calculated with the HCM, computed with FEM and measured with the DIC for three different load levels is given.
The comparison confirms the accuracy of the HCM in predicting both shape and amplitude of the COD distribution. Moreover, the very good agreement between the FEM results and the experimental measures validates the accuracy of the simulation work.

CONCLUSIONS

In this work the hodograph cone method for estimating the COD shape and amplitude distribution of off-center cracks in pipe under bending for LBB analysis has been verified by means of an extensive finite element simulation work and performing experimental measures by means of DIC technique. The agreement between the experimental measures and FEM results was found to be very good confirming the accuracy of the FEM model in reproducing the expected three dimensional displacement field along the crack. The comparison between the FEM and the HCM showed that although it predicts fairly well the COD shape as well as crack closure event, the HCM solution overestimates the effective COD amplitude for given load in the off-center configurations. The accuracy of the HCM can be easily improved scaling the maximum amplitude according to the solution proposed here. Finally, the HCM is found to be a very effective analytical tool to predict the COD, and consequently the COA, over a wide range of crack lengths and off-center angles for LBB structural integrity analysis.

REFERENCES

[1] V. Kumar, M. German, W. Wilkening, W. Andrews, H. de Lorenzi, D. Mowbray, EPRI Final Report NP-3607, Palo Alto, CA: Electric Power Research Institute.P. Kurath, (1984)
[2] S. D. Downing, D. R. Galliart, In: Multiaxial Fatigue, Edited by G. E. Leese and D. F. Socie. Society of Automotive Engineers, AE-14 (1989) 13.
[3] S. Rahman, N. Ghadiali, G. M. Wilkowski, N. Bonora, Nuclear Engineering and Design, 2 (1996) 55.
[4] N. Bonora, International Journal of Fracture, 75 (1996) 1.
[5] S. Rahman, N. Ghadiali, G.M. Wilkowsky, B. Brickstad, International of Pressure Vessels and Piping, 75(5) (1998) 397.
[6] R. Firmature, S. Rahman, Engineering Fracture Mechanics, 66 (1) (2000) 15.
[7] P.C. Paris, H. Tada, NUREG/CR-3464. Washington, DC: US Nuclear Regulatory Commission (1983).
[8] R. Klecker, F. Brust, G. Wilkowski, NUREG/CR-4572, Washington, DC: US Nuclear Regulatory Commission. (1986).