Predicting fatigue lifetime of engineering components under variable amplitude multiaxial fatigue loading

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ABSTRACT. The aim of the present paper is to systematically investigate the accuracy of the so-called Modified Wöhler Curve Method (MWCM) in estimating fatigue lifetime of engineering components damaged by in-service variable amplitude multiaxial load histories. In more detail, the MWCM, which is a bi-parametrical critical plane approach, postulates that initiation and Stage I propagation of fatigue cracks occur on those material planes experiencing the maximum shear stress amplitude (this being assumed to be always true independently from the degree of multiaxiality of the applied loading path). Further, the fatigue damage extent is hypothesised to depend also on the maximum stress perpendicular to the critical plane, the mean normal stress being corrected through the so-called mean stress sensitivity index (i.e., a material constant capable of quantifying the sensitivity of the assessed material to the presence of superimposed static stresses). To extend the use of the above criterion to those situations involving variable amplitude loadings, the MWCM is suggested here as being applied by defining the critical plane through that direction experiencing the maximum variance of the resolved shear stress: since the resolved shear stress is a monodimensional quantity, stress cycles are directly counted by the classical Rain-Flow method. In the present investigation, the overall accuracy of the MWCM in estimating high-cycle fatigue strength was checked through several experimental results taken from the literature and generated, under variable amplitude multiaxial fatigue loading, by testing plain samples of commercial steels. Such systematic validation exercise allowed us to prove that the MWCM is highly accurate, resulting in estimates falling within the error scatter bands associated with the experimental data used to calibrate the method itself.

SOMMARIO. Lo scopo del presente lavoro è quello di indagare sistematicamente l'accuratezza del *Modified Wöhler Curve Method* (MWCM) nella stima della vita a fatica di componenti meccanici sollecitati da storie di carico multiassiali ad ampiezza variabile. In particolare, il MWCM, che non è altro che un approccio bi-parametrico di piano critico, postula che l'innesco e la prima fase della propagazione di cricche di fatica avvengano sempre sui piani che sperimentando la massima ampiezza della tensione di taglio (e questo indipendentemente dal grado di multiassialità della storia di carico applicata). Inoltre, il livello di danneggiamento viene assunto dipendere anche dalla massima tensione perpendicolare al piano critico stesso, dove la tensione normale media viene corretta tramite il cosiddetto *mean stress sensitivity index*. Per estendere l'uso di tale criterio anche al caso di carichi multiassiali ad ampiezza variabile, nel presente lavoro si mostra come il MWCM possa essere applicato definendo il piano critico attraverso la direzione che sperimenta la massima varianza dello sforzo di taglio risolto: visto che, per definizione, la tensione tangenziale risolta è una quantità monodimensionale, i cicli di fatica possono essere pertanto direttamente contati mediante il classico metodo del Rain-Flow. Infine, l'accuratezza e l'affidabilità del MWCM nella stima della vita a fatica è stata verificata attraverso una serie di risultati sperimentali tratti dalla letteratura e ottenuti sollecitando provini lisci in acciaio con storie di carico multiassiali ad ampiezza variabile. Tale validazione sistematica ha consentito di dimostrare come l'uso del MWCM risulti in stime della vita a fatica molto accurate, ovvero con previsioni caratterizzate da un livello di dispersione statistica comparabile con quello delle curve di fatica utilizzate per calibrare lo stesso MWCM.

KEYWORDS. Critical plane; Multiaxial loading; Variable amplitude; Rain-Flow counting.

INTRODUCTION

Estimating fatigue damage in engineering materials subjected to variable amplitude (VA) multiaxial fatigue loading is a problem as complex as important to be solved, since, in situations of practical interest, mechanical components experience, at their critical locations, stress/strain states which are not only multiaxial, but also vary randomly. In light of the importance of such a structural integrity issue, since the beginning of the 80s many different attempts have been made in order to formalise sound procedures suitable for estimating fatigue damage under such circumstances [1-20]. In this complex scenario, the present paper summarises then an attempt of extending the use of the so-called Modified Wöhler Curve Method (MWCM) [21], a bi-parametrical critical plane approach on which we have been working over the last decade systematically, to those situations in which engineering materials are subjected to in-field VA multiaxial load histories, by formalising, at the same time, an alternative multiaxial cycle counting strategy based on the classical Rain-Flow method.

FUNDAMENTALS OF THE MWCM UNDER CONSTANT AMPLITUDE LOADING

The MWCM is a critical plane approach which estimates multiaxial fatigue damage through the maximum shear stress amplitude, τ_a, as well as through the mean value, $\sigma_{n,m}$, and the amplitude, $\sigma_{n,a}$, of the stress perpendicular to the critical plane. According to the fatigue damage model the MWCM is based on [21], the critical plane is defined as that material plane experiencing the maximum shear stress amplitude, τ_a. From a practical point of view, the combined effect of both τ_a, $\sigma_{n,m}$ and $\sigma_{n,a}$ are taken into account simultaneously through the following stress index [22]:

$$\rho_{eff} = \frac{m \cdot \sigma_{n,m} + \sigma_{n,a}}{\tau_a} \tag{1}$$

In the above identity, mean stress sensitivity index m is a material property to be determined experimentally [21, 22]. As to ratio ρ_{eff} instead, thanks the way it is defined, such a stress index is seen to be sensitive not only to the presence of superimposed static stresses, but also to the degree of non-proportionality of the applied loading path [21].

Turning back to the MWCM, the way it estimates fatigue damage under multiaxial fatigue loading is schematically shown by the modified Wöhler diagram reported in Fig. 1a. The above log-log diagram plots the shear stress amplitude relative to the critical plane, τ_a, against the number of cycles to failure, N_f. By performing a systematic reanalysis based on numerous experimental data [21-24], it was proven that, as index ρ_{eff} varies, different fatigue curves are obtained (Fig. 1a). In particular, it was observed that fatigue damage tends to increase as ρ_{eff} increases: this results in the fact that the corresponding fatigue curve tends to shift downward in the above diagram with increasing of ρ_{eff} (Fig. 1a). According to the classical log-log schematisation used to summarise fatigue data, the position and the negative inverse slope of any Modified Wöhler curve can unambiguously be defined through the following linear relationships [21, 24]:

$$k_\tau(\rho) = \alpha \cdot \rho + \beta \tag{2}$$

$$\tau_{Ref}(\rho) = a \cdot \rho + b \tag{3}$$

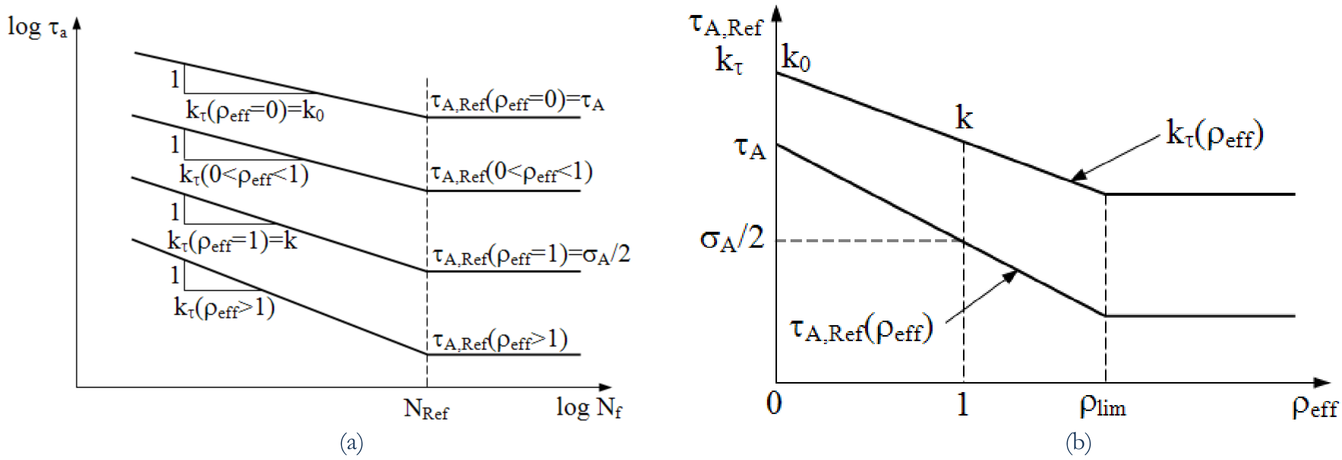


Figure 1: Modified Wöhler diagram (a) and MWCM's governing equations (b).

In the above definitions, $k_\tau(\rho_{eff})$ is the negative inverse slope, while $\tau_{Ref}(\rho_{eff})$ is the reference shear stress amplitude extrapolated at N_A cycles to failure (see Fig. 1a). Further, α, β, a and b are material constants to be determined experimentally. In particular, by remembering that ρ_{eff} is equal to unity under fully-reversed loading and to zero under torsional loading [21], the constants in Eqs (2) and (3) can be calculated directly as follows:

$$k_\tau(\rho_{eff}) = (k - k_0)\cdot \rho_{eff} + k_0 \qquad \text{for } \rho_{eff} \le \rho_{lim} \qquad (4)$$

$$\tau_{A,Ref}(\rho_{eff}) = \left(\frac{\sigma_A}{2} - \tau_A\right)\cdot \rho_{eff} + \tau_A, \qquad \text{for } \rho_{eff} \le \rho_{lim} \qquad (5)$$

the meaning of the adopted symbols being explained in Fig. 1. For the sake of clarity, relationships (4) and (5) are also plotted in Fig. 1b. The latter schematic chart allows us to briefly discuss another important hypothesis which is usually formed to efficiently handle those situations involving large values of ratio ρ_{eff}. In particular, the above diagram shows that, for $\rho_{eff}>\rho_{lim}$, $k_\tau(\rho_{eff})$ and $\tau_{A,Ref}(\rho_{eff})$ take on the following values [21, 22]:

$$k_\tau(\rho_{eff}) = (k - k_0)\cdot \rho_{lim} + k_0 = const \qquad \text{for } \rho_{eff} \le \rho_{lim} \qquad (6)$$

$$\tau_{A,Ref}(\rho_{eff}) = \left(\frac{\sigma_A}{2} - \tau_A\right)\cdot \rho_{lim} + \tau_A = const \qquad \text{for } \rho_{eff} \le \rho_{lim} \qquad (7)$$

where [22]

$$\rho_{lim} = \frac{\tau_A}{2\tau_A - \sigma_A} \qquad (8)$$

To conclude, it is worth observing that the above correction, which plays a fundamental role in the overall accuracy of the MWCM, was introduced in light of the fact that, under large values of ratio ρ_{eff}, the predictions made by the MWCM were seen to become too conservative [25]. According to the experimental results due to Kaufman and Topper [26], such a high degree of conservatism was ascribed to the fact that, when micro/meso cracks are fully open, an increase of the normal mean stress does not result in a further increase of fatigue damage.

OUR VA FATIGUE DESIGN METHODOLOGY

Consider a component subjected to a complex system of time-variable forces resulting in a multiaxial stress state at the assumed section point (Fig. 1a). From the above stress state, the orientation of the critical plane at point O can directly be determined by locating that plane containing the direction experiencing the maximum variance of the resolved shear stress (direction MV in Fig. 1b) [27]. The shear stress amplitude relative to the critical plane, τ_a, can then be calculated from the variance of stress signal $\tau_{MV}(t)$, i.e. [27]:

$$\tau_m = \frac{1}{T}\int_0^T \tau_{MV}(t)\cdot dt$$

$$Var[\tau_{MV}(t)] = \frac{1}{T}\int_0^T [\tau_{MV}(t) - \tau_m]^2 \cdot dt \;\Rightarrow\; \tau_a = \sqrt{2\cdot Var[\tau_{MV}(t)]} \tag{9}$$

where $\tau_{MV}(t)$ is the shear stress resolved along direction MV (Fig. 1b), whereas T is the time interval over which the assessed load history is defined (Fig. 1c). In a similar way, the mean value, $\sigma_{n,m}$, and the amplitude, $\sigma_{n,a}$, of the stress, $\sigma_n(t)$, normal to the critical plane take on the following values [27] (Fig. 1d):

$$\sigma_{n,m} = \frac{1}{T}\int_0^T \sigma_n(t)\cdot dt \tag{10}$$

$$Var[\sigma_n(t)] = \frac{1}{T}\int_0^T [\sigma_n(t) - \sigma_{n,m}]^2 \cdot dt \;\Rightarrow\; \sigma_{n,a} = \sqrt{2\cdot Var[\sigma_n(t)]} \tag{11}$$

As soon as both τ_a, $\sigma_{n,m}$ and $\sigma_{n,a}$ are known, the degree of multiaxiality and non-proportionality of the assessed load history can directly be evaluated in terms of index ρ_{eff}, Eq. (1) - (Fig. 1e), the position of the corresponding modified Wöhler curve being estimated according to relationships (4) to (7) - (Fig. 1f). By taking full advantage of the classical Rain-Flow method, the resolved shear stress cycles can now be counted (Figs 1c and 1g) to build the corresponding load spectrum (Fig. 1h). Finally, the calculated load spectrum has to be used to estimate the damage content associated with any counted shear stress level (Figs. 1h and 1f), the estimated number of blocks to failure being equal to:

$$N_b = \frac{D_{cr}(\rho_{eff})}{D} = \frac{D_{cr}(\rho_{eff})}{\sum_{i=1}^{j}\frac{n_i}{N_{f,i}}} \tag{12}$$

In the above relationship $D_{cr}(\rho_{eff})$ is the critical value of damage sum D and such an index is proposed here to be defined as follows:

$$D_{cr}(\rho_{eff}) = d_1\cdot\rho_{eff} + d_2 \tag{13}$$

d_1 and d_2 being material fatigue properties to be determined experimentally. According to identity (13), the hypothesis is formed then that, in the most general case, the critical value of the damage sum can vary as the degree of multiaxiality and non-proportionality, evaluated through ratio ρ_{eff}, of the stress state at the assumed crack initiation site varies. To conclude, it can be observed here that, if fatigue lifetime is estimated instead according to the classical rule due to Palmgren and Miner, then the critical value of the damage sum, D_{cr}, can directly be taken invariably equal to unity.

Material	Ref.	a	b	α *[MPa]*	β *[MPa]*	m	ρ_{lim}	N_{Ref} *[Cycles]*	d_1	d_2
35NCD16	[28]	-4.6	14.1	-140.9	337.4	1	1.197	$2\cdot10^6$	0.18	0.09
SAE 1045	[29, 30]	-6.2	12.5	-62.3	156.6	1	1.257	$2\cdot10^6$	0	0.2
S460N	[31, 32]	-1.5	9.6	-17.5	116.7	1	3.325	$2\cdot10^6$	0	0.2

Table 1: Constants in the MWCM's governing equations for the plain and notched materials being investigated.

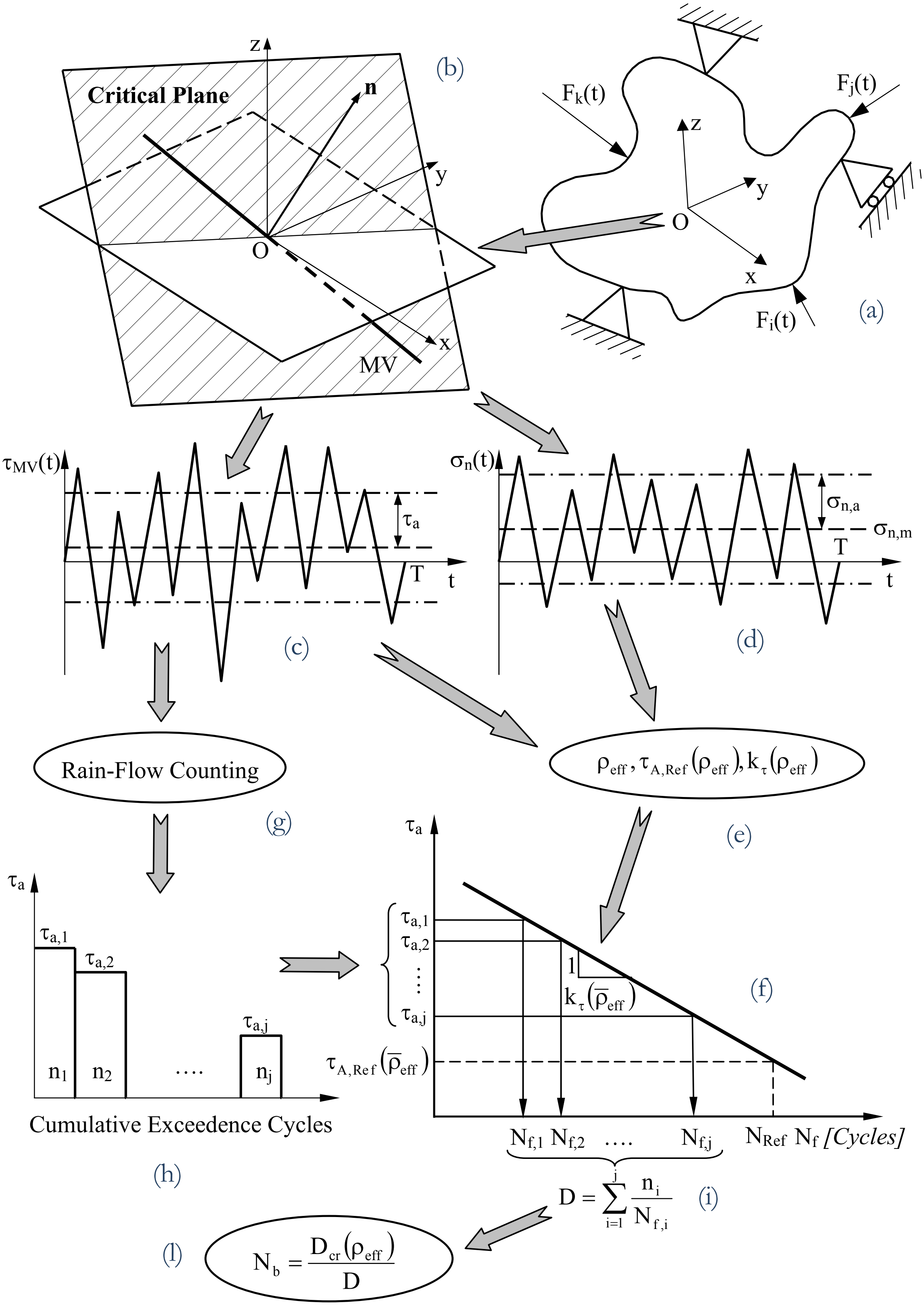


Figure 2: In-field use of the MWCM to perform the fatigue assessment under VA multiaxial fatigue loading.

ACCURACY IN ESTIMATING FATIGUE LIFETIME UNDER VA MULTIAXIAL FATIGUE LOADING

In order to check the accuracy and reliability of the proposed approach in estimating fatigue damage under VA multiaxial fatigue loading, a systematic bibliographical investigation was carried out to select a set of suitable experimental results. The fatigue constants relative to the investigated plain and notched samples are summarised in Tab. 1, where, since all the investigated load histories were characterised by superimposed static stress either equal to zero or, in any case, very close to zero, the mean stress sensitivity index, m, was assumed to be equal to unity for all the considered materials.

Before moving to the validation exercise discussed in the present section, it is worth observing that the scatter bands plotted in Figs. 3c and 4c were calculated under the hypothesis of a long-normal distribution of the number of cycles to failure at any stress level, with a confidence value equal to 95%.

Fig. 3 summarises the results obtained from the reanalysis of the experimental results generated by Franck Morel during his PhD work [28]. In particular, he tested plain samples of high strength steel 35NCD16 under in-phase VA tension-compression and torsion. The adopted VA load histories were directly derived from standard sequence CARLOS lateral [33] by filtering it in order to speed up the experimental investigation. In more detail, Morel considered two omission levels resulting in two filtered sequences called CARLOS-f1 (13568 extrema) and CARLOS-f2 (46656 extrema) [28]. As to the above results, it is worth observing that, as pointed out by Morel himself, the two omission levels did not affected the experimental results significantly, so that, in theory, all the generated results could have been reanalysed together by considering the original VA sequence. However, in our validation exercise we rigorously considered the two VA load histories obtained by applying the two filtering strategies, i.e., ω=0.4 and ω=0.6 omission level [28]. The chart reported in Fig. 3a shows the resolved shear stress spectrum built, under both uniaxial (ρ_{eff}=1.06), torsional (ρ_{eff}=0), and in-phase biaxial loading (ρ_{eff}=0.762), by employing the Rain-Flow method to count the resolved shear stress cycles.

In order to accurately estimate the results generated by Morel under biaxial loading, the first problem to be addressed was the correct evaluation of the critical value of the damage sum. According to the D_{exp} vs. ρ diagram reported in Fig. 3b, the resulting constants in Eq. (13) were then as follows: d_1=0.18 and d_2=0.09. The estimated, $N_{b,e}$, vs. experimental blocks to failure, N_b, diagram of Fig. 3c makes it evident that the usage of the MWCM resulted in estimates falling within the widest scatter band between the two relative to the CA fatigue curves used to calibrate the MWCM itself, the above results being calculated by adopting the 2k-1 correction to handle those cycles of low shear stress amplitude [34].

Subsequently, we checked the accuracy and reliability of our VA multiaxial fatigue lifetime estimation technique by reanalysing the results generated, in the ambit of the SAE Biaxial Fatigue Program, by testing notched samples of SAE 1045 under a particular VA in-phase bending and torsion load history called “Ag Tractor Bending” [29, 30]. Initially, it is worth remembering here that the MWCM can be applied to perform the fatigue assessment of notched components in terms of not only local, but also nominal quantities [21]. In particular, in the latter case the constants in the MWCM’s governing equations have to be calculated from the uniaxial and torsional fatigue curves generated by testing the same geometrical feature as the one to be designed against multiaxial fatigue loading. Since the fatigue curves generated under both fully-reversed CA bending and fully-reversed CA torsion were reported in Ref. [29] for the specific case of the SAE notched samples, all the pieces of experimental information necessary for correctly calibrating the MWCM were directly available in the original source. It is worth remembering here also that the notched samples investigated in the SAE Biaxial Program were nothing but conventional cylindrical shafts with fillet, where the notch root radius of 5mm resulted in a stress concentration factor of 1.42 under bending and of 1.23 under torsion. The values of the constants in the MWCM’s governing equation, calculated with respect to the net nominal section, are listed in Tab. 1. The investigated VA load history was gathered from an instrumented drive axle of an agriculture tractor and it was reported in Ref. [29] in terms of a Markovian matrix. In order to reconstruct the input load history, the algorithm summarised in Ref. [29] was adopted obtaining the shear stress spectrum shown in Fig. 4a, where such a spectrum was determined by using the Rain-Flow counting method to extract the cycles from the load history plotted in terms of shear stress resolved along the direction experiencing the maximum variance of the resolved shear stress. As clearly reported in the chart of Fig. 4a, the calculated value for ρ_{eff} was equal to 0.993 under bending, whereas it was equal to zero under torsion and to 0.763 under in-phase bending and torsion. In order to correctly calculate the critical value of the damage sum, as done for Morel’s data, also in this case the VA results generated under pure bending as well as under pure torsion were considered, obtaining, as shown in Fig. 4b, an average value for D_{cr} equal to approximately 0.2. To conclude, the estimated, $N_{b,e}$, vs. experimental blocks to failure, N_b, chart reported in Fig. 4c shows that, also in this case, the use of the MWCM, formalised to specifically address the VA multiaxial fatigue problem, resulted in highly accurate estimates, such a results being obtained by adopting the 2k-1 correction to account for the damaging effect of the shear stress cycles of low stress amplitude [34].

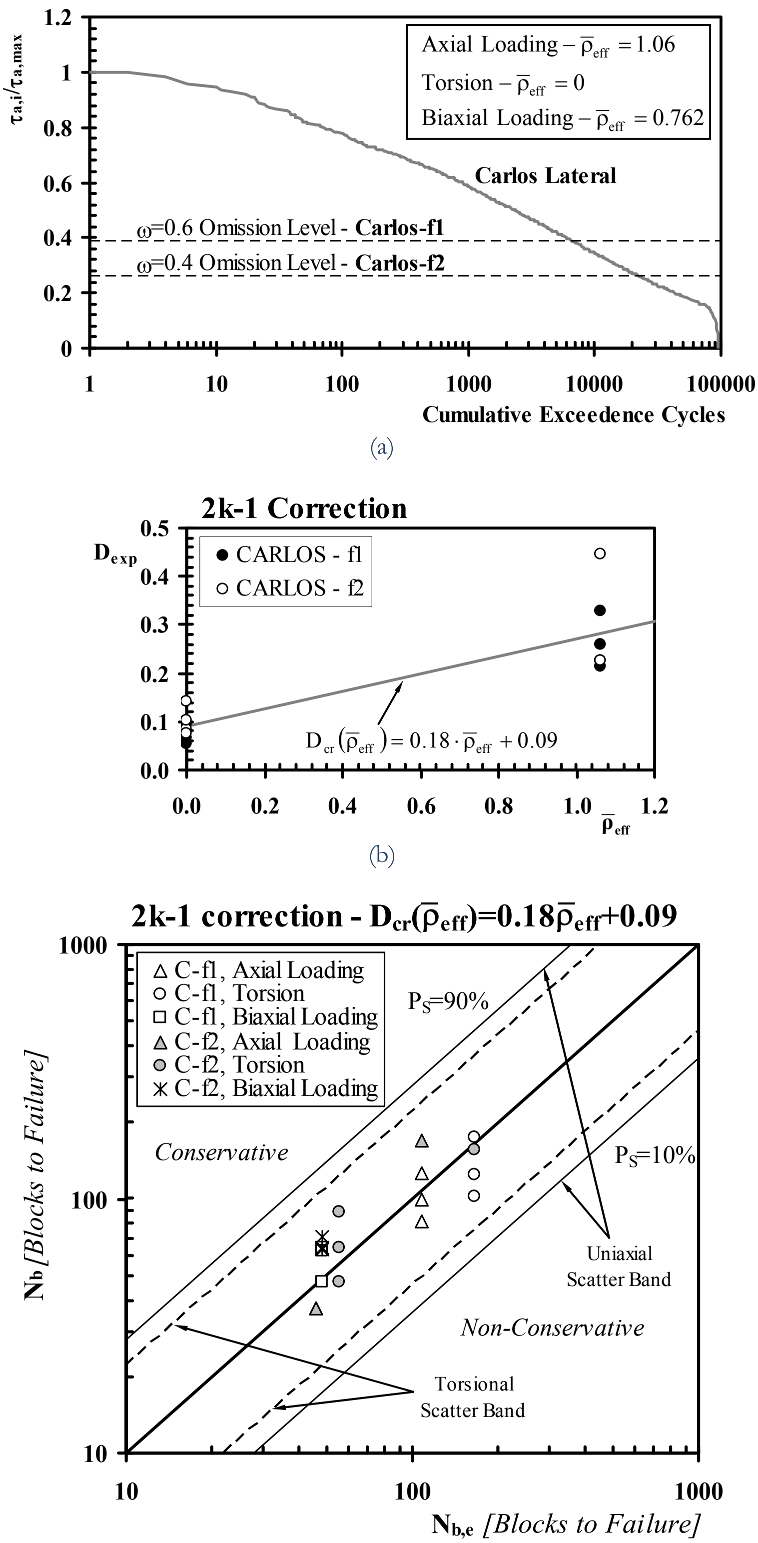


Figure 3: MWCM's accuracy in estimating the fatigue lifetime of plain samples made of 35NCD16 and subjected to VA multiaxial fatigue loading (Data taken from Ref. [28]).

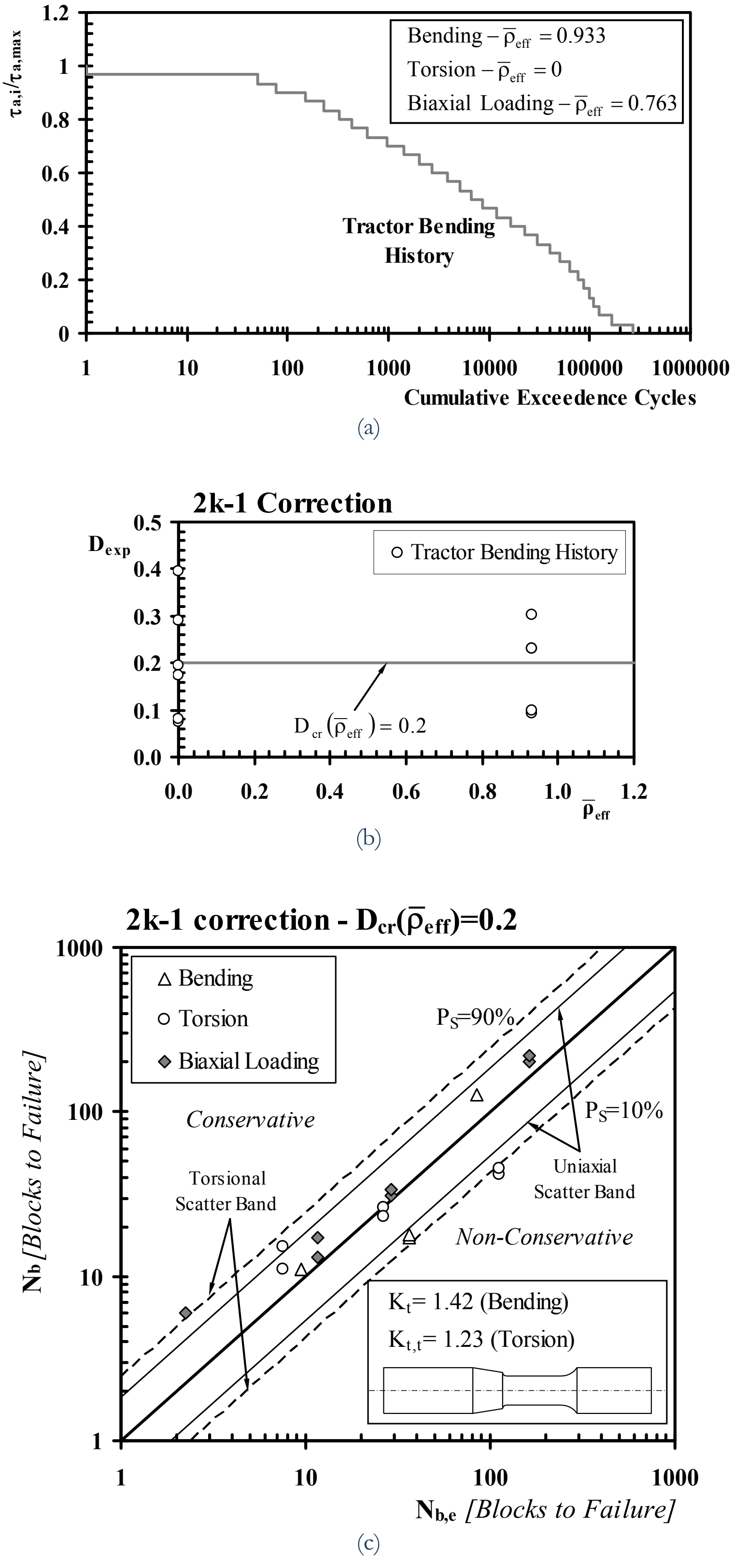
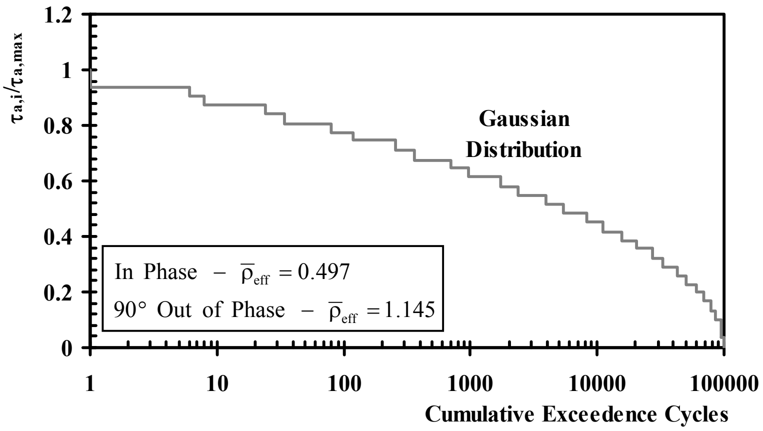
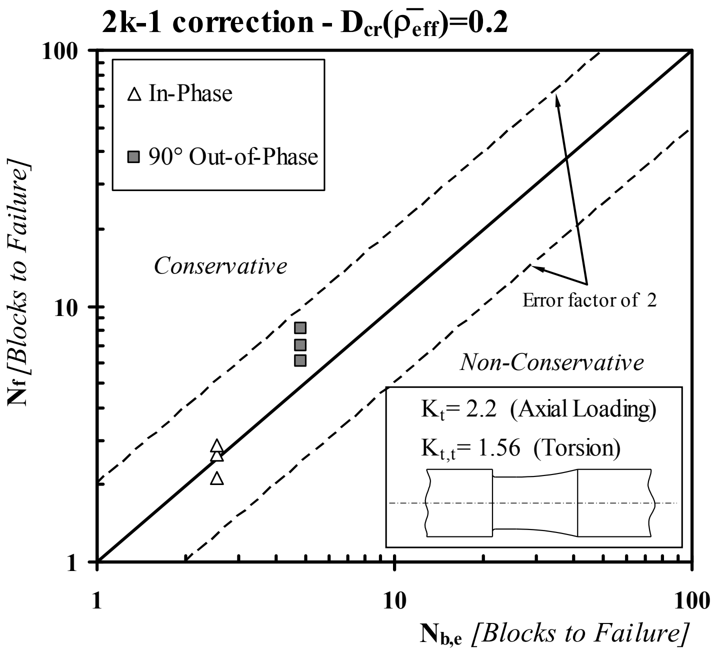


Figure 4: MWCM's accuracy in estimating the fatigue lifetime of notched samples made of SAE 1045 and subjected to VA multiaxial fatigue loading (Data taken from Refs [29, 30]).



(a)



(b)

Figure 5: MWCM's accuracy in estimating the fatigue lifetime of notched samples made of S460N and subjected to in-phase and 90° out-of-phase VA multiaxial fatigue loading (Data taken from Refs [31, 32]).

The last data set investigated in the present section to check the accuracy of our method in estimating VA fatigue lifetime was the one generated by Vormwald and co-workers [31, 32] by testing, under in-phase and 90° out-of-phase tension-compression and torsion, samples with shoulder fillet of fine grain steel S460N, the input VA load histories being generated by adopting a Gaussian spectrum (Fig. 5a). As done for the SAE samples, also in this case all the calculations were carried out in terms of net nominal stresses and the necessary material fatigue constants (see Table 1) were estimated by using the only two experimental CA fatigue curves available for the above notched samples (i.e., the one generated under in-phase CA biaxial loading and the one obtained under 90° out-of-phase CA biaxial loading). This was possible since, as explained in Ref. [21] in great detail, if the mean stress sensitivity index is taken equal to unity, the MWCM's governing equations can directly be calibrated through two fatigue curves generated under two different values of ratio ρ_{eff}. Finally, owing to the fact that, for the specific material/geometrical configuration under investigation, it was not possible to determine a reference value of the critical damage sum from the experiments, D_{cr} was taken equal to 0.2 as done for the SAE samples: the estimated, $N_{b,e}$, vs. experimental blocks to failure, N_b, chart reported in Fig. 5b proves that the use of the 2k-1 correction based formalisation of the MWCM resulted in estimates within an error factor (in lifetime) of 2.

CONCLUSIONS

1) The reformulation of the MWCM proposed in the present paper is seen to be successful in estimating lifetime under VA multiaxial fatigue loading.
2) Since the MWCM estimates fatigue damage through the shear stress resolved along the direction of maximum variance of the resolved shear stress, the cycle counting under multiaxial fatigue loading can directly be performed by using the classical Rain-Flow Method.
3) 4) According not only to our calculations, but also to the results reported in the technical literature [35], in the presence of long VA load histories the critical value of the damage sum is suggested as being taken always lower than 0.25, of course, if it cannot be evaluated by running appropriate experiments.

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