Transient creep modeling based on the dependence of the activation energy on the internal stress

Nicola Bonora, Luca Esposito
University of Cassino, Cassino, I-03044, Italy

ABSTRACT. Although primary stage is often neglected in creep design, in several alloys for high temperature applications, most of the allowable design strain can occurs before reaching the minimum creep rate. The kinetic of the primary creep stage is important since it determines the microstuctural conditions for the subsequent creep regime. Under rapid stress change creep transient occurs and, according to the developed dislocation substructure, a stress dependence of the creep rate is observed. In order to account for these features in the material response, an appropriate modelling is required. Assuming that the current creep rate in the transient regime can be expressed as a function of the steady state creep rate, a model based on the evolution of the internal stress is derived. The proposed model does not rely on any particular formulation for the steady state creep and therefore is of general applicability. In the present paper, a mechanism based secondary creep stage model, with an explicit dependence of the creep exponent n on stress, as proposed by the authors, has been used [1]. An application to polycrystalline copper is presented.

KEYWORDS. Transient creep; Internal stress; Activation energy; Activation volume; Copper.

INTRODUCTION

Traditionally, creep modeling and design are limited to the study of the secondary creep stage while transitory processes, such as those occurring during primary creep, are usually neglected. In the literature, only a limited number of physical based models for primary and transient creep have been proposed [2]. Most of them are phenomenological in nature [3] and make use of additive formulations [4, 5]: primary and secondary creep terms are treated separately, giving for each one a strain or time-hardening law.
However, it is well known that transient creep behaviour of crystalline solids is directly related to microstructural changes that occur with time [6, 7]. Creep transients, such as primary creep or those occurring after stress (or temperature) changes, have been correlated with variations in the dislocation density, dislocation arrangements and subgrain formations. The steady state creep is a special condition of microstructural dynamical equilibrium where the structure remains constant with time and the creep rate remains at its minimum value. According to this, primary stage, secondary stage, and each transitory phase should be described by the evolution of continuous variables where the effect of structural changes is responsible for the resulting creep rate. An helpful variable for describing the overall effect of microstructure evolution on the creep response is the mean internal stress. This is defined as the quantity characterizing the local internal stresses acting against the dislocation motion [8]. In this work, taking into account the evolution of the apparent activation energy with the internal stress, a formulation for transient creep is derived.

TRANSIENT MODEL

The Orowan's equation allows the determination of the creep rate as follow:

$$\dot{\varepsilon} = b\rho_m \bar{v}_m \tag{1}$$

where b is the Burgers vector, ρ_m is the mobile dislocation density and $\bar{v}_m$ is the average velocity of mobile dislocations. In transient creep, both the dislocation density and the velocity vary with time reaching a constant value at the steady state where the Eq. (1) becomes:

$$\dot{\varepsilon}_{ss} = b\rho_{ss} \bar{v}_{ss} \tag{2}$$

Combining Eq. (1) and Eq. (2), it is possible to write the current creep rate as function of the creep rate at the steady state:

$$\dot{\varepsilon} = \left(\frac{\rho_m \bar{v}_m}{\rho_{ss} \bar{v}_{ss}} \right) \cdot \dot{\varepsilon}_{ss} \tag{3}$$

Assuming that in the transient the dislocation density evolves similarly to the dislocation velocity:

$$\frac{\rho_m}{\rho_{ss}} = \left(\frac{\bar{v}_m}{\bar{v}_{ss}} \right)^{\beta} \tag{4}$$

the first term on the right hand side of equation (3) becomes:

$$\left(\frac{\rho_m \bar{v}_m}{\rho_{ss} \bar{v}_{ss}} \right) = \left[\frac{\exp\left(\frac{-Q}{RT} \right)}{\exp\left(\frac{-Q_{ss}}{RT} \right)} \right]^{\beta+1} \tag{5}$$

Where Q is the apparent activation energy that reaches the constant value Q_{ss} only at the steady state [19]. Out of this condition Q varies and should show a dependence on the average internal stress and the activation volume.
Jeong et al. [9] proposed the use of the following expression,

$$Q = Q_0 - \Omega\sigma^* \tag{6}$$

where Ω is the activation volume and σ^* is the effective stress defined as the difference between the applied stress and the average internal stress,

$$\sigma^* = \sigma - \bar{\sigma}_i \tag{7}$$

The average internal stress accounts for the local stresses which oppose to the dislocation motions as a result of the presence of obstacles, long range barriers and strain hardening events.
The evolution law for the internal stress can be derived from dislocation mechanics. Several expressions have been proposed in literature [10, 11].
According to Estrin-Mecking kinetics, the evolution for the internal stress can be derived from the evolution of the dislocation density as a function of the creep strain [12] leading to,

$$\frac{d\bar{\sigma}_i}{d\varepsilon} = \frac{1}{\varepsilon_0}\left(\bar{\sigma}_{iss} - \bar{\sigma}_i \right) \tag{8}$$

where $\bar{\sigma}_{iss}$ is the the internal stress value at the steady state. Integrating the Eq. (8) we get:

$$\bar{\sigma}_i = \bar{\sigma}_{iss}\left[1 - \exp\left(-\frac{\varepsilon}{\varepsilon_0} \right) \right] \tag{9}$$

Thus, substituting Eq. (7) and (8) in Eq. (10) leads to,

$$Q = \left(Q_0 - \Omega\sigma + \Omega\bar{\sigma}_{iss}\right) - \Omega\bar{\sigma}_{iss}\exp\left(-\frac{\varepsilon}{\varepsilon_0}\right) \tag{10}$$

Recognizing that the first right hand side term is Q_{ss}, we get,

$$Q = Q_{ss} - \Omega\bar{\sigma}_{iss}\exp\left(-\frac{\varepsilon}{\varepsilon_0}\right) \tag{11}$$

and substituting in Eq. (5) we can finally rewrite the Eq. (3),

$$\frac{\dot{\varepsilon}}{\dot{\varepsilon}_{ss}} = \exp\left[\frac{(\beta+1)}{RT}\Omega\bar{\sigma}_{iss}\exp\left(-\frac{\varepsilon}{\varepsilon_0}\right)\right] \tag{12}$$

In order to simplify the parameters identification, the following rearrangement is proposed:
a) the internal stress at the steady state, $\bar{\sigma}_{iss}$, is expressed as a fraction of the applied stress, $\bar{\sigma}_{iss} = \phi\sigma$, being ϕ also a function of stress $\phi = \phi(\sigma)$.
b) the term $(\beta+1)$, ϕ and Ω, are grouped in the scaled activation volume $\bar{\Omega}(\sigma) = \phi(\sigma)\cdot\Omega(\sigma)\cdot(1+\beta)$.
Under these assumptions, equation (12) can be rewritten in terms of external applied stress:

$$\frac{\dot{\varepsilon}}{\dot{\varepsilon}_{ss}} = \exp\left[\frac{\bar{\Omega}}{RT}\sigma\exp\left(-\frac{\varepsilon}{\varepsilon_0}\right)\right] \tag{13}$$

Multiple stress change model

The expression in equation (13) allows the description of a single transient such as the primary creep stage. To reproduce multiple transient phases caused by consecutive stress changes, a reset of the variables at the beginning of each stress change is neessary. Thus, the equation (13) can be generalized for multiple stress changes as follows:

$$\frac{\dot{\varepsilon}}{\dot{\varepsilon}_{ss}} = \exp\left[\frac{\bar{\Omega}}{RT}(\sigma - \tilde{\sigma})\exp\left(-\frac{(\varepsilon - \tilde{\varepsilon})}{\varepsilon_0}\right)\right] \tag{14}$$

where and are the stress and the strain at the time at which the stress change occurs. Eq. (14) remains valid for primary creep where both and are zero.
In Fig. 1 the qualitative evolution of the internal stress, as inferred in Eq. (14), during a generic stress change test is shown.

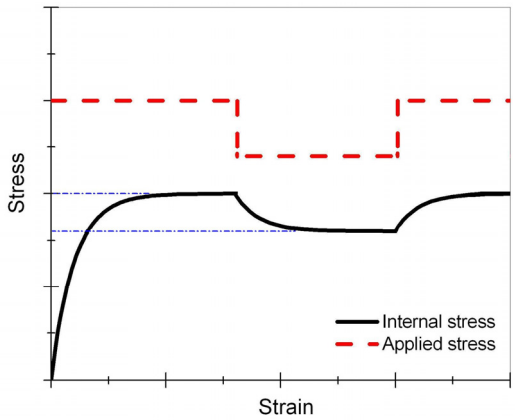

Figure 1: Qualitative evolution of internal stress.

STEADY STATE MODEL

Accordingly to the range of stress and temperature, creep deformation can occur mainly either by dislocation glide (or glide-plus-climb) or by diffusional flow. For what concerns steady state creep stage, for each deformation mechanism specific models, based on dislocation mechanics theory, are available [13]. Recently the authors [1] proposed a creep model formulation that preserve a power-law type description, in which the change in the creep mechanism can be accounted for through an explicit dependence of the exponent n on stress,

$$\dot{\varepsilon}_{ss} = A_0 \cdot \exp\left(-\frac{Q_{ss}}{RT}\right)\left(\frac{\sigma}{\sigma_0}\right)^{\exp\left[\left(\frac{\sigma}{\sigma_0}\right)^m\right]} \tag{15}$$

This model allows to predict the non linearity in the $\ln(\dot{\varepsilon}_{ss})$ vs $\ln(\sigma)$ plot which is an essential requisite to investigate the material behavior on wide stress range.

APPLICATION TO POLYCRYSTALLINE COPPER

The model proposed in this work has been applied to a fine grain copper tested at 686-824K over large stress range. The experimental data are taken from Wilshire and Battenbought [7] who performed both standard creep and change stress tests. Some tests were done above the initial yield stress, σ_{0y}, of 20, 18 and 15MPa at 686, 723 and 824K respectively. Similarly to [7] the following plastic flow rule has been used:

$$\sigma_y = \sigma_{0y} + \chi \varepsilon_p^{1/2} \tag{16}$$

The steady state model parameters have been identified by a non linear fitting of the experimental temperature compensated minimum creep rate using an activation energy equal to 110 kJ/mol. In Fig. 2 the resulting fit curve is given. The steady state parameters are summarized in Tab. 1. In Fig. 3 the comparison of the minimum creep rate prediction with the experimental data is shown.

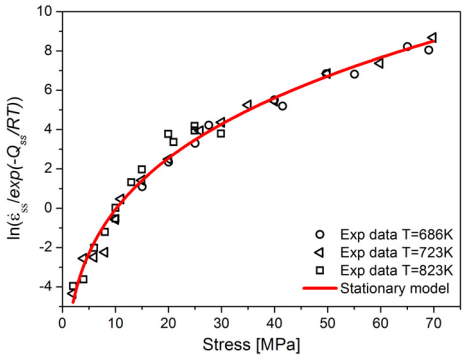

Figure 2: Steady state model parameters identification.

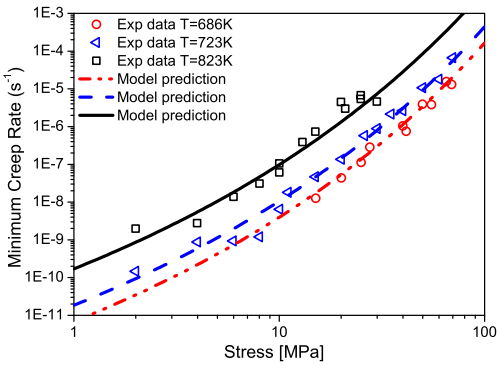

Figure 3: Comparison of experimental minimum creep rate with Eq. (15) prediction.

A_0 [s^{-1}]	σ_0 [MPa]	m	Q_{ss} [J/mol]
2.74e-2	3.1629	0.1	110000

Table 1: Steady state parameters.

In Fig. 4 is shown the variations of the creep strain rate as a function of the increasing inelastic strain (plastic plus creep) at 723K, is given. Here, firstly the minimum creep rate for 40 MPa is reached; then a stress reduction down to 30MPa is applied and hold for 14 ks after which the stress is again increased up to 40 MPa and hold.

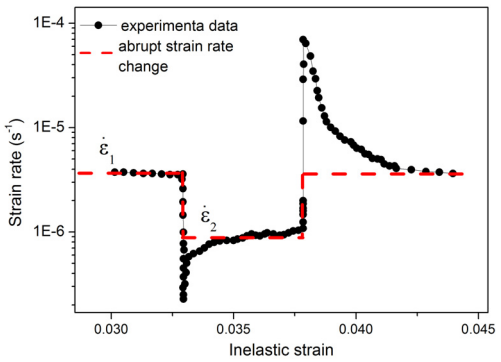

Figure 4: Stress change test ranging from 40 to 30 MPa at 723K.

Immediately after the stress drop, the creep rates are low because dislocations must move at 30MPa within the substructure developed at 40MPa [7]. On reloading, dislocations move easily within the recovered substructure formed at 30MPa, resulting in a higher creep rate that gradually decreases to the 40 MPa value. Equation 14 has been used to reproduce this behavior. The parameters $\bar{\Omega}$ and ε_0, identified on the reloading, are 1.6643 mm^3/mole and 1.08e-3 respectively. Must be underlined that the scaled activation volume expressed in mm^3/mole must be multiplied by a factor of one thousand in Eq. (14) to obtain the activation energy in Joules units.
The finite element simulation of the creep stress change test in terms of creep rate versus the inelastic strain is shown in Fig. 5. In the simulation as in the experiment the stress reduction was maintained for 14 ks.
Fig. 6 shows the accumulation of the total strain versus time calculated with the proposed model. The corresponding total strain accumulation in the case of abrupt change of strain rates is also reported. In that case to reproduce the experimental strain accumulation the stress reduction have to be maintained for about 6 ks.

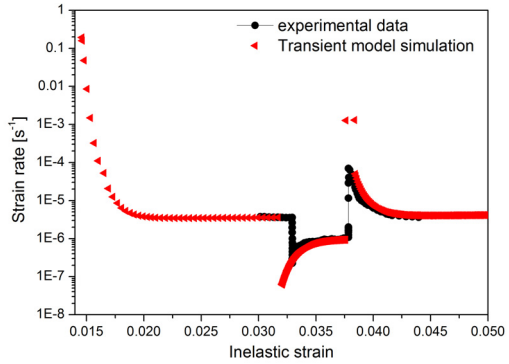

Figure 5: Comparison of transient model simulation with the stress change test.

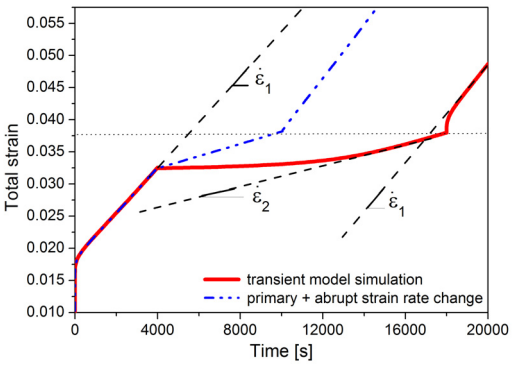

Figure 6: Total strain accumulation versus time obtained in the stress change test simulation.

CONCLUSION

In this work a model for transient creep based on the dependence of the activation energy on the evolution of the internal stress has been derived. The proposed formulation has the following features: a multiplicative formulation where the transient creep rate is given a fraction of the steady state creep rate is found; the model is physically based; a limited number of parameters, of easy determination, is needed. The proposed formulation can be used to reproduce multiple creep transients. The model has been applied to fine grain size copper and the comparison with the experimental data is very good. The accuracy of the model predictions has been verified on multiple creep stress change test data.

REFERENCES

[1] N. Bonora, L. Esposito: J. Eng. Mater. and Technol.- Trans. ASME, 132 (2010) 021013.
[2] B. Derby, M.F. Ashby, Acta Metall., 35 (1987) 1349.
[3] S.V. Raj, A.D. Freed, Material Science and Engineering A, 283 (2000) 196.
[4] E.N.d.C. Andrade, In: Proceedings of Royal Society of London A, 84 (1910) 1.
[5] R.W. Evans, B. Wilshire, Creep of Metals and Alloys, The Institute of Metals, London, (1985).
[6] C.N. Ahlquist, R. Gasca-Neri, W.D. Nix: Acta Metallurgica, 18 (1970) 663.
[7] B. Wilshire, A.J. Battenbough, Material Science and Engineering A, 443 (2007) 156.
[8] F. Dobeš, Scripta Metallurgica, 12 (1978) 63.
[9] C.Y. Jeong, S.W. Nam, J. Ginsztler, Material Science and Engineering A, 264 (1999) 188.
[10] W.G. Jhonston, J.J. Gilman, Journal of Applied Physics, 30 (1959) 129.
[11] A. Orlová, K. Milička, F. Dobeš: Material Science and Engineering A, 194 (1995) 9.
[12] Estrin Y., Mecking H., Acta Metallurgica, 32 (1984) 57.
[13] H. J. Frost, M. F. Ashby, Deformation-Mechanism Maps, Pergamon press, New York, (1981).