A new approach for the description of fracture in laminated glass beams

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ABSTRACT. Failure of laminated glass units is characterized by the growth and propagation of interfaces that can arise in unpredictable location of the layers. The paper presents the application of a theoretical model suitable in predicting the fracture path across the layers of laminated glass units when subjected to a static transversal load. The model falls in the context of the Strong Discontinuities Approach (SDA). All the relevant equations of the model derive from a variational principle formulated in a general context, thus allowing also for nonlinear continua. The numerical implementation in the Finite Element Method is based on Elements with Embedded Discontinuities concept. Relevant applications to laminated glass beams are presented and the results are compared with other theories.

KEYWORDS. Interfaces; Strong Discontinuities; Laminated Glass Units.

INTRODUCTION

Laminated glass elements for structural applications consist of two or more ply of glass bonded together by a thin interlayer, usually polyvinyl butyral (PVB). The assembly presents some advantages over monolithic glass of the same nominal thickness with respect to impact resistance and post fracture behaviour. The failure is dominated by a lot of factors such as thickness of the ply and of the interlayer, temperature, composition of the interlayer.

Despite the increased use of laminated glass, research has focused mainly on monolithic glass, while fewer efforts were devoted to laminated glass. More specifically, to date, experimental data on laminated glass exist, while theoretical models are scarce. Moreover, existing theoretical analyses are restricted by additional simplifying assumptions based on the intuitive evaluation that the actual structural behaviour of the laminated glass beam lies somewhere between two limiting cases: the layered limit and the monolithic limit.

Analytical models that predict stress development and ultimate strength of laminated glass beams and plates have been presented in [1,2,3,4]. However the analysis of breakage and the prediction of crack paths is not frequent in the literature. For instance, in [5] an analytical model based on cumulative damage theory is discussed for the prediction of the cumulative probabilities of inner glass ply breakage.

Taking into account that on the whole glass units exhibit a damaging-fracturing behaviour and discontinuities in the displacement field can arise in unpredictable locations, it seems possible that the problem can be studied in a different area. Indeed, this phenomena can be effectively described by means of mechanical models that incorporate the kinematics of strong discontinuities obtained by an enrichment of the displacement field with a discontinuous term. Consequently, the strain field is decomposed into a compatible and an enhanced term.

The paper shows the application in the context of the Strong Discontinuities Approach (SDA) [6,7] of the model presented in [8, 9] to simply supported laminated glass beams. Simple numerical simulations of three point bending test are reported. In the applications both the flexural behaviour of the beams and the growth and propagation of interfaces inside the glass layers and across the polymeric interlayer are investigated.

THE MODEL

This paper presents the application of a theoretical model suitable in predicting the fracture path across the layers of laminated glass beams when subjected to a static transversal load. The mechanical model is based on the kinematics of the Strong Discontinuities Approach. The basic equations are derived following a variational approach, which is fully described in [10]. The general formulation, derived for an elastic plastic damaging continuum, is specialized in the case under examination and specific constitutive hypotheses for the glass and the interlayer are assumed.

Kinematics

In [8] the classical kinematics of the Strong Discontinuities Approach is used to develop a structural model for the simulation of growth and propagation of interfaces inside a continuum medium.

Let S be an interface embedded within a continuous body occupying the domain Ω. Let Ω_φ be a subset of Ω containing the discontinuity and such that S divides Ω_φ into two subdomains, Ω_φ^+ and Ω_φ^- respectively. The normal $\mathbf{n}$ is oriented toward the interior of Ω_φ^+. The boundary of Ω_φ is divided by the surface S in two parts. According to the position of the interface part of the boundary of Ω_φ can belong to the boundary of Ω. The geometry of the problem is depicted in Fig. 1.

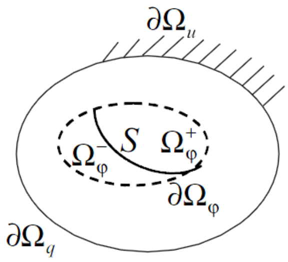


Figure 1: Domain Ω and discontinuity surface S.

Across the interface S the displacement field is discontinuous and the jump is denoted by $[\![\mathbf{u}]\!]$. The displacement field is usually given by the sum of a continuous differentiable function $\bar{\mathbf{u}}$ defined in Ω plus function $\tilde{\mathbf{u}}$ continuous and differentiable everywhere except on the interface S, so that the kinematics of the Strong Discontinuities is ruled by the following equations:

$$\mathbf{u}(\mathbf{x},t) = \bar{\mathbf{u}}(\mathbf{x},t) + \tilde{\mathbf{u}}(\mathbf{x},t)$$

$$\begin{aligned} \tilde{\mathbf{u}}^+(\mathbf{x},t) \quad &- \quad \tilde{\mathbf{u}}^-(\mathbf{x},t) = [\![\mathbf{u}]\!]_S(\mathbf{x},t) \quad \forall \mathbf{x} \in S \\ \tilde{\mathbf{u}}(\mathbf{x},t) \quad &= \quad 0 \quad \text{on} \quad \partial\Omega_\varphi \end{aligned}$$

$$\tilde{\mathbf{u}}(\mathbf{x},t) = \bar{M}_S(\mathbf{x})[\![\mathbf{u}]\!](\mathbf{x},t)$$

The enhanced enrichment function $\overline{M}_S$ vanishes on the boundary of Ω_φ and on the restrained boundary of Ω and presents an unit jump across S. Function $[\![\mathbf{u}]\!]$ is a regular function, such that $[\![\mathbf{u}]\!] = [\![\mathbf{u}]\!]_S$ on S.

The F.E. form of the displacement field leads to the following expression of the Strong Discontinuity

$$\mathbf{u}(\mathbf{x}) = \sum_{i \in N_m} N_i(\mathbf{x})\hat{\mathbf{u}}_i + [\![\mathbf{u}]\!] \left[H_S(\mathbf{x}) - \sum_{j \in S_m{}^+} N_j(\mathbf{x}) \right] \bar{N}_S(\mathbf{x})$$

where N_i are the shape functions defining the approximation of the displacement field, $\hat{\mathbf{u}}_i$ are nodal degrees of freedom, the first sum is extended over the set of all the nodes of the finite element mesh, while S_m^+ is the set of the enriched nodes belonging to Ω_φ^+. H_S is the Heaviside function on Ω_φ.

The domain Ω_φ coincides with the band of elements that are cut by the discontinuity and the interpolation of function $[\![\mathbf{u}]\!]$ is made element-wise. In this way, the nodal degrees of freedom coincide with the nodal displacements and the jump function can be treated as an internal variable. Function $[\![\mathbf{u}]\!]$ is supposed to be constant inside the element. Function $\overline{N}_S$ plays the role of annihilating the enriched component of the displacement field on the restrained portion of the boundary.

Equilibrium, compatibility and constitutive equations

All the state equations are derived by means a variational approach. A general formulation in which the medium and the interface are ruled by different constitutive equations, defined by distinct free energy and dissipation functionals is considered. The variational statement of the problem is derived starting from a generalized mixed multi-fields Hu-Washizu functional considering an elastic-plastic damaging behaviour for both the bulk and the interface as described in [10].

All the equations are particularized to the case of an elastic continuum medium in which interfaces have a dissipative behaviour. The growth and the propagation of interfaces is ruled by specific activation functions for each material of the glass units, based on cohesive fracture like criteria.

NUMERICAL PROCEDURE

The numerical procedure used allows one to predict the fracture path inside and across the layers. All the relevant equations of the models are discretized.

The Finite Element implementation of the algorithm is based on the Elements with Embedded Discontinuities [11]. It follows recently developed strategies exploiting the formal analogy between the equations of the enriched continuum and the theory of classical plasticity [12]. It obtains a structure of the numerical algorithm that allows the use of the procedure inside classical F.E. codes for the equilibrium problem of elastic-plastic solids. As a special note, the equilibrium condition at the interface is satisfied in a weak sense, leading to the classical equations of Statical Kinematical Optimal Nonsymmetric formulation of SDA [11], obtained under the hypotheses that the jump field $[\![\mathbf{u}]\!]$ be constant and that a Petrov-Galerkin approximation of the incompatible strain is used in the orthogonality condition between stresses and enhanced strain.

APPLICATION TO LAMINATED GLASS BEAMS

The numerical response of simply supported two-ply laminated glass beams under a static transversal load has been investigated. The interfaces growth is assimilated to flexural cracks opening in the glass and to shear slip in the interlayer.

The geometry of the laminated composite beam, characterized by two layers of thin glasses and one layer of PVB, is shown in Fig. 2.

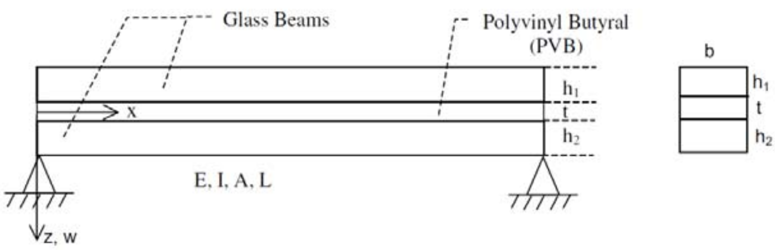


Figure 2: Laminated glass beam.

The geometrical and material data have been assumed as in [13]. The span length L of the beam is 0.8 m; the cross section width is 10 cm, the glass thickness is 5 mm for both the layers and the interlayer thickness is 0.38 mm. Glass elastic modulus and interlayer shear modulus are taken as 64.5 GPa and 1287 kPa, respectively. The Poisson's ratio of glass and PVB are taken to be 0.23 and 0.49, respectively. A numerical three point bending test has been performed by increasing monotonically a prescribed displacement applied at mid-span.
The behaviour of the interfaces is ruled by two activation function, for the glass and PVB respectively, as it is reported in Tab 1. In this simple application a softening behaviour characterizes both the materials.

glass	$f(\mathbf{t}_S, \chi_S) = \mathbf{t}_{S_n} - f_{tu} + \chi_S$	$f_{tu} = 40$
PVB	$f(\mathbf{t}_S, \chi_S) = [\mathbf{t}^2_{S_m}]^{0.5} - f_{tu} + \chi_S$	$f_{tu} = 1$

Table 1: Unit materials activation function.

In Tab. 1 $\mathbf{t}_{S_n}$ and $\mathbf{t}_{S_m}$ are the normal and tangential component of the stress vector on the surface *S*, χ_S is the internal scalar force defined only on the surface S that rules the evolution of cracks, conjugated to the internal kinematic variable α_{S_e}, and f_{tu} is the tensile strength of the material.
In Fig. 3 the evolution of cracks in the layers is shown. Specifically the zoom in the mid-span of the internal softening variable $\alpha_{S_p} = -\alpha_{S_e}$ is represented without any interpolation inside the element so that the graph reports the actual value of the variable at the Gauss points. The interfaces rise at the midpoint of the bottom beam and propagate along the vertical direction in the top beam. At a certain stage the shear force in the interlayer exceeds its limit value and the irreversible mutual shear displacement takes place in the PVB as well.

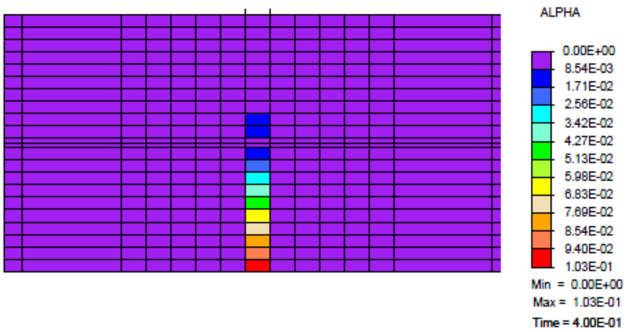


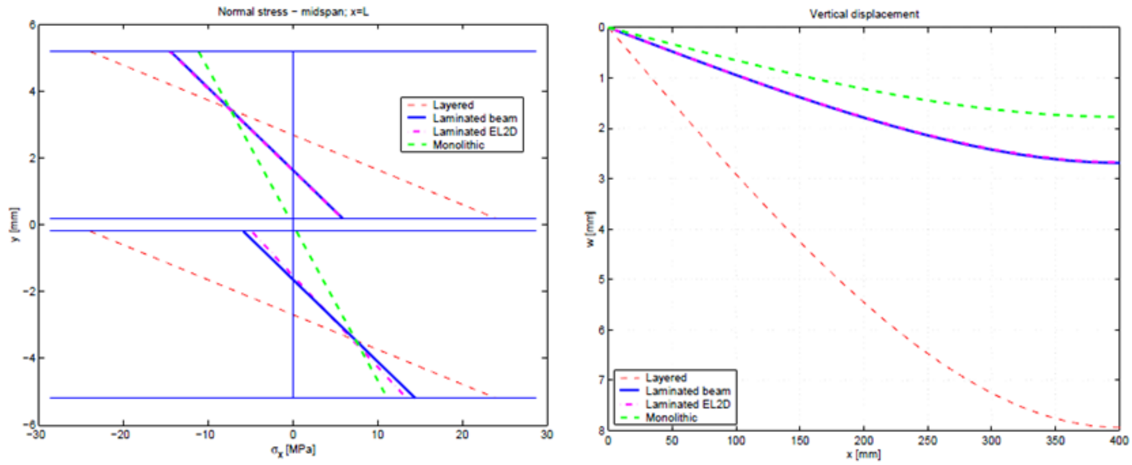
Figure 3: Laminated beam - SDA prediction of the post post-critic behaviour. Initial crack at midspan of the bottom glass beam and diffusion of cracks in the top glass beam.

In order to verify the correct prediction of the initial location of interfaces in the glass layers and in the interlayer, the results are firstly compared with those of a 2D finite element model developed and solved with ADINA code under the hypothesis of perfect elasticity. A 8-nodes discretization of 800 x (10+2+10) plane stress elements has been used. Indeed, the material has an elastic behaviour until interfaces occur, so that, using the activation function in Tab. 1, that at the first stage when $\chi_S = 0$ is a Rankine-like criterion for glass, cracks rise where the maximum value of the tensile stress is achieved. This first happens at bottom surface of the bottom glass beam.
The results are also commented with reference to the prediction of the mathematical model for the behaviour of laminated glass beams of Aşik and Tezcan [13]. The model, derived by using large deflection theory, predicts that the behaviour of simply supported laminated glass beams is bounded by two limiting cases which are monolithic and layered behaviour.
The analytic solution for a simply supported beam gives the following stresses at the surfaces of the plies:

$$\sigma_1^{top} = -\frac{M}{I}\frac{h_1}{2} + \frac{N_1}{A_1} \qquad \sigma_2^{top} = -\frac{M}{I}\frac{h_2}{2} + \frac{N_2}{A_2}$$

$$\sigma_1^{bottom} = \frac{M}{I}\frac{h_1}{2} + \frac{N_1}{A_1} \qquad \sigma_2^{bottom} = \frac{M}{I}\frac{h_2}{2} + \frac{N_2}{A_2}$$

N_1 being the axial force in the top glass layer, equal to N_2, M the bending moment, A_1, and A_2 the cross sections of the glass layers, I the sum of the moments of inertia of the two glass layers, h_1 and h_2 the glass layers thickness respectively.
The behaviour for monolithic, laminated and layered beams is illustrated in Fig. 4, the transition being influenced by the thickness of the interlayer and its stiffness. The comparison with ADINA simulation is also reported, denoted in the pictures legend as "Laminated EL2D". In the case of coupled response (Laminated) both the analytical model and the ADINA numerical simulation predict that the activation candidate point is the midpoint of the bottom surface, as it is predicted by the SDA simulation of Fig. 3.



(a) (b)

Figure 4: Aşik and Tezcan model and ADINA solution.
(a) Normal stress at midspan. (b) Vertical displacement at midspan.

The model in [13] predicts the same curvature for both the glass beams. This result derives from the kinematic hypothesis and is independent on the interlayer stiffness and beam slenderness. Actually, the top beam exhibits a greater curvature than the bottom beam, the difference becoming larger with the decrease of the slenderness and of the interlayer stiffness. It can be shown that the behaviour is strongly influenced by the stiffness of the interlayer, ranging from layered to monolithic for increasing values of the shear modulus. Specifically, in the case of deep beams and low interlayer shear modulus, the behaviour tends to the layered one and the maximum of the tensile stress is reached at the bottom surface of the top beams. In this situation the propagation of cracks starts from the upper beam, as it is predicted by the SDA model. The same results is given by the ADINA simulation. Successively cracks also arise in the bottom beam and follows the same evolution in both the beams, as it is shown in Fig. 5.

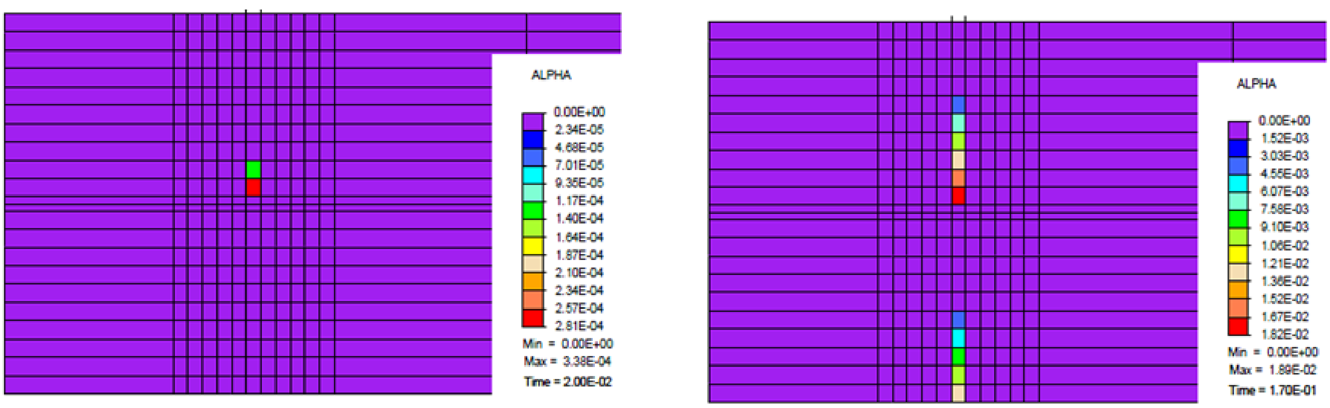


Figure 5: Laminated deep beam - SDA prediction of the post post-critic behaviour.
Initial crack at midspan of the top glass beam and diffusion of cracks in the bottom glass beam.

CONCLUSIONS

The paper concerns the application of a theoretical model based on the Strong Discontinuities Approach to laminated glass beams for simulating the growth and propagation of fracture. Specifically the numerical response in a three point bending test has been presented and compared with other solutions.
The hypothesis of elastic behaviour for both the materials of the composite beam has been formulated, until interface occur, where an elastic-softening behaviour is introduced. This is realistic for the glass because it behaves in an elastic-fracturing manner. However, the PVB presents a viscous behaviour, that should be enclosed in the model. Nevertheless, the model is able to predict the growth and evolution of cracks inside the layers and gives a good approximation of the stresses in the materials. This initial attempt has to be improved considering more refined constitutive equations and more complex loading conditions.

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