



A probabilistic model for fatigue strength of hydraulic cylinders' housings

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RIASSUNTO. L'articolo presenta un modello probabilistico per la valutazione dell'affidabilità dei cilindri idraulici in diverse condizioni di sollecitazione. I cilindri idraulici sono componenti meccanici che convertono energia idraulica in energia meccanica. In questo lavoro abbiamo considerato cilindri sottoposti a più di 2 milioni di cicli di pressione interna applicata durante la loro vita. Per questo motivo il progetto deve garantire una vita a fatica infinita.

Secondo le ipotesi formulate, una vita a fatica infinita è garantita quando le condizioni di sollecitazione non permettono la nascita e la crescita di cricche. Le cricche di solito nascono da discontinuità superficiali già presenti nel materiale, che sono molto dannose per questa applicazione. Per questo motivo una variabile fondamentale del modello proposto è la distribuzione delle imperfezioni sulla superficie del cilindro, che è stata ottenuta con misure sperimentali.

La soglia per la propagazione delle cricche è stata modellata tenendo in considerazione l'effetto delle cricche corte.

L'analisi statistica proposta considera inoltre alcune ipotesi sulla probabilità di individuare disomogeneità superficiali di profondità superiore ad una certa soglia attraverso i controlli non distruttivi.

Il metodo di Monte Carlo viene usato per valutare l'affidabilità di un componente particolare in diverse condizioni.

ABSTRACT. The paper presents a probabilistic model used to investigate the effect of different variables upon the failure probability of hydraulic cylinder housings subjected to fatigue loads.

The inputs of the model are: i) an experimental distribution of surface discontinuities; ii) a threshold model obtained by experiments on micronotched specimens; iii) geometric tolerances of the tubes; iv) POD curve of the NDT control. A key input in the proposed model is the surface quality, which is modeled considering the distribution of longitudinal surface discontinuities obtained from experimental measurements. Crack propagation threshold has been modeled taking into consideration the effect of short cracks. The probability of detection of surface inhomogeneities during ordinary non destructive tests have been modeled according to data reported in the literature.

Finally the Monte Carlo method has been implemented to assess the reliability of a component in different conditions.

KEYWORDS. Reliability, hydraulic cylinders, fatigue, short cracks.



INTRODUCTION

The hydraulic cylinders are mechanical components which convert hydraulic energy in mechanical energy. In this article hydraulic cylinders subjected to more than 2 million internal pressure cycles in their life will be considered. For this reason the design must guarantee an infinite fatigue life. In Fig. 1 an example of a hydraulic cylinder is reported.

A fatigue assessment method, based on a fracture mechanics approach, to assess the reliability of tubes for hydraulic cylinder housings under different service conditions is presented. Infinite fatigue life is guaranteed when service conditions do not allow the creation and growth of cracks. Surface discontinuities are detrimental for this application, as cracks usually start from pre-existing surface imperfections [1, 2]. Therefore, a key input in the proposed model is the surface quality, which is modeled considering the distribution of longitudinal surface discontinuities obtained from experimental measurements.

The statistical analysis herein proposed has considered some hypotheses on the probability of detection of surface inhomogeneities during ordinary non destructive tests [3]. Finally the Monte Carlo method [4] has been used to assess the reliability of a component with certain geometry under different load conditions.



Figure 1: Example of a hydraulic cylinder.

THRESHOLD MODEL

According to the well-known Murakami's theories on fatigue [5], flaws having a depth lower than 1 mm can be considered as small cracks. In this case, the fatigue strength in presence of surface discontinuities is characterized by the so-called Kitagawa diagram [6], as reported in Fig. 2.

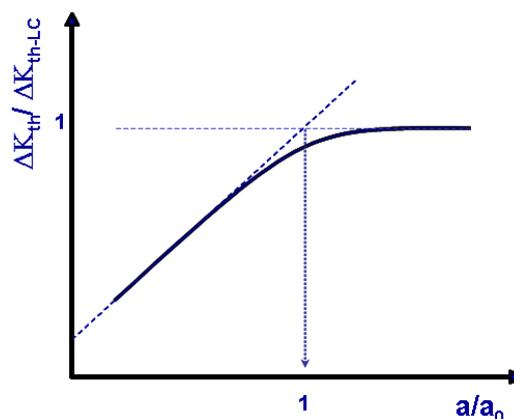


Figure 2: Kitagawa diagram.



The diagram shows the dependence of the stress intensity factor (SIF) threshold, ΔK_{th} , with the defect size. It can be noticed that the SIF threshold increases if defect size increases, until, for a defect size equal to a_0 , the SIF reaches the threshold for long cracks, $\Delta K_{th,LC}$. Therefore, in order to determine the fatigue behaviour in presence of surface discontinuities, the relation between the applied SIF, the flaw size and the SIF threshold for the crack growth might be used. The verification of infinite fatigue life can be carried out comparing the SIF applied to the component and the SIF threshold for crack growth, as shown in Eq. (1).

$$\Delta K_{applied} \leq \Delta K_{th} \quad (1)$$

If Eq. (1) is fulfilled, cracks are not able to grow and the component has an infinite fatigue life. $\Delta K_{applied}$ depends on the crack geometry according to relation in Eq. (2):

$$\Delta K_{applied} = 0.65 \cdot \Delta \sigma_{applied} \cdot \sqrt{\pi \cdot \sqrt{A}} \quad (2)$$

where A is the area of the defect projected onto a plane perpendicular to the applied stress, and 0.65 is the geometric parameter describing the defect shape [5]¹. Previous research on fatigue of hydraulic cylinder tubes has shown that fatigue strength is controlled by the presence of surface inhomogeneities and that ΔK_{th} can be expressed as [2]:

$$\Delta K_{th} = \Delta K_{th,LC} \sqrt{\frac{\sqrt{A}}{\sqrt{A} + \sqrt{A_0}}} \quad (3)$$

where $\sqrt{A_0}$ is obtained by fatigue limit tests on micronotched specimens [1, 2].

According to this approach, the verification for an infinite fatigue life can be carried out if the following variables are known:

- the applied stress: $\Delta \sigma_{applied}$
- the geometry of the defect, \sqrt{A} , directly related to a .

For the purpose of this study the SIF threshold for long cracks $\Delta K_{th,LC}$ and the El-Haddad parameter $\sqrt{A_0}$ are considered constant.

ASSESSMENT OF PROBABILITY OF FAILURE

To carry out a reliability analysis, the actual value of an input must be compared with the limit value. In many cases both values are not fixed but are statistically distributed. For example, as described in the previous section, the applied SIF can be compared with the threshold SIF. The reliability is the probability that the applied SIF is lower than the threshold SIF, applying an infinite fatigue life criterion as expressed in Eq. 1. A reliability function can be defined as in Eq. (4):

$$g(\Delta \sigma, a) = \Delta K_{applied} - \Delta K_{th} \quad (4)$$

The limit condition is given when $g(\Delta \sigma, a) = 0$. Supposing that $\Delta \sigma$ and a are statistically distributed uncorrelated variables, the function defines two domains in the $(\Delta \sigma, a)$ space: if $g(\Delta \sigma, a) \leq 0$ no failure occurs (safe region) while for $g(\Delta \sigma, a) > 0$ failure occurs (unsafe region). The probability of failure of a component can be calculated solving the integral reported in Eq. (5) [7], where $f_{\Delta \sigma}$ and f_a are the probability density functions respectively of the applied stress and of the defects depth.

$$P_f = \int_{g < 0} f_{\Delta \sigma}(\Delta \sigma_{max}) \cdot f_a(a) da d\Delta \sigma \quad (5)$$

¹ For semi-elliptical surface discontinuities with a ratio $a/c \gg 5$, as those found during experimental measurements, $\sqrt{A} = \sqrt{10} \cdot a$ [5] (see paragraph *Research of surface imperfections and assessment of flaws distribution* for definition of a and c).



Alternatively a Monte Carlo simulation might be applied if the distribution of the applied stress and the distribution of the surface flaws are known [4].

The probability of failure calculated according to Eq. (5) is valid if only one defect is present in each component. The average number of imperfections present in a component must be taken into consideration when calculating the probability of failure. In the case study herein presented experimental data show that not all components will present a flaw, as the density of surface flaws is small compared to the hypothetical surface of a hydraulic cylinder. Therefore the probability of a flaw to be present in the component must be considered in order to avoid an overestimation of the probability of failure. Thus, the final probability of failure is calculated according to Eq. (6):

$$P_f = P_{f,flaw} \cdot Prob\{flaw\} + P_{f,noflaw} \cdot Prob\{noflaw\} \quad (6)$$

$P_{f,flaw}$ and $P_{f,noflaw}$ are the probabilities of failure if respectively one flaw and no flaws are present in the component, $Prob\{flaw\}$ and $Prob\{noflaw\}$ are the probability of respectively one flaw and no flaws to be present on the component's surface. For the aim of this study $P_{f,noflaw}$ has been considered zero, thus the probability of failure can be obtained from Eq. (7).

$$P_f = P_{f,flaw} \cdot Prob\{flaw\} \quad (7)$$

RESEARCH OF SURFACE IMPERFECTIONS AND ASSESSMENT OF FLAWS DISTRIBUTION

We have inspected longitudinal discontinuities present on tubes using magnetic powder. We have measured length and depth of the flaws. We have measured the length using a pocket rule. We have then grinded the imperfections and we have measured their depth with a micrometer gauge, by measuring the difference between the outer diameter of the tube close to the defect (OD_1) and the outer diameter on the repaired defect (OD_2), as shown in Fig. (3). The total surface we have examined is 411m².

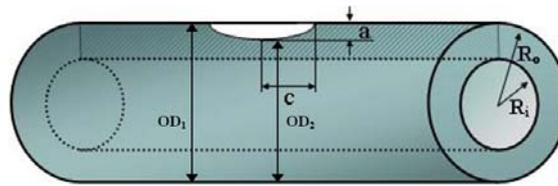


Figure 3: Scheme of a tube with a surface flaw.

Experimental data permit to obtain the average density of imperfections on the surface. The value of $Prob\{flaw\}$ used in Eq. (6) and (7) can be obtained when the component surface is known.

In Tab. 1 the data concerning the inspection of the different lots are reported. For each lot the total examined surface and the tubes wall thickness are indicated.

lot	#defects	Total examined surface [sqm]	WT [mm]
1	62	86	12.5
2	11	63	10
3	0	48	12.5
4	18	33	15
5	3	63	12.5
6	2	42	17.5
7	9	48	10
8	2	28	12.5
total	107	411	

Table 1: Characteristics of inspected lots



All examined surface discontinuities have a ratio $\frac{c}{a} > 5$, which means they are long and shallow longitudinal flaws.

Therefore we have calculated the parameter \sqrt{A} of each defect with the equation $\sqrt{A} = \sqrt{10} \cdot a$ [5].

We have evaluated the distribution of a considering the whole set of data and the data from the tubes batches which presented a sufficient number of defects. In some batches we found very few defects and no evaluation of their distribution has been possible. We have interpolated the experimental values with different distributions. The best fitting distribution has been found through the method of the probability paper and by applying the Kolgomorov-Smirnov test to compare the goodness of fit [4, 8]. The whole set of data, as well as the data concerning single lots, are well described by a LEVD (Largest Extreme Value Distribution), whose cumulative density function is reported in Eq. (8).

$$F(a) = \exp\left\{-\exp\left[-\left(\frac{a - \lambda}{\delta}\right)\right]\right\} \quad (8)$$

In Fig. 4 we show the probability papers for the whole set of data (black) and for data concerning single lots (coloured). On the x-axis we show the values of a normalized in respect to its average, \bar{a} (normalized a).

With the maximum likelihood method [4] we have obtained the best fitting parameters, λ and δ , and their confidence interval for the data of single lots and for the whole set of data.

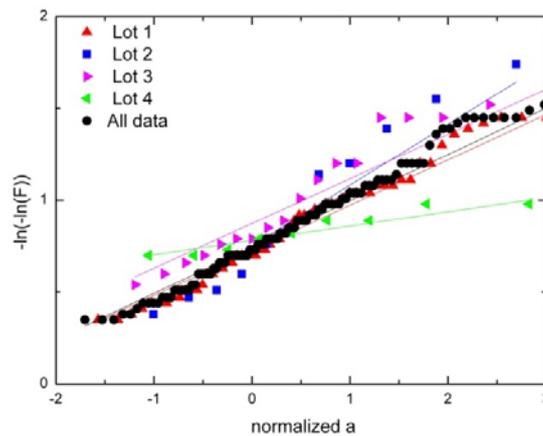


Figure 4: Probability papers for the flaws depth, a , normalized in respect to its average, \bar{a} (normalized a)

We assume that the parameters obtained from the whole set of data, $\bar{\lambda}$ and $\bar{\delta}$, are representative of the defects distribution on any batch of tubes.

The fact that surface discontinuities are well described with a LEVD distribution suggests that those flaws, that have been detected with magnetic powder, are the largest values of the surface imperfections present onto the tubes surface. This is in agreement with the method used to measure the depth of imperfections, in fact the magnetic powder permits to detect only quite large and deep longitudinal flaws.

The distribution, that describes the defects, must take into consideration the application of standard nondestructive tests, that eliminate discontinuities which depth exceeds a certain value. For this reason we have corrected the LEVD defect distribution, $f(a)$, by formulating a hypothesis on the probability of detection (POD) of the flaws, which has been modelled as a lognormal cumulative density function distribution [3], as reported in Eq. (9). The final flaws distribution we have obtained is reported in Eq. (10).

$$POD(a) = \Phi\left(\frac{\ln(a) - \theta}{\zeta}\right) \quad (9)$$

where

$$\zeta^2 = \ln\left[1 + \left(\frac{\sigma_{POD}}{\mu_{POD}}\right)^2\right]; \quad \theta = \ln(\mu_{POD}) - \frac{1}{2}\zeta^2$$



$$f'(a) = \frac{d}{da} \left\{ \frac{\int_0^a f(a) \cdot [1 - POD(a)]}{\int_0^\infty f(a) \cdot [1 - POD(a)]} \right\} \quad (10)$$

In Fig. 5 we show an example of the initial defect distribution, $f(a)$, the lognormal cumulative density function, $POD(a)$, and the final defects probability density function, $f'(a)$. On the x-axis we show the values of a normalized in respect to its average, \bar{a} .

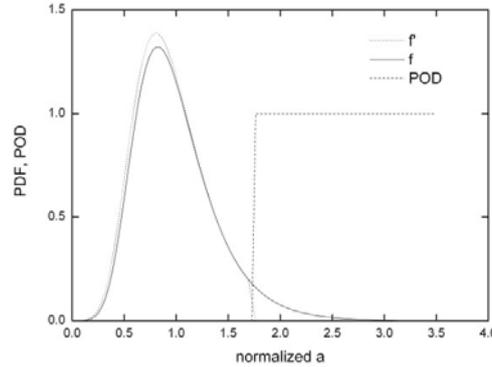


Figure 5: Distributions $f'(a)$, $f(a)$, and $POD(a)$.

CASE STUDY

In this section we describe the reliability analysis of a specific component. For the evaluation of the reliability we used the Monte Carlo method. This method requires an assumption for the distribution of the input variables. We have modeled the variables as follows:

- Geometry:
 - Cylinder inner diameter, ID : normally distributed with a mean value of 115 mm and a tolerance (3 times the standard deviation) equal to 0.4% of the nominal value.
 - Wall thickness, WT : normally distributed with a mean value of 10 mm and a tolerance (3 times the standard deviation) equal to 8.4% of the nominal value.
- Surface flaws, a : LEVD distributed with parameters $\bar{\lambda}$ and $\bar{\delta}$, obtained by fitting the whole set of experimental data. Other LEVD distributions have been considered, maintaining the ratio $\bar{\lambda}/\bar{\delta}$ constant.
- Probability of Detection, $POD(a)$: lognormal cumulative density function. We considered different values of POD mean, μ_{POD} , expressed as a percentage of the tube wall thickness. The standard deviation of the defect depth is always equal to 0.025 mm.
- Cylinder surface, S : constant equal to 0.4 m².
- Flaws density: constant, according to experimental data.
- The following material properties are assumed constant and an hypothesis on their values is formulated:
 - $\Delta K_{th,LC}$, SIF threshold for long cracks;
 - $\sqrt{A_0}$, El-Haddad parameter.
- Applied internal pressure, ΔP : constant in the range from 20 to 35 MPa. We have considered the ratio $R = P_{min}/P_{max}$ equal to 0.1.

Given the internal pressure, the applied stress can be calculated according to Eq. (11), valid for small wall thickness tubes

($\frac{wt}{ID} \leq 0.1$):

$$\Delta\sigma_{applied} = \frac{\Delta P \cdot ID}{2 \cdot wt} \quad (11)$$



Since $\Delta\sigma_{\text{applied}}$ is a combination of normal variables, it is also normally distributed. Mean and standard deviation of the applied stress can be calculated using the formula according to random variables theory [4].

RESULTS

We have calculated the probability of failure using the Monte Carlo method with 10^6 simulations. In Tab. 2 we show the input values for the different cases as well as the probability of failure normalized in respect to the reference case ($P_f / P_{f,\text{reference case}}$). The Monte Carlo routine randomly extracts values of inner diameter, wall thickness and surface flaw depth. The maximum error obtained with 10^6 simulations is less than 10% [4].

In Fig. 6 we plot the values of the normalized probability of failure ($P_f / P_{f,\text{reference case}}$) versus the applied pressure, ΔP , the normalized position parameter of the defects LEVD distribution, $\lambda / \bar{\lambda}$, and the mean of the lognormal probability of detection, μ_{POD} . The plots show that the value of the applied pressure has a strong influence on the system reliability, as well as a change in the probability density function of the defects. On the other side, an increase of the mean threshold of nondestructive tests does not increase significantly the probability of failure of the system, while its reduction clearly improves the system performance.

	P [MPa]	λ_{defects}	μ_{POD} [%wt]	$P_f/P_{f,\text{reference case}}$
reference case	30	$\bar{\lambda}$	5%	1
case 1	20	$\bar{\lambda}$	5%	0
case 2	25	$\bar{\lambda}$	5%	0
case 3	32	$\bar{\lambda}$	5%	1.96
case 4	35	$\bar{\lambda}$	5%	3.63
case 5	30	$0.5 \bar{\lambda}$	5%	0.02
case 6	30	$2 \bar{\lambda}$	5%	4.20
case 7	30	$0.1 \bar{\lambda}$	5%	0
case 8	30	$3 \bar{\lambda}$	5%	5.40
case 9	30	$5 \bar{\lambda}$	5%	5.82
case 10	30	$\bar{\lambda}$	3%	0
case 11	30	$\bar{\lambda}$	3.5%	0.06
case 12	30	$\bar{\lambda}$	4%	0.35
case 13	30	$\bar{\lambda}$	4.5%	0.74
case 14	30	$\bar{\lambda}$	6%	1.24

Table 2: Results.

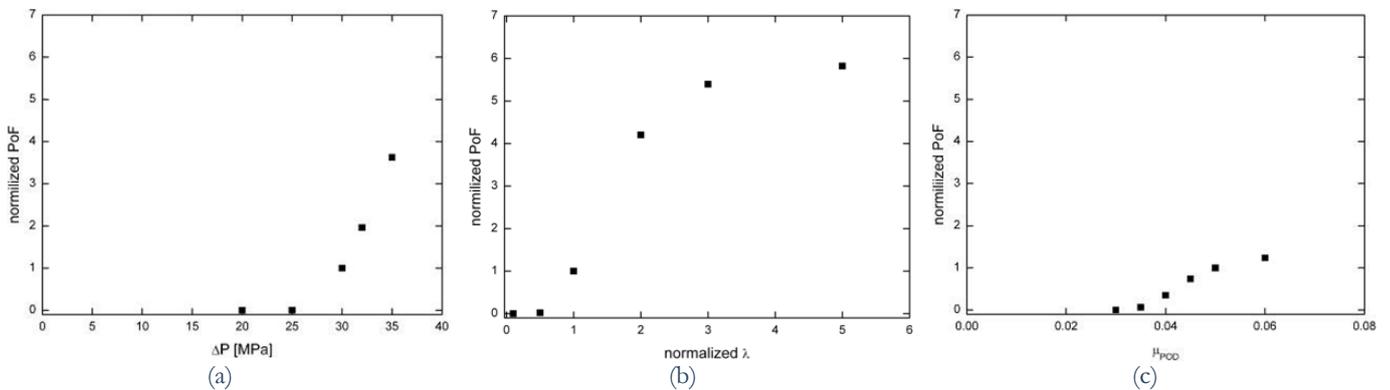


Figure 6: Normalized probability of failure as a function of a) the applied pressure ΔP , b) the normalized position parameter of the defects LEVD distribution $\lambda / \bar{\lambda}$, c) the mean of the lognormal probability of detection, μ_{POD} .



CONCLUSIONS

In this paper a simple probabilistic model for the fatigue strength of hydraulic cylinder housings has been shown. The inputs of the model are: i) an experimental distribution of surface discontinuities; ii) a threshold model obtained by experiments on micronotched specimens; iii) geometric tolerances of the tubes; iv) POD curve of the NDT control. The model has allowed to investigate the effect of different variables upon the failure probability of hydraulic cylinder housings.

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