# EXPERIMENTAL DETERMINATION OF *T*-STRESS IN MODE II SPECIMENS

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**SOMMARIO**

Dalla definizione classica della deformazione di cricche in piano, il termine *T* costante nello sviluppo in serie del fattore di intensificazione degli sforzi esiste solo in presenza del modo I di sollecitazione. Tuttavia, recenti studi mostrano che il *T*-stress può esistere anche in situazione di sollecitazione secondo il modo II, e modificare significativamente il campo di sforzi elastici presenti nell'intorno dell'apice della cricca. Questi effetti possono essere visualizzati e testati sperimentalmente col metodo della fotoelasticità. Basandosi sugli studi analitici, la presenza del *T*-stress in cricche sollecitate con modo II modifica la forma delle frange isocromatiche da anelli chiusi e simmetrici in una forma asimmetrica e discontinua.
In questo lavoro è proposto uno studio sull'influenza del *T*-stress in cricche sollecitate secondo il modo II e i suoi effetti sul campo di frange visibili sperimentalmente. I provini utilizzati sono dischi, chiamati *Brazilian disks*, al cui interno sono contenute le cricche centrate rispetto al cilindro da analizzare. Una volta generate le cricche, gli sforzi residui presenti nei provini sono rimossi attraverso un ciclo termico. Successivamente è applicato il carico, secondo angoli prefissati, in modo da ottenere deformazioni in puro modo II negli apici della cricca. I risultati sperimentali indicano che questi tipi di provini contengono valori negativi di *T*-stress in condizione di puro modo II. Le frange isocromatiche osservate sperimentalmente mostrano un andamento in buon accordo con quanto predetto dalla formulazione analitica.

**ABSTRACT**

According to the classical definition for in-plane modes of crack deformation, the constant stress term *T* exists only in the presence of mode I. However, recent studies show that this term can exist in mode II conditions as well; and significantly affect the elastic stress field around the crack tip. These effects can be visualized using the experimental method of photoelasticity. Based on the analytical studies, presence of the *T*-stress in mode II cracks transforms the isochromatic fringe patterns from symmetric closed loops to asymmetric and discontinuous shapes. In this paper, presence of the *T*-stress in mode II cracks and its effects on the fringe patterns is investigated experimentally. The test specimens are Brazilian disks containing very sharp central cracks. After crack generation, all residual stresses are removed by performing a thermal process on the specimens. Then, compressive load is applied in specific angles to induce mode II deformation in the crack tips. Experimental results indicate that this specimen contains negative values of *T*-stress in pure mode II condition. The observed isochromatic fringes show very good agreement with theoretical predictions.

## 1. INTRODUCTION

Many structural materials are subected to crack fo rming and propagation during their service life. These cracks influence the stress distribution in the component and can result in significant decrease of its strength. Because of the importance of safety and reliability, the crack problem has been of interest to a large number of researchers.
Elastic stress field around a crack tip is usually written as a set of infinite series expansions as [1]:

$$\sigma_{xx} = \frac{K_I}{\sqrt{2\pi r}}\cos\left(\frac{\theta}{2}\right)\left[1-\sin\left(\frac{\theta}{2}\right)\sin\left(\frac{3\theta}{2}\right)\right] + \frac{K_{II}}{\sqrt{2\pi r}}\sin\left(\frac{-\theta}{2}\right)\left[2+\cos\left(\frac{\theta}{2}\right)\cos\left(\frac{3\theta}{2}\right)\right] + T + O\left(r^{1/2}\right)$$

$$\sigma_{yy} = \frac{K_I}{\sqrt{2\pi r}}\cos\left(\frac{\theta}{2}\right)\left[1+\sin\left(\frac{\theta}{2}\right)\sin\left(\frac{3\theta}{2}\right)\right] + \frac{K_{II}}{\sqrt{2\pi r}}\sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\theta}{2}\right)\cos\left(\frac{3\theta}{2}\right) + O\left(r^{1/2}\right) \quad (1)$$

$$\sigma_{xy} = \frac{K_I}{\sqrt{2\pi r}}\cos\left(\frac{\theta}{2}\right)\sin\left(\frac{\theta}{2}\right)\sin\left(\frac{3\theta}{2}\right) + \frac{K_{II}}{\sqrt{2\pi r}}\cos\left(\frac{\theta}{2}\right)\left[1-\sin\left(\frac{\theta}{2}\right)\sin\left(\frac{3\theta}{2}\right)\right] + O\left(r^{1/2}\right)$$

where r and $\theta$ are the polar coordinates centered at the crack tip (Fig. 1). The two first terms in each expansion are the singular stresses, depending on the mode I and mode II stress intensity factors $K_I$ and $K_{II}$ . The second term, often called the *T*-stress, is constant and independent of the distance *r* from the crack tip. The next terms of this expansion, represented by $O(r^{1/2})$, are higher order terms which are usually neglected in the singularity dominated zone.

Based on the classical definition of crack deformation modes [1], *T*-stress exists only in mode I or combinations of mode I and II, and it vanishes in pure mode II condition. However, some published results of several analytical and numerical researches indicate that this term can also exist in mode II problems [2-4], and ignoring its effect can introduce significant inaccuracies in predicting mode II brittle fracture.
The constant stress term *T* acts over a large distance from the crack tip. The amounts of this stress and its sign have an important effect on the brittle fracture of engineering materials, whether in predominantly linear elastic materials or elastic-plastic cases. It has been shown that the sign of *T*-stress influences the stability and direction of fracture path. Presence of the negative *T*-stress in mode I leads the crack to grow along its plane, while when the *T* is positive, the crack deviates from its initial plane [5]. This effect is not restricted to mode I conditions. Ayatollahi and Abbasi [6] have shown that the *T*-stress can affect considerably the angle between the crack line and fracture path in mode II as well. Presence of *T*-stress also affects the mode II fracture toughness. It has been shown that *T* is the most important parameter for describing the crack tip constraint in constrained yielding [7]. For mode II specimens exhibiting small to moderate scale yielding around the crack tip, Ayatollahi et al. [8] have shown that *T* affects the size and shape of the plastic zone. The stresses inside the plastic zone are also influenced significantly by a remote *T*-stress. Thus, ignoring the *T*-term in mode II can introduce considerable inaccuracies in studying of mode II brittle fracture.
Considering this point, the elastic stress components near the crack tip in mode II can be expressed in Cartesian coordinate system as:

$$\sigma_{xx} = \frac{K_{II}}{\sqrt{2\pi r}}\sin\left(\frac{-\theta}{2}\right)\left[2+\cos\left(\frac{\theta}{2}\right)\cos\left(\frac{3\theta}{2}\right)\right] + T + O\left(r^{1/2}\right)$$

$$\sigma_{yy} = \frac{K_{II}}{\sqrt{2\pi r}}\sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\theta}{2}\right)\cos\left(\frac{3\theta}{2}\right) + O\left(r^{1/2}\right) \quad (2)$$

$$\sigma_{xy} = \frac{K_{II}}{\sqrt{2\pi r}}\cos\left(\frac{\theta}{2}\right)\left[1-\sin\left(\frac{\theta}{2}\right)\sin\left(\frac{3\theta}{2}\right)\right] + O\left(r^{1/2}\right)$$

The crack parameters $K_{II}$ and *T* in these equations can be determined using different analytical, numerical, and experimental methods. Among the experimental techniques, method of photoelasticity has been frequently used for calculating the crack parameters in various cracked specimens [e.g. 9,10]. Also, several procedures have been suggested and utilized to determine $K_I$, $K_{II}$ and *T* from photoelastic fringe patterns.

Using the stress series expansion (Eq. 1) and the fundamental optic equations for an isochromatic fringe [11], in general, a non-linear equation is obtained in terms of three unknown parameters $K_I$ , $K_{II}$ , and $T$. Different methods have been suggested to solve this equation among which, the over-deterministic technique is able to provide a more accurate analysis [11]. Because it is a full-field method that can use the coordinates $r$ and $\theta$ from four or more arbitrary points on given isochromatic fringes. The resultant non-linear equations are solved numerically, and the fitting process involves both the Newton-Raphson method and the method of least squares. Another full-field technique is proposed by Nurse and Patterson [12], based on complex Fourier analysis which is more complicated.

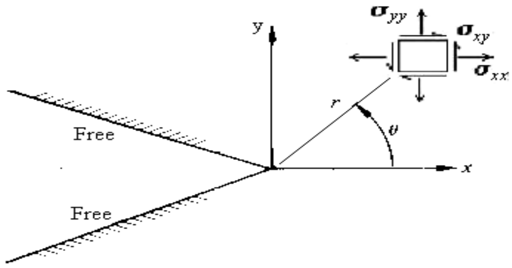


Fig. 1. Crack tip coordinates and stress components.

Although the full-field methods generate solutions with high degree of accuracy for mode I and mixed mode I/II problems, sometimes the results are not satisfactory for pure mode II. On the other hand, the theoretical results suggest that the fringe patterns are always symmetric in mode II conditions [11]. However, asymmetric fringes have been observed in some of the previous experiments [13]. As described previous researches [2,13] this inconsistency between the theory and experiments can be due to neglecting the effect of $T$-stress in some mode II specimens. The main obecti ve of this paper is to investigate the presence of $T$ in a mode II specimen, and its effects on the isochromatic fringe patterns around the crack tip by using the experimental method of photoelasticity. In the following, a brief review on the analytical relations [13] is presented. Then, different steps of the performed experimental program is described and the observed fringe patterns are compared with theoretical predictions. Also, calculated valued for crack parameters are validated by using the results from finite element analysis (FEM).

## 2. MATHEMATICAL RELATIONS OF ISOCHROMATIC FRINGES

Based on the classical concepts of photoelasticity, locus of an isochromatic fringe around the crack tip is expressed as [11]:

$$2\tau_m = \frac{Nf}{h} \tag{3}$$

where $\tau_m$ is maximum in-plane shear stress. $N$ and $f$ are the fringe order and material fringe value, respectively, and $h$ is the thickness of specimen. Also, the maximum shear stress $\tau_m$ is related to the Cartesian stress components with this equation [11]:

$$(2\tau_m)^2 = (\sigma_{xx} - \sigma_{yy})^2 + 4\sigma_{xy}^2 \tag{4}$$

Substituting stress terms from Eqs. (2,3) in Eq. (4), the mathematical equation for a fringe loop developing around a mode II crack tip is written in a simple form presented in ref. [13] and defining three dimensionless parameters:

$$S = \left(\frac{Nf}{hT}\right)^2 \quad , \quad B = \frac{T\sqrt{\pi a}}{K_{II}} \quad , \quad r' = r/2a \tag{5}$$

In which *a* is the crack length for edge cracks and semi-crack length for central cracks, a quadratic algebraic equation is obtained. Solving this equation, the locus of isochromatic fringes in presence of *T*-stress is determined as:

$$\sqrt{r'} = \frac{b \pm \sqrt{b^2 + (S-1)(4 - 3\sin^2\theta)}}{2B(1-S)}$$
$$\text{where} \quad b = \left( \sin\theta \cos\frac{3\theta}{2} + 2\sin\frac{\theta}{2} \right) \tag{6}$$

This equation predicts asymmetric fringes which are not continuous along the crack edges (see Fig. 2-a). Meanwhile, in the case of zero *T*-stress, the locus of isochromatic fringes is obtained as Eq.(7) that suggests a set of closed loops, symmetric about directions $\theta = 0°$ and $\theta = 90°$, similar to the earlier analytical results presented in [11,14]. A typical scheme of these loops is shown in Fig. 2-b.

$$r = \frac{1}{2\pi} \left[ \frac{hK_{II}}{Nf} \right]^2 \left( 4 - 3\sin^2\theta \right) \tag{7}$$

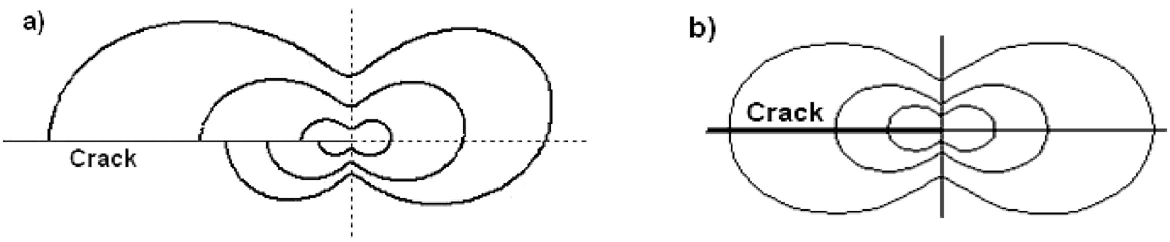


Fig. 2. Typical isochromatic fringes around a mode II crack tip: a) $T \neq 0$ , b) $T = 0$.

## 3. EXPERIMENTAL PROCEDURE

### 3.1. Specimen Preparation.

For experimental investigation of *T*-stress effects on the elastic stress field, two Brazilian disk specimens were utilized. The disks were made from a polycarbonate sheet of thickness *t*=10mm. For creation of the central cracks, first an initial small notch was made by using the water et technolog y. The notched sheet was put in liquid nitrogen of -196°C temperature for 15 minutes to become completely brittle, and then the crack was created by applying a mechanical shock on the notched zone. The crack obtained in this way is very close to a natural crack with sharp tips (Fig. 3-a). In our tests, the total crack lengths were 2*a*=58.8mm and 2*a*=60mm. Then the sheet was cut in the form of two disks of radius *R*=66.5mm (specimen N-1) and *R*=60mm (specimen N-2), respectively. It is notable that the crack tips generated in this way are not perfectly straight through the thickness, and there is a curvature which may affect the results specially in the case of thick sheets (Fig. 3-b).
Since the specimens should be stress-free before loading, all residual stresses induced during the cracking and cutting process were removed by using a thermal treatment according to Fig. 4. It is seen from the figure that the heating rate is decreased in temperatures higher than 145°C, which is the minimum glass transition temperature for polycarbonate [15]. The specimens were placed in the furnace for 59 hours and then, stresses were checked in the polariscope machine. It was observed that the disks are almost stress-free.

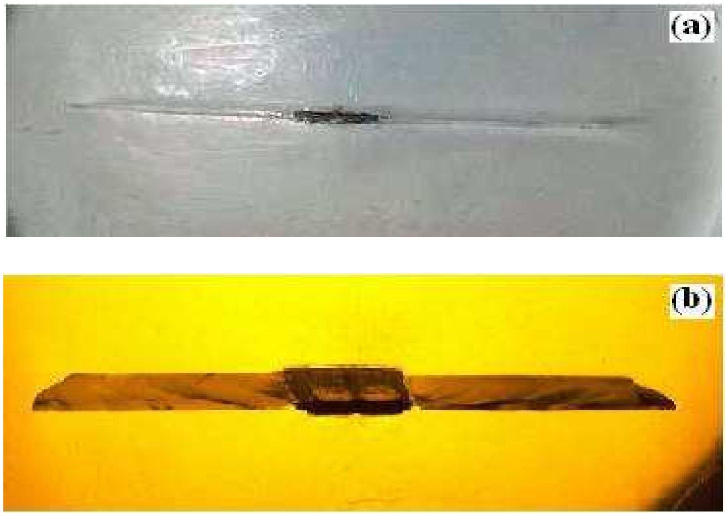


Fig. 3. Created semi-natural cracks: a) front view, b) through the sheet thickness.

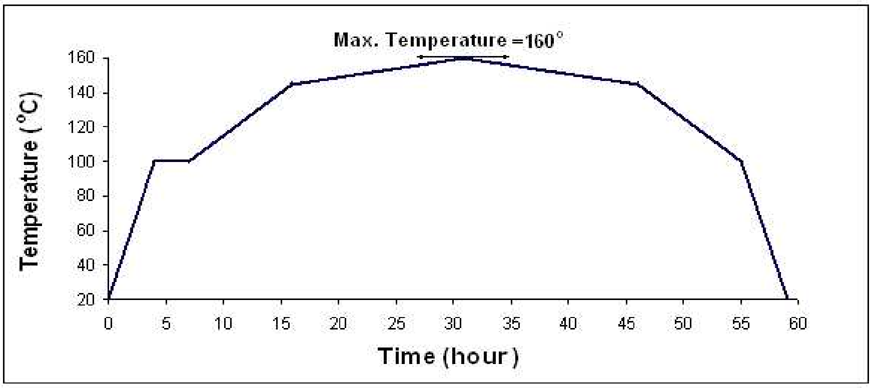


Fig. 4. Applied thermal cycle to remove residual stresses from test specimens.

### 3.2. Determination of Material Fringe Value.

As an optical property of photoelastic materials, the fringe value for an intact polycarbonate is about $f$ =7 ${}^{N/mm}\!/_{fringe}$ [11]. Since our specimens were exposed to heat treatment, a material calibration test was performed to determine the fringe value after thermal process. For this purpose, a disk of diameter 50mm from the same material was put in the furnace along with the main specimens. This disk was then employed for a calibration test under diagonal compressive load according to [16]. Test was conducted in two steps including loading and unloading, and considering both cases, the fringe value was obtained as $f$ =6.9 ${}^{N/mm}\!/_{fringe}$. Fig. 5 shows the isochromatic fringe patterns in the calibration disk when the two fringes of order $N$=8 are oining together in the centre of disk.

### 3.3. Photoelastic Tests.

The cracked Brazilian disks were sited in the loading frame as shown in Fig. 6. Compressive loads were applied by using the loading screw and the gage shows the force amount. The loads were selected as P=525N and P=367.5N for specimens N-1 and N-2, respectively. It should be mentioned that using the earlier FEM results [4], the angle $\alpha$ between the crack line and loading direction (see Fig. 6 ) was such selected that the crack was exposed to mode II condition. These angles were $\alpha = 24.5°$ for N-1 (a/R=0.44), and $\alpha = 23.15°$ for N-2 (a/R=0.5). Fig. 7 shows the resultant isochromatic fringes around the crack tips for the two disks.

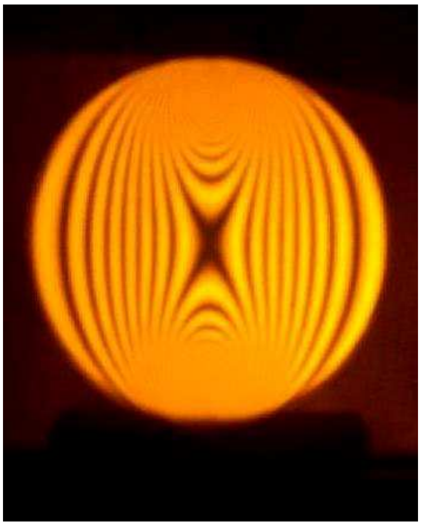

Fig. 5. Isochromatic fringes in calibration disk.

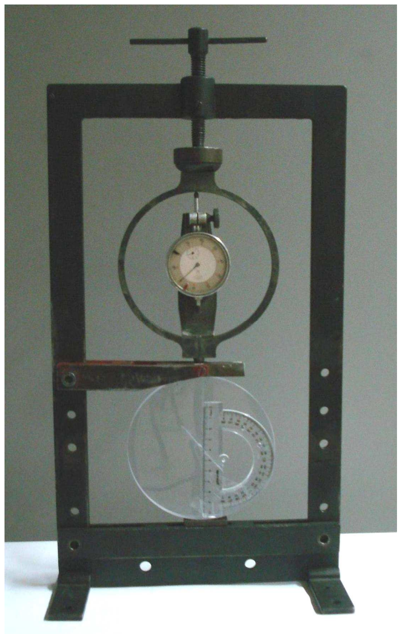

Fig. 6. Loading frame employed to apply compressive load on the disks.

Table 1- Experimental results obtained from the Brazilian disks, compared with FEM results [4].

| | $K_{II}$ [MPa $\sqrt{mm}$ ] | | | *T*-Stress [MPa] | | |
|---|---|---|---|---|---|---|
| Test Specimen | Experiment | FEM | Error | Experiment | FEM | Error |
| Specimen N-1 | 4.34 | 4.71 | 7.9% | -0.389 | -0.40 | 2.7% |
| Specimen N-2 | 4.82 | 5.13 | 6.0% | -0.262 | -0.27 | 2.9% |

## 4. EXPERIMENTAL RESULTS

As shown in Fig. 7, the obtained photoelastic fringes around the crack tips are asymmetric in both cases. The observed discontinuous loops are in good consistency with our theoretical predictions, and confirm the existence of the *T*-stress in mode II conditions. This stress term can be quantified by using the common methods for calculating the crack tip parameters, $K_I$, $K_{II}$, and *T*. For this aim, the obtained isochromatic fringes were analysed using a computer code prepared in MATLAB software; and the Image Processing Toolbox was employed to collect some data points from fringe loops of different orders. These data were utilized in another MATLAB program prepared for a full field analysis based on the over-deterministic technique [11]. In this technique, the resultant non-linear equations are solved numerically, and the fitting process involves both the Newton-Raphson method and the method of least squares. Finally, the unknown parameters $K_{II}$, and *T* were calculated as presented in Tab. 1.
It should be mentioned that $K_I$ was very small with respect to $K_{II}$ in both cases. Hence it could be assumed that the crack tips was subected to mode I I conditions.

## 5. DISCUSSION

Experimental findings presented in Tab. 1 indicate that the investigated Brazilian disks contain negative values of *T*-stress in pure mode II condition. In order to validate these results, they are compared with numerical results [4] obtained from FEM analysis (see Tab. 1). It is seen that there is a good agreement between the results from two methods, though there are some minor errors especially in the case of $K_{II}$. This problem may be due to the curved crack front through the specimen thickness which was not taken into consideration in the FEM models. Furthermore, since these cracks were very close to natural cracks, the crack faces are completely overlapped and there may be contact between them depending on the loading conditions. This contact can affect the results and increase the difference between experimental and numerical findings. The role of crack tip curvature and probable contact effects can be studied in our future research works. However, FEM results confirm the values calculated for *T*-stress in these specimens.

## 6. CONCLUSION

In this research, presence of the *T*–stress and its effects on the elastic stress field around a mode II crack tip were studied experimentally. Photoelastic experiments were conducted on the Brazilian disks, and the observed isochromatic fringe patterns revealed that the specimens had negative *T*–stresses in mode II condition. The experimental results were consistent very well with theoretical predictions in that the *T*-stress significantly affects the symmetric shape of the fringe loops, and causes the loops to become asymmetric and discontinuous along the crack edges. The results presented in this paper were validated by comparing with the previous numerical results from FEM analysis.

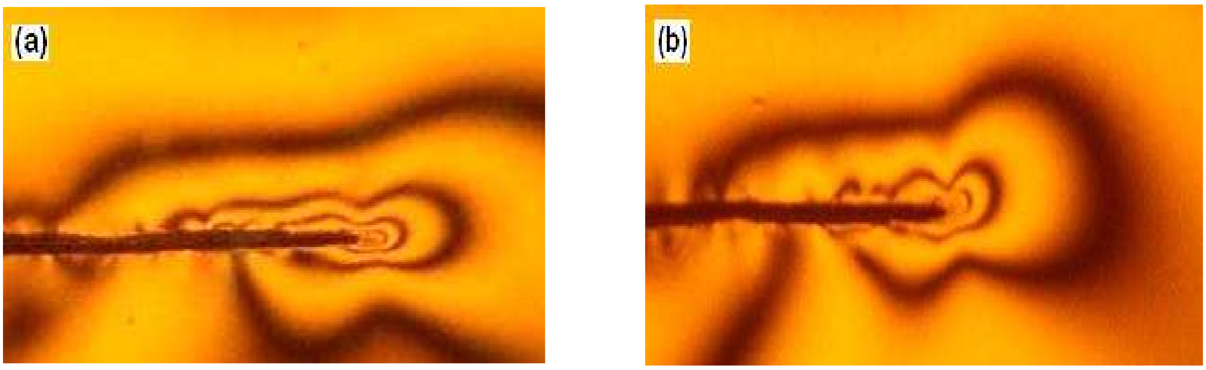


Fig. 7. Isochromatic fringes around the crack tips of the disks: (a) N-1, (b) N-2.

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