

STRESS INTENSITY FACTORS FOR A SUBSURFACE CRACK IN A TWO DIMENSIONAL HALF-SPACE

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ABSTRACT

Mode I and II Stress Intensity Factors (SIFs) under general loading conditions were determined for a crack parallel to the surface in a two-dimensional half space. To this purpose, a parametric Finite Element (FE) analysis of the subsurface crack was set up, and the analyses were conducted for independent loading conditions. Symmetric and anti-symmetric for both normal and tangential loads were considered and the corresponding SIFs were determined at the two crack tips. Several distances of the crack with respect to the surface were analysed and general functions representing the SIFs versus this parameter were obtained by interpolating the FE results. For both K_I and K_{II} , influence coefficients were defined reproducing the FE results with a satisfactory accuracy. When the crack approaches the surface the loss of symmetry was demonstrated to produce coupling effects between modes I and II of fracture. This effect is quantified and some consequences for a general loading condition discussed.

1 INTRODUCTION

Subsurface crack growth is responsible for several mechanisms of failure such as spalling in rolling contact fatigue [1-3] or fatigue failure from subsurface defects in giga-cycle fatigue [4,5].

These problems have been studied by several authors and different approaches have been proposed to predict crack nucleation and growth. As a first approximation, if the distance of the defect from the surface is sufficiently high as compared to its length, the Fracture Mechanics (FM) problem can be solved by assuming the Griffith crack model. However, this approximation turns out to be questionable for subsurface cracks lying immediately below the surface, as those defects observed in many contact fatigue problems. For this kind of phenomenon, some theoretical and FE analyses have been proposed to identify the parameters influencing the crack behaviour, to evaluate the SIF [6-8] and also to predict the crack propagation path [9,10]. In those analysis, many FM analyses have to be carried out for predicting crack growth and the FE analysis is usually a time-consuming technique, therefore, the development of more efficient approaches seems to be useful.

The present paper is aimed at carrying out a preliminary investigation, based on an elaboration of FE analyses, of the general properties of the FM solution for a subsurface crack parallel to the external surface in a two-dimensional half space under general loading conditions. To this purpose, very general loading conditions were considered in order to discuss any relevant aspects of this problem: effect of the free surface, mixed mode loading, normal and tangential loading on the cracks, presence of two tips. The present results can also be considered the basis for the development of a general Weight Function for this particular geometry which is not found in the literature. The correctness and the accuracy of the SIFs calculated by the FE analysis was estimated by comparison with the exact solution of the reference Griffith problem, that can be also considered the asymptotical solution for the analysed problem when the crack is far away from the surface.

The FE analyses were performed for different ratios between the crack distance from the surface and the crack length. This dimensionless quantity represents the only geometrical parameter governing the problem. The effects of symmetrical and anti-symmetrical loading on the behaviours at the two crack tips was studied. Analytical expressions of the SIFs were determined by interpolation of these results, thus defining a set of influence coefficients as functions of the ratio between crack length and distance from the surface. The loss of

symmetry of the problem when the crack is near the surface was discussed, by highlighting the dependence of the coupling effects between Mode I and II of fracture, on the crack position.

2 PROBLEM DEFINITION

An embedded crack in a semi-plane having length $2a$ and distance b from the semi-plane surface is represented in fig.1. In order to rigorously define the FM parameters (in particular regarding the sign of K_{II} of which a general definition is not usually adopted), a local Cartesian reference system is defined at each crack tip A and B. The two local reference systems shear the y -axis while they have opposite x -axes (x_A and x_B respect.) each of them pointing toward the direction of crack propagation at the two tips. It is worth noting that the y -axis is the only axis of symmetry for the problem, its versus is univocally defined as it is chosen toward the nearest free surface. This is not the only possible choice, indeed the two local reference frames could be obtained by a complete rotation of both the axes thus producing two couple of opposite axis (both x and y) for the two tips. However, the adopted systems is useful for representing the axial symmetry of the problem, in which the quantities referred to the y -axis are the same for the two opposite parts of the crack.

As discussed in [11], the relative displacements of the points C^+ (D^+) and C^- (D^-) located on the upper ($y+$) and lower ($y-$) crack edge respectively, define the sign of the SIFs when C (or D) approaches the tip A (or B). In particular: K_I for the tip A (B) is positive if the y component of the displacement of point C^+ (D^+) is higher than the correspondent component of point C^- (D^-). Similarly, K_{II} at the tip A (B) is positive if the x_A (x_B) displacement component of point C^+ (D^+) is higher than the corresponding component of the point C^- (D^-).

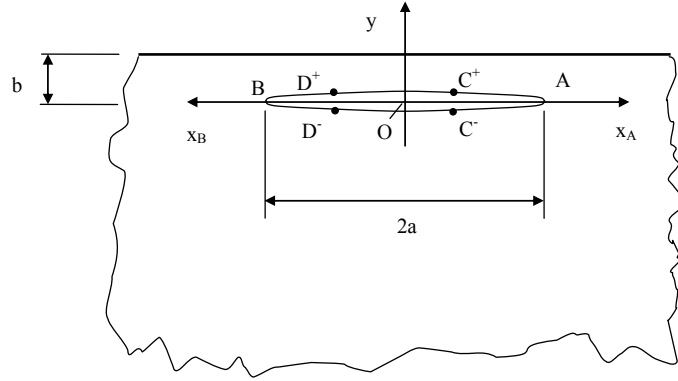


Fig.1 Subsurface crack parallel to the semiplane surface

The Griffith crack in an infinite body can be considered as an asymptotical condition of the subsurface crack when the ratio a/b tends to zero. Under symmetrical uniform ($\sigma=\sigma_0$ or $\tau=\tau_0$) or linearly variable ($\sigma=|x/a|\cdot\sigma_0$ or $\tau=|x/a|\cdot\tau_0$) nominal stress applied on the crack faces, the SIF of the Griffith crack can be expressed for both tips A and B as:

$$\begin{pmatrix} K_I \\ K_{II} \end{pmatrix}_A = \begin{pmatrix} \sigma_0 \\ \tau_0 \end{pmatrix} \sqrt{\pi a} \quad (\text{uniform stress}) \quad (1a)$$

$$\begin{pmatrix} K_I \\ K_{II} \end{pmatrix}_A = \frac{2}{\pi} \cdot \begin{pmatrix} \sigma_0 \\ \tau_0 \end{pmatrix} \cdot \sqrt{\pi a} \quad (\text{linearly variable stress}) \quad (1b)$$

As a consequence of the definition of the local reference frames, the SIFS for symmetrical loadings are the same for the two tips.

Under anti-symmetrical nominal stress applied on the crack faces, either uniform: $\sigma=\sigma_0$ (resp. $\tau=\tau_0$) on segment OA and $\sigma=-\sigma_0$ ($\tau=-\tau_0$) on segment BO, or linearly variable: $\sigma=x/a\sigma_0$ (resp. $\tau=x/a\tau_0$) on segment OA $\sigma=-x/a\sigma_0$ (resp. $\tau=-x/a\tau_0$) on segment BO, the SIF of the Griffith Crack can be expressed as follows:

$$\begin{pmatrix} K_I \\ K_{II} \end{pmatrix}_A = \frac{2}{\pi} \cdot \begin{pmatrix} \sigma_0 \\ \tau_0 \end{pmatrix} \cdot \sqrt{\pi a} \quad \begin{pmatrix} K_I \\ K_{II} \end{pmatrix}_B = - \begin{pmatrix} K_I \\ K_{II} \end{pmatrix}_A \quad (\text{uniform stress}) \quad (2a)$$

$$\begin{pmatrix} K_I \\ K_{II} \end{pmatrix}_A = \frac{1}{2} \begin{pmatrix} \sigma_0 \\ \tau_0 \end{pmatrix} \sqrt{\pi a} \quad \begin{pmatrix} K_I \\ K_{II} \end{pmatrix}_B = - \begin{pmatrix} K_I \\ K_{II} \end{pmatrix}_A \quad (\text{linearly variable stress}) \quad (2b)$$

Under anti-symmetrical loadings, both SIFs at the two crack tips have opposite sign.

For a crack located relatively near to the surface ($a/b \neq 0$), the geometrical parameter controlling the problem is the ratio a/b , so that influence coefficients depending on a/b have to be introduced in the SIF expression. Moreover, the coupling effect between Modes I and II due to the loss of symmetry of the problem when the crack approaches the surface has to be accounted for. Therefore, by combining the effects of symmetrical and anti-symmetrical loads, the following expression can be introduced, respectively for the tips A and B of the crack:

$$\begin{pmatrix} K_I(a) \\ K_{II}(a) \end{pmatrix}_A = \sqrt{\pi a} \cdot \left[\begin{pmatrix} F_{I,\sigma} & F_{I,\tau} \\ F_{II,\sigma} & F_{II,\tau} \end{pmatrix}^S \cdot \begin{pmatrix} \sigma \\ \tau \end{pmatrix}^S + \begin{pmatrix} F_{I,\sigma} & F_{I,\tau} \\ F_{II,\sigma} & F_{II,\tau} \end{pmatrix}^E \cdot \begin{pmatrix} \sigma \\ \tau \end{pmatrix}^E \right] \quad (3a)$$

$$\begin{pmatrix} K_I(a) \\ K_{II}(a) \end{pmatrix}_B = \sqrt{\pi a} \cdot \left[\begin{pmatrix} F_{I,\sigma} & F_{I,\tau} \\ F_{II,\sigma} & F_{II,\tau} \end{pmatrix}^S \cdot \begin{pmatrix} \sigma \\ \tau \end{pmatrix}^S - \begin{pmatrix} F_{I,\sigma} & F_{I,\tau} \\ F_{II,\sigma} & F_{II,\tau} \end{pmatrix}^E \cdot \begin{pmatrix} \sigma \\ \tau \end{pmatrix}^E \right] \quad (3b)$$

The influence coefficients have to be evaluated using four independent loading conditions for each of the considered loading case (i.e. uniform stress, linearly variable stress, etc.).

In order to determine the influence coefficients, a FE model was developed by the Ansys 5.7 computer program using linear elastic 8-nodes iso-parametric plane strain elements. The crack tip region was modelled by 36 equal quarter points elements surrounding the crack tip, each having a radial dimension of $0.02a$. The SIFs were obtained by the quarter node displacements. The FE model with a detail of the crack tip region is shown in Fig. 2. The model has been applied to study the Griffith crack. For every loading condition considered in the analysis, an accuracy better than 0.1 % in the evaluation of either K_I or K_{II} was found.

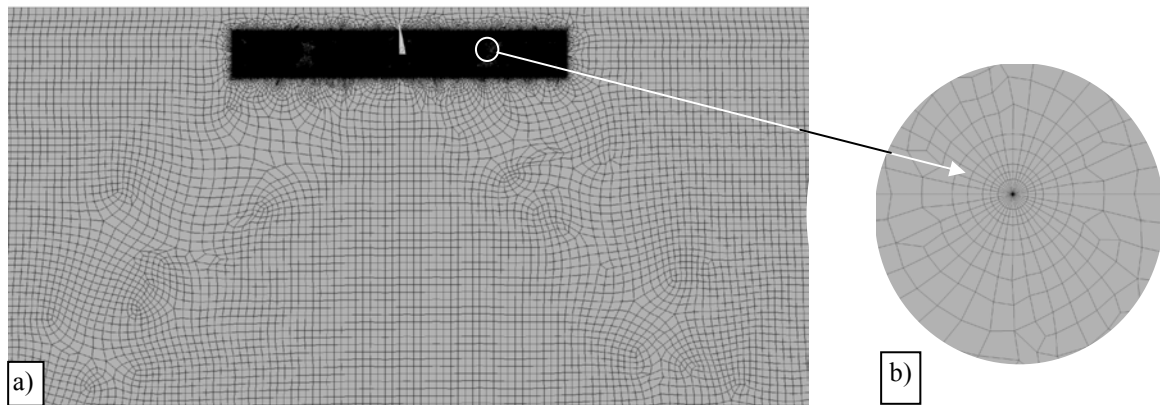
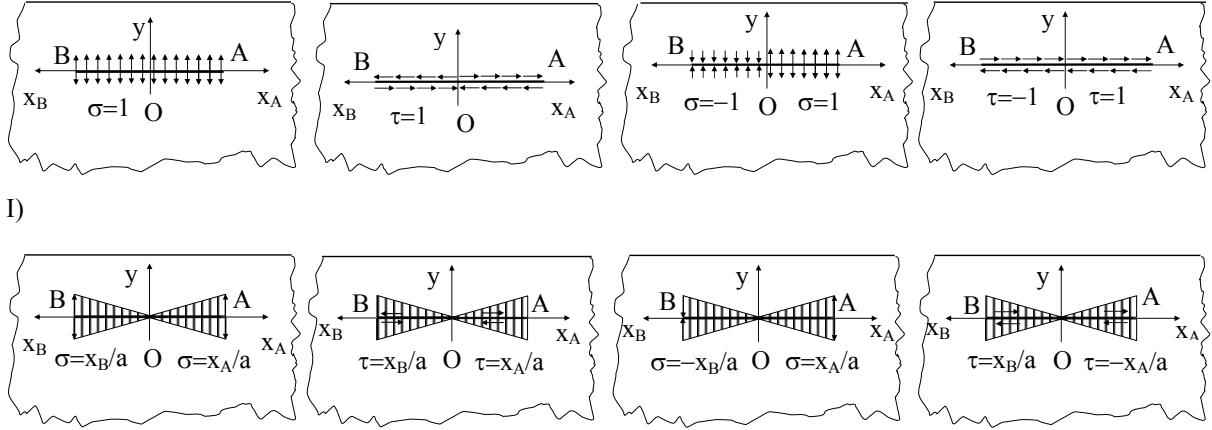


Fig. 2: a) finite element model used for the analysis; b) particular of the crack tip mesh

The solutions for uniform and the linearly variable stress applied to the crack faces are presented in the following. In order to evaluate the influence coefficients of Eqns. 3a and 3b, for uniform stress the following conditions were considered: symmetrical normal ($\sigma=1$) or tangential ($\tau=1$) stress on the segment AB and anti-symmetrical normal $\sigma=1$ (resp. tangential $\tau=1$) stress on segment OA and $\sigma=-1$ (resp. $\tau=-1$) on segment BO as represented in Fig. 3 I. The linearly variable loading condition was represented by: $\sigma=x_A/a$, $\sigma=x_B/a$ (or $\tau=x_A/a$, $\tau=x_B/a$) for symmetrical loading and $\sigma=x_A/a$, $\sigma=-x_B/a$ (or $\tau=x_A/a$, $\tau=-x_B/a$) for anti-symmetrical loading (Fig 3 II)

The SIF values were normalized by the corresponding SIFs of the Griffith crack under the same loading condition (as expressed by Eqn. 1-2). The dependence of the influence coefficients on the geometrical parameter a/b was represented by a truncated power series as follows:

$$\frac{F_{M\mu}^k(a/b)}{F_G} = \sum_{j=0}^m \lambda_{M\mu}^k \cdot \left(\frac{a}{b}\right)^j \quad (m=3) \quad (4)$$



I)

II)

Fig. 3: loading conditions used in the analysis. I) uniform loading, II) linearly variable conditions

Where M indicates the Mode (I or II), μ the type of loading (σ or τ) and k the symmetric or emi-symmetric condition (S or E). The scaling factor F_G is deduced by the SIFs of the Griffith crack as it follows: $F_G=1$ for symmetrical uniform stress (eqn. 1a), $F_G=2/\pi$ for linearly variable symmetrical stress (eqn. 1b) and anti-symmetrical uniform stress (eqn. 2a), $F_G=1/2$ for linearly variable symmetrical stress (eqn. 2b). In this way the asymptotical properties of the solutions can be better appreciated, indeed $\lambda_{M\mu 0}^k=1$ for the diagonal terms (i.e. $M\mu=I\sigma$ or $II\tau$), whereas it is $\lambda_{M\mu 0}^k=0$ for the off-diagonal terms (i.e. $M\mu=II\sigma$ or $I\tau$)

3 RESULTS

The values of the $\lambda_{M\mu j}^k$ coefficients are reported in Table I for the uniform loading and in Table II. for the linearly variable loading

j	Symmetrical terms				Anti-symmetrical terms			
	$M\mu=I\sigma$	$II\sigma$	$I\tau$	$II\tau$	$I\sigma$	$II\sigma$	$I\tau$	$II\tau$
0	1	0	0	1	1	0	0	1
1	0.2198	0.0276	-0.0505	0.1063	-0.0882	-0.0087	-0.0367	-0.0173
2	0.3194	-0.2393	-0.1004	-0.0479	0.2852	-0.0391	-0.0563	0.0944
3	-0.0423	0.0256	0.0228	0.0077	-0.0354	0.0891	0.0124	-0.0172

Tab. I: values of the interpolating coefficients for the uniform loading condition

j	Symmetrical terms				Anti-symmetrical terms			
	$M\mu=I\sigma$	$II\sigma$	$I\tau$	$II\tau$	$I\sigma$	$II\sigma$	$I\tau$	$II\tau$
0	1	0	0	1	1	0	0	1
1	0.1457	0.0090	-0.0384	0.0606	-0.0631	0.0619	-0.0338	-0.0122
2	0.2014	-0.1368	-0.0488	-0.0212	0.2051	-0.0236	-0.0375	0.0679
3	-0.0271	0.0145	0.0120	0.0040	-0.0256	-0.0065	0.0080	-0.0120

Tab. II: values of the interpolating coefficients for the linearly variable loading condition

The comparison between the SIFs obtained by FE analysis and those calculated using the influence coefficients defined by eqn. 3-4 are reported versus a/b in figs. 4 and 5. It can be verified that, as a consequence of the local reference frames defined at each crack tip, the SIFs at tip A coincides with those at crack tip B under symmetrical loading, whereas they have opposite sign under anti-symmetrical loading. In Fig. 4 and 5, the values for tip A are shown, using the corresponding values of SIF for the Griffith as normalising factor.

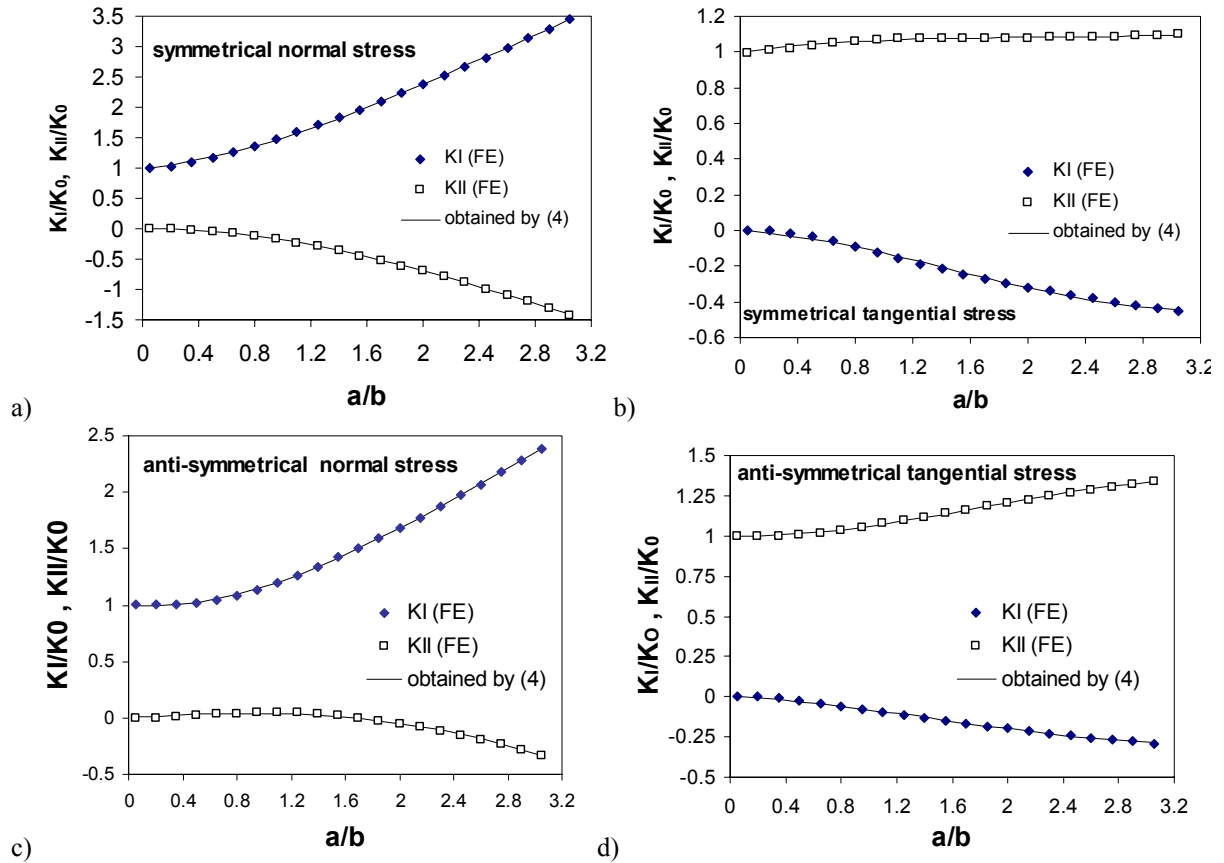


Fig.4: K_I and K_{II} at the crack tips A and B vs. a/b for different uniform loading conditions: symmetrical normal (a), tangential (b) stress, anti-symmetrical normal (c), tangential (d) stress

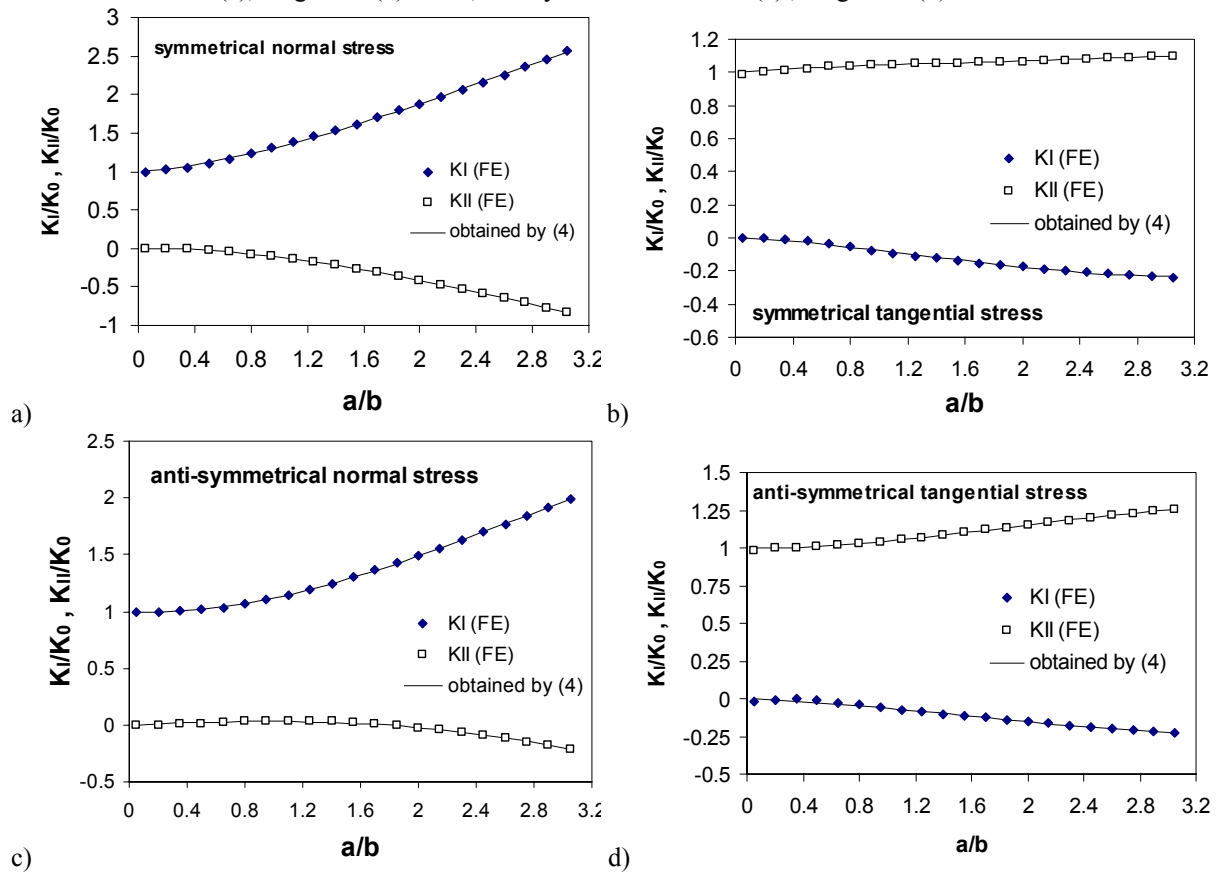


Fig.5: K_I and K_{II} at the crack tips A and B vs. a/b for linearly variable loading conditions: symmetrical normal (a), tangential (b) stress, anti-symmetrical normal (c), tangential (d) stress

In the analysis, values of $K_I < 0$ were obtained for some loading conditions. These values have only a mathematical meaning and are not acceptable from a physical point of view. Indeed, the effects of the contact forces arising between crack edges in these cases, were not accounted for in this preliminary analysis.

A good agreement between the FE solutions and those obtained using Eqns.3-4 can be verified: the relative differences are within a few percents over the entire a/b range for any studied loading condition, thus confirming the adequacy of the adopted interpolating expression.

When a/b approaches zero, the subsurface crack can be regarded as a Griffith crack. The errors introduced by assuming a Griffith crack can be considered acceptable up to $a/b=0.3$ as the relative differences being lower than (2-4)% depending on the loading conditions. At higher values of a/b the loss of symmetry of the problem leads to the onset of the coupling effects between Mode I and II. This effect is originated by the different compliance experienced by the upper and the lower crack edges when loaded.

3 CONCLUSIONS

The properties of a subsurface crack parallel to the external surface of a two-dimensional half space were studied. A FE model was set up to study the Mode I and II SIFs under two independent loading conditions: uniform and linearly variable stress applied on the crack faces. The analysis was performed for different distances of the crack with respect to the surface for symmetrical and anti-symmetrical loading.

Analytical expressions of the SIFs as a function of the ratio between crack length and crack distance from the surface were determined by interpolating the FE results. It was verified that the effect of the free surface on the FM analysis cannot be neglected when the length of the semi crack is greater than three times the distance from the surface. For cracks located near the surface, the coupling effects between Mode I and II of fracture, due to the lack of symmetry, were discussed and included in the analytical expressions for SIFs. A complete set of influence coefficients was defined in the crack reference frame for the two considered loading conditions.

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