FRACTURE RESPONSE OF CRACKED ORTHOTROPIC PLATES

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ABSTRACT

The solution of the elastostatic problem of an orthotropic body having a central inclined crack and subjected at infinity to a uniform biaxial load has been employed. It is assumed that the crack line does not coincide with an axis of elastic symmetry of the body. The stress filed is reported.

The topic of the present paper is the extension of the maximum Circumferential Tensile Stress Criterion to orthotropic materials, in order to obtain the crack initiation angle, pointing out the effects of orthotropy and load biaxiality. The influence of the non singular terms on the crack initiation angle is also investigated.

1 INTRODUCTION

The elastostatic problem of an orthotropic body having a central inclined crack and subjected at infinity to a uniform biaxial load has been studied in [1]. It is assumed that the crack line does not coincide with an axis of elastic symmetry of the body. The problem must be considered as one of general orthotropy, due in particular to the fact that the elastic coefficients of the material change with rotation of the reference system. The original problem has been transformed with respect to the rotated system of coordinates which is proper of the crack.

One of the main purposes of this paper is to predict the initial crack growth. The Maximum Circumferential Tensile Stress Criterion [2] has been used for the isotropic solids and applied by Buczek and Herakovich [3], Saouma et al. [4], Beuth and Herakovich [5] Ayari and Zhiming [6] and Carloni et al. [7] to orthotropic plates with a crack aligned with one of the axes of elastic symmetry. In this work, the Maximum Circumferential Tensile Stress Criterion is extended to the general case with the crack not aligned to one of the direction of elastic symmetry. It is assumed that the critical stress intensity factor for Mode-I has a polar variation [3-7]. The crack initiation is determined via minimization of the ratio of the circumferential stress over the material critical stress intensity factor for Mode-I, pointing out the effects of orthotropy and load biaxiality on the near tip elastic fields and on the angle of incipient crack propagation.

2 THE INCLINED CRACK PROBLEM

Consider an homogeneous, orthotropic and infinite plate, having a central crack, of length 2ℓ, inclined of an angle φ with respect to the x_1-axis of the Cartesian co-ordinates system O (x_1,x_2) (Fig. 1). Suppose that the crack is not aligned with one of the orthogonal axes of elastic symmetry of the body, coincident with the co-ordinates system O (x_1,x_2). Moreover, admit that the orthotropic body is subjected at infinity to a uniform biaxial load, applied along x_1 and x_2-directions.

The uniform load at infinity $\sigma_{11}^{*(\infty)} \equiv T_1$, $\sigma_{22}^{*(\infty)} \equiv T_2$, $\sigma_{12}^{*(\infty)} \equiv T_3$ referred to x_1^*, x_2^*, is a function of the crack inclination angle:

$$(\sigma_{11}^{*})^{(\infty)} \equiv T_1 = \frac{T}{2}\left[(1+k)-(1-k)\cos 2\varphi\right] \qquad (1)$$

$$(\sigma_{22}^{*})^{(\infty)} \equiv \mathrm{T}_2 = \frac{\mathrm{T}}{2}\left[(1+\mathrm{k})+(1-\mathrm{k})\cos 2\varphi\right] \quad (2)$$

$$(\sigma_{12}^{*})^{(\infty)} \equiv \mathrm{T}_3 = \frac{\mathrm{T}}{2}(1-\mathrm{k})\sin 2\varphi \quad (3)$$

k is the biaxial load parameter.

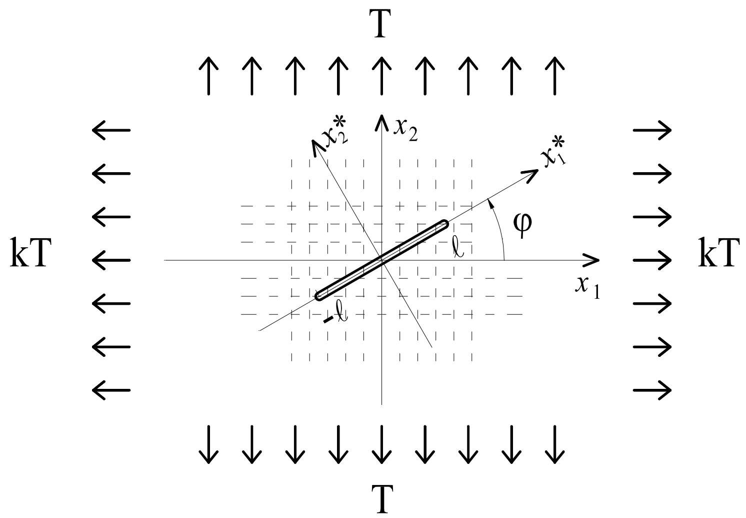

Fig. 1: The inclined crack geometry

The fracture response of an orthotropic plate having a crack not aligned with one of the axes of elastic symmetry has been carried out by Tsukrov and Kachanov [8], Prabhu and Lambros [9] and Nobile et al. [1], among others. In terms of polar coordinates (r, ϑ) centred at the crack tip, the stress field near the crack tip is expressed by [10,11]:

$$\begin{aligned}\sigma_{11}^{*} = {} & \frac{\mathrm{K_I}}{\sqrt{\pi\ell}}\left[\lambda\tilde{\sigma}_{11}^{0} + h_{11}^{1}\right] + \frac{\mathrm{K_{II}}}{\sqrt{\pi\ell}} h_{11}^{2} + \\ & + \frac{\mathrm{K_I}}{\sqrt{2\pi r}}\left[\frac{1}{\sqrt{g_1(\vartheta)}}\left(\bar{a}_1\cos\frac{\vartheta_1}{2} - \bar{a}_2\sin\frac{\vartheta_1}{2}\right) - \frac{1}{\sqrt{g_2(\vartheta)}}\left(\bar{a}_3\cos\frac{\vartheta_2}{2} - \bar{a}_4\sin\frac{\vartheta_2}{2}\right)\right] + \\ & + \frac{\mathrm{K_{II}}}{\sqrt{2\pi r}}\left[\frac{1}{\sqrt{g_1(\vartheta)}}\left(\bar{\bar{a}}_1\cos\frac{\vartheta_1}{2} - \bar{\bar{a}}_2\sin\frac{\vartheta_1}{2}\right) - \frac{1}{\sqrt{g_2(\vartheta)}}\left(\bar{\bar{a}}_3\cos\frac{\vartheta_2}{2} - \bar{\bar{a}}_4\sin\frac{\vartheta_2}{2}\right)\right]\end{aligned} \quad (4)$$

$$\begin{aligned}\sigma_{22}^{*} = {} & \frac{\mathrm{K_I}}{\sqrt{\pi\ell}}\left[\lambda\tilde{\sigma}_{22}^{0} + h_{22}^{1}\right] + \frac{\mathrm{K_{II}}}{\sqrt{\pi\ell}} h_{22}^{2} + \\ & + \frac{\mathrm{K_I}}{\sqrt{2\pi r}}\left[\frac{1}{\sqrt{g_1(\vartheta)}}\left(\bar{b}_1\cos\frac{\vartheta_1}{2} - \bar{b}_2\sin\frac{\vartheta_1}{2}\right) - \frac{1}{\sqrt{g_2(\vartheta)}}\left(\bar{b}_3\cos\frac{\vartheta_2}{2} - \bar{b}_4\sin\frac{\vartheta_2}{2}\right)\right] + \\ & + \frac{\mathrm{K_{II}}}{\sqrt{2\pi r}}\left[\frac{1}{\sqrt{g_1(\vartheta)}}\left(\bar{\bar{b}}_1\cos\frac{\vartheta_1}{2} - \bar{\bar{b}}_2\sin\frac{\vartheta_1}{2}\right) - \frac{1}{\sqrt{g_2(\vartheta)}}\left(\bar{\bar{b}}_3\cos\frac{\vartheta_2}{2} - \bar{\bar{b}}_4\sin\frac{\vartheta_2}{2}\right)\right]\end{aligned} \quad (5)$$

$$\sigma_{12}^{*}=\frac{\mathrm{K_I}}{\sqrt{\pi\ell}}\left[\lambda\tilde{\sigma}_{12}^{0}+h_{12}^{1}\right]+\frac{\mathrm{K_{II}}}{\sqrt{\pi\ell}}h_{12}^{2}+$$
$$+\frac{\mathrm{K_I}}{\sqrt{2\pi r}}\left[\frac{1}{\sqrt{g_1(\vartheta)}}\left(\bar{c}_1\cos\frac{\vartheta_1}{2}-\bar{c}_2\sin\frac{\vartheta_1}{2}\right)-\frac{1}{\sqrt{g_2(\vartheta)}}\left(\bar{c}_3\cos\frac{\vartheta_2}{2}-\bar{c}_4\sin\frac{\vartheta_2}{2}\right)\right]+ \quad (6)$$
$$+\frac{\mathrm{K_{II}}}{\sqrt{2\pi r}}\left[\frac{1}{\sqrt{g_1(\vartheta)}}\left(\bar{\bar{c}}_1\cos\frac{\vartheta_1}{2}-\bar{\bar{c}}_2\sin\frac{\vartheta_1}{2}\right)-\frac{1}{\sqrt{g_2(\vartheta)}}\left(\bar{\bar{c}}_3\cos\frac{\vartheta_2}{2}-\bar{\bar{c}}_4\sin\frac{\vartheta_2}{2}\right)\right]$$

where λ=T$_1$/T$_2$, and $\mathrm{K_I}=\mathrm{T_2}\sqrt{\pi\ell}$, $\mathrm{K_{II}}=\mathrm{T_2}\sqrt{\pi\ell}$ are the stress intensity factor for Mode-I and Mode-II respectively. The coefficients appearing in Eqs. (4)-(6) are reported in [11]. Note that the stress intensity factors correspond to the ones of the isotropic case. This means that the expressions of the stress intensity factors do not depend on the inclination of the crack respect to the directions of elastic symmetry of the material. The stress intensity factors depend only on the geometry.
Note that also the non-singular terms are included. They affect all the stress components σ_{ij}^{*}, despite of the isotropic case.

3 MAXIMUM CIRCUMFERENTIAL TENSILE STRESS CRITERION

For isotropic materials, the circumferential stress σ_{ϑ}^{*} defined as:

$$\sigma_{\vartheta}^{*}=\sigma_{11}^{*}\sin^2\vartheta+\sigma_{22}^{*}\cos^2\vartheta-\sigma_{12}^{*}\sin 2\vartheta \qquad (7)$$

can be studied, following the criterion proposed by Erdogan and Sih [12], for analysing the crack extension angle. The criterion states that the direction of crack initiation is the one defined by the maximum of the circumferential tensile stress. Referring to the above direction, the crack extension begins as soon as the circumferential stress reaches a critical value, related to the critical stress intensity factor for Mode-I. For isotropic materials the fracture toughness is a constant function of the polar angle ϑ.
The maximum circumferential stress criterion, used for isotropic material, is inadequate to predict the crack extension angle for orthotropic materials. In fact, in this case the critical stress intensity factor for Mode-I varies with polar angle ϑ. If $\mathrm{K_{IC}^{x_1}}$ and $\mathrm{K_{IC}^{x_2}}$ are the critical stress intensity factors for Mode–I along the axes of elastic symmetry x_1 and x_2, the critical stress intensity factor on the ϑ plane will be:

$$\mathrm{K_{IC}^{\vartheta}}=\mathrm{K_{IC}^{x_1}}\sin^2\left(\vartheta+\varphi\right)+\mathrm{K_{IC}^{x_2}}\cos^2\left(\vartheta+\varphi\right) \qquad (8)$$

The criterion, proposed by Saouma et al. [3] Buczek and Herakovich [4] and Carloni et al. [7] consists of finding the maximum of the following function:

$$R_{\vartheta}=\frac{\sigma_{\vartheta}\sqrt{2\pi r}}{\mathrm{K_{IC}^{\vartheta}}} \qquad (9)$$

The direction, corresponding to the value of ϑ for which R_{ϑ} becomes maximum, is the crack extension direction. The mentioned value of ϑ, defining the crack growth direction, is indicated with ϑ_0. In order to find the value of ϑ_0, it is convenient to analyse the maximum of the following normalised function:

$$\bar{R}_\vartheta = \frac{\bar{\sigma}_\vartheta}{\mathrm{K}_{\mathrm{IC}}^{\vartheta} / \mathrm{K}_{\mathrm{IC}}^{x_2}} = \frac{\frac{\sigma_\vartheta^*}{\mathrm{T}}\sqrt{\frac{2r}{\ell}}}{\mathrm{K}_{\mathrm{IC}}^{\vartheta} / \mathrm{K}_{\mathrm{IC}}^{x_2}} = \frac{\frac{\left(\sigma_{11}^*\sin^2\vartheta + \sigma_{22}^*\cos^2\vartheta - \sigma_{12}^*\sin 2\vartheta\right)}{\mathrm{T}}\sqrt{\frac{2r}{\ell}}}{\frac{\mathrm{K}_{\mathrm{IC}}^{x_1}}{\mathrm{K}_{\mathrm{IC}}^{x_2}}\sin^2\left(\vartheta+\varphi\right)+\cos^2\left(\vartheta+\varphi\right)} \qquad (10)$$

considering that critical stress intensity factor for Mode-I is calculated referring to x_1, x_2.
For studying the maximum problem it necessary to fix the value of r/ℓ. In the present study r/ℓ was taken as 0.01. It was also assumed that the critical stress intensity factors ratio $\mathrm{K}_{\mathrm{IC}}^{x_1} / \mathrm{K}_{\mathrm{IC}}^{x_2}$ can be evaluated by the elastic moduli ratio E_1 / E_2.
Referring to three different orthotropic materials (Arcisz and Sih [12]), Steel-Aluminium (Fig. 2), Glass-Epoxy (Fig. 3) and Graphite-Epoxy (Fig. 4), and using the mentioned criterion, the crack extension angle ϑ_0 versus the crack inclination angle φ, is obtained for different values of the biaxial load parameter, as shown in figures 2-4.
Note that the crack initiation angle depends on the elastic properties of the orthotropic material. In particular, when the elastic moduli ratio E_1/E_2 becomes close to one, the crack extension angles, predicted by the Maximum Circumferential Tensile Stress, approximate the values obtained in the isotropic case. This means that the value of the crack extension angle depends not only on the biaxial load parameter but also on the orthotropic behaviour of the material. For a fixed value of the crack inclination angle φ, the crack initiation angle is an increasing function of s. Note also that for k>1 the crack initiation angle can be negative.

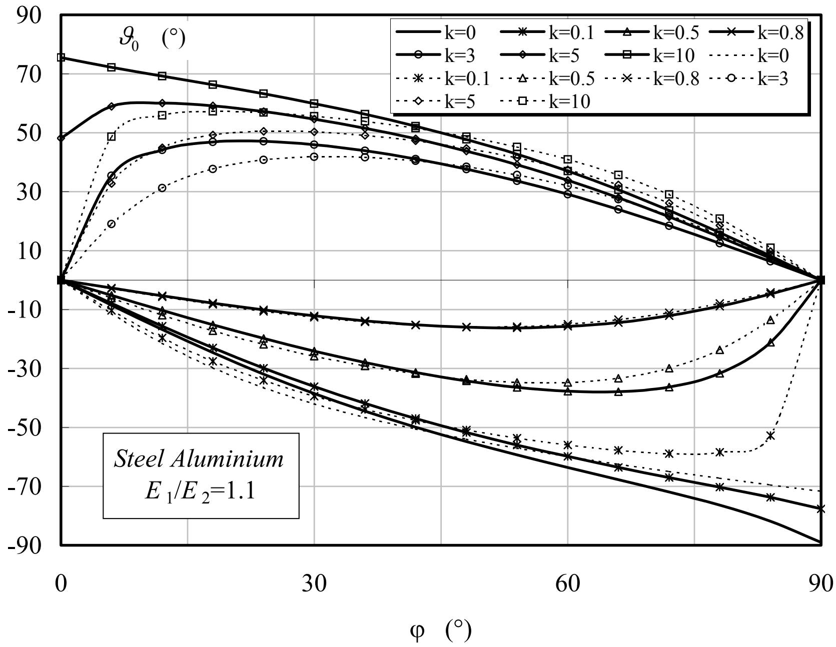

Fig. 2: Steel Aluminium: crack initiation angle ϑ_0 vs. crack inclination angle φ, for different values of the biaxial load parameter. Evaluation of the non singular terms effect.

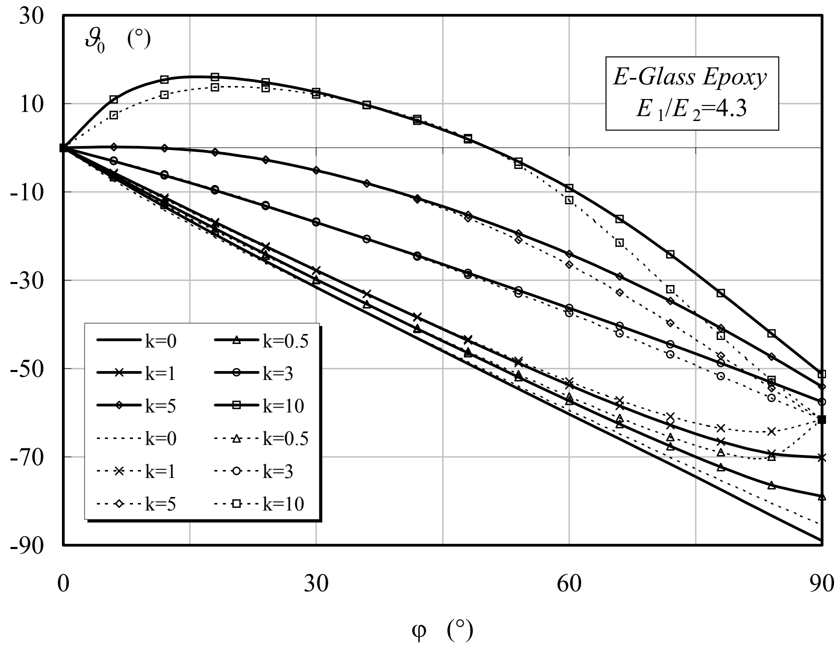

Fig. 3: E-Glass Epoxy: crack initiation angle ϑ_0 vs. crack inclination angle φ, for different values of the biaxial load parameter. Evaluation of the non singular terms effect.

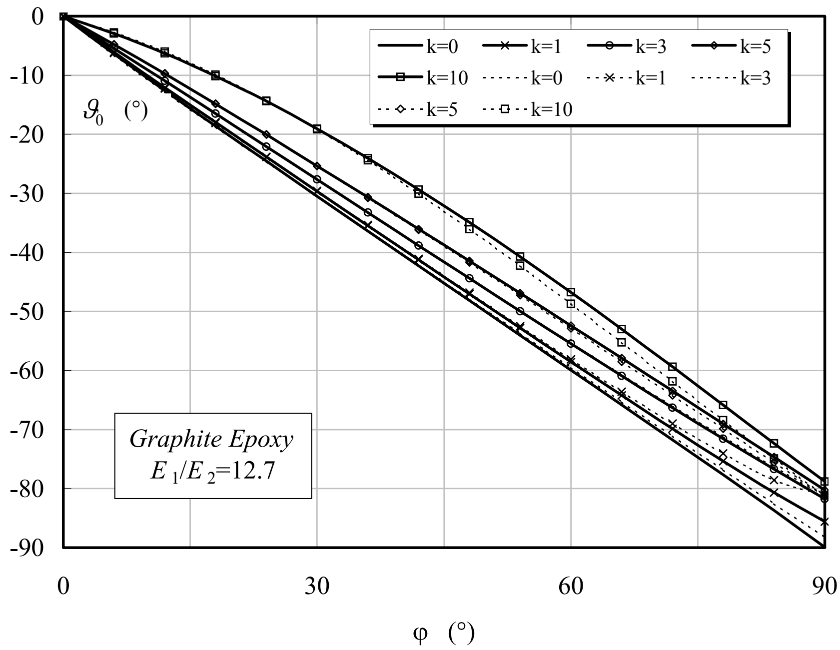

Fig. 4. Graphite Epoxy: crack initiation angle ϑ_0 vs. crack inclination angle φ, for different values of the biaxial load parameter. Evaluation of the non singular terms effect.

Figures 2-4 point out the effect of the non-singular terms on the crack initiation angle. The angle of incipient growth is also obtained neglecting the non-singular terms inside the stress components (dot lines). The influence of the non-singular terms is more evident when the mechanical behaviour of the orthotropic material is close to the isotropic one. Besides the effect of the non-singular terms depends on the value of k. For k>1 the effect of the non-singular terms increases when k increases. For k<1 the effect of the non-singular terms increases when k decreases. For E-Glass Epoxy and Graphite Epoxy the influence of the non singular terms seems to be emphasised when the crack inclination angle is close to 90°. The reason of this fact can be found in equations (4)-(6). The non-singular terms affect all the stress components. This means that the relationship between the biaxial load parameter and the influence of the non-singular terms is quite different from the isotropic case.

4 CONCLUSIONS

The elastostatic problem of an orthotropic body having a central inclined crack and subjected at infinity to a uniform biaxial load has been reported. The crack line does not coincide with an axis of elastic symmetry of the body. The Maximum Circumferential Tensile Stress Criterion has been extended to orthotropic materials in order to predict the crack initiation angle, defining a polar variation for the critical intensity factor for Mode-I. The crack initiation angle has been represented as a function of the crack inclination angle, for different values of the biaxial load parameter s. The dependence of crack initiation angle on the elastic properties of the orthotropic material and on the biaxial load parameter are underlined, noticing that when the elastic moduli ratio becomes close to one, the crack extension angles approximate the values of isotropic case.
The influence of the non-singular terms is also related to the orthotropic behaviour of the material.

5 REFERENCES

[1] Nobile L, Piva A, Viola E (2003). On the crack problem in an orthotropic medium. *Engng. Fract. Mech.*, **71**, 529-546.
[2] Erdogan F, Sih GC (1963). On the Crack Extension in Plates under Plane Loading and Transverse Shear. *J. Bas. Engng.*, **85**, 519-527.
[3] Buczek MB, Herakovich CT (1985). A Normal Stress Criterion for Crack Extension Direction in Orthotropic Materials. *J. Comp. Mat.*, **19**, 544-553.
[4] Saouma V, Ayari ML, Leavell D (1987). Mixed mode crack propagation in homogeneous anisotropic solids. *Engng. Fract. Mech.*, **27**, 171-184.
[5] Beuth JL, Herakovich CT (1989). Analysis of crack extension in anisotropic materials based on local stress. *Theor. Appl. Fract. Mech.*, **11**, 27-46.
[6] Ayari ML, Zhiming Ye (1995). Maximum strain theory for mixed mode crack propagation in anisotropic solids. *Engng. Fract. Mech.*, **52**, 389-400.
[7] Carloni C, Piva A, Viola E (2003). An Alternative complex variable formulation for an inclined crack in an orthotropic medium. *Engng. Fract. Mech.*, **70**, 2033-2058.
[8] Tsukrov I., Kachanov M. (1998). Anisotropic material with arbitrarily oriented cracks and elliptical holes: effective elastic moduli, *Int. J. Fract.*, **92**, L9-L14.
[9] Prabhu S., Lambros J. (2002). Mixed mode asymptotic crack tip fields in orthotropic materials: Derivation and range of dominance, *Int. J. Fract.*, **118**, 339-361.
[10] Nobile L, Carloni C (2003). Crack initiation in orthotropic solids. Technical Report n° 118, DISTART Department, University of Bologna.
[11] Carloni C., *Mixed Mode Fracture Response of Orthotropic Matrials*, PhD Thesis, University of Bologna, 2004.
[12] Arcisz M, Sih GC (1984). Effect of orthotropy on crack propagation. *Theor. Appl. Fract. Mech.*, **1**, 225-238.