SCALE-INDEPENDENT CONSTITUTIVE LAW FOR CONCRETE IN COMPRESSION

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ABSTRACT

The size effects in compression on drilled cylindrical concrete specimens obtained from a unique concrete block over a large scale range (1:19) are analyzed. The experimental results show scale effects on dissipated energy density rather than on the compressive strength. A theoretical explanation for such a phenomenon is presented, assuming a noninteger physical dimension of the sub-domain where dissipation occurs. A comparison between experimental and theoretical values is discussed and a renormalization procedure to obtain a scale independent constitutive law is presented. A scale-independent constitutive law in compression is put forward, which permits to define a unique relationship for softening in concrete. This goal is achieved by defining a fractal strain (or dilatation) whose fractal dimension is related to the sub-domain in which energy dissipation occurs.

INTRODUCTION

The variation of the compressive strength with size and height-diameter (or slenderness) ratio is relevant when the rigid test machine platens are in direct contact with the concrete specimen, the lateral deformation of concrete being restrained at the specimen ends. In this context, a wide investigation has been carried out by Carpinteri et al. [1]. When, instead, the friction at the specimen ends is reduced, the strength variation is less evident.

Van Vliet and van Mier [2], using improved experimental techniques of axial displacement control and lubricated end platens as well as variable height to diameter ratios, observed that post-peak data from uniaxial compression experiments on plain concrete suggest a stress-displacement rather than a stress-strain relation.

An experimental investigation on geometrically similar cylindrical concrete specimens, obtained by a unique concrete block in compression over a very large scale range (1:19), will be brifly reported citecompressione1 and the obtained scale effects will be herein discussed. It will be shown how, avoiding friction, the strength is almost independent of specimen dimension while strong variations are observed for dissipated energy density. This phenomenon can be interpreted by considering the fragmentation and the comminution theories. In this field, Fractal Geometry represents a very helpful tool to explain such a phenomenon.

A theoretical explanation for the scale effects on the dissipated energy density in compression, is discussed and applied to the experimental results. From the theory it can be evidenced how, in the scale range of the tested specimens, the energy dissipation occurs in a sub-domain with a noninteger physical dimension.

EXPERIMENTAL EVIDENCE

In this section, the experimental tests performed at the Politecnico di Torino are briefly presented. All the cylinder were obtained by drilling from a unique concrete block with sizes 800 $\times$ 500 $\times$200 mm. The microconcrete used for the specimens is characterized by a maximum aggregate size of 4 mm, with a compression strength, obtained by cubes (150 $\times$ 150 $\times$ 150 mm) after 28 days, equal to 33 N/mm^2. The water-cement ratio was equal to 0.65.

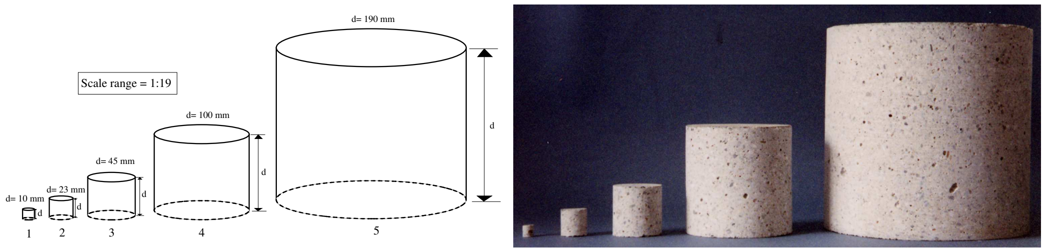

Figure 1. (a) Geometries of the five different concrete specimens; (b) overall view of the five specimen sizes.

Five different diameters were considered in relation to the disposable drilling core-bits in a scale range of 1:19. The specimens were cylinders with a height-diameter ratio h/d =1 and d chosen as characteristic dimension equal to 10, 23, 45, 100, 190 mm, respectively. Six specimens have been tested for d= 10, 23 and 45 mm and four specimens for d= 100 and 190 mm. The geometries of the tested specimens are presented in Fig. 1. For the three smallest sizes, the tests were carried out on a uniaxial compression machine with a capacity of 100 kN. The machine was controlled by a closed-loop servo-hydraulic system. All compression tests with this machine have been performed under displacement control, by imposing a constant rate of the displacement of the upper loading platen. For the two remaining specimen sizes, d=100 (C4) and 190 mm (C5), a manual load controlled uniaxial compression machine with a capacity of 3000 kN was used.

The system adopted in the present compression tests for reducing friction at the ends of the specimens comes out from the analysis of the RILEM Technical Committee 148 SSC (1997) results. These results suggested us to use two teflon layers of 150 μm thickness with oil in between and a specimen slenderness equal to one.

The experimental load vs. displacement diagrams can be found in [3]. The values of the peak-stresses, which are commonly called *compressive strength*, can be deduced from Fig.3a by varying the specimen sizes. It can be noticed how, reducing friction, a marked size effect does not come out, as instead can be evidenced in tension [4, 5, 6] or in compression when localization is present [1]. The scatter in the results is not pronounced and even for the smallest size the values are comparable to the compressive strength of standard cubes. This permits to affirm that, if friction is avoided or drastically reduced, the compressive strength of an existing concrete structure can be evaluated using very small drilling core specimens.

The dissipated energy density can be evaluated by considering the area under the $P - \delta$ curve divided by the volume of the specimen. This is equivalent to consider the area under the stress-strain curve. The values of the dissipated energy density are plotted versus the characteristic specimen size in Fig. 2. They undergo severe scale effects. The trend is a decrease by increasing the specimen dimension.

FRACTAL EXPLANATION OF SIZE EFFECT ON DISSIPATED ENERGY DENSITY IN COMPRESSION

The performed compression tests have shown an evident decrease of dissipated energy density with increasing specimen dimension (Fig. 2). This interesting phenomenon can be interpreted by considering the fragmentation and the comminution theories. In this field, Fractal Geometry represents a very helpful tool. Turcotte [7] in the formulation of his fragmentation theory explains the difficulties in developing comprehensive theories. A primary reason is that fragmentation involves initiation and propagation of fractures. Fracture propagation is a highly nonlinear process requiring complex models even for the simplest configuration. Fragmentation involves the interaction between fractures over a wide range of scales. If fragments are produced over a wide range of sizes and if natural scales are not associated with the fragmented material, fractal distribution of number versus size would seem to be expected. The statistical number-size distribution for a large number of objects can be fractal [7, 8].

Let us consider a concrete specimen which undergoes a compression test. In the post-peak softening regime the specimen is characterized by the generation of a large number of fragments. After fragmentation, the number of fragments N with a characteristic linear dimension greater than r should satisfy the relation:

$$N = \frac{B}{r^D}, \tag{1}$$

where B is a constant of proportionality, and D is the fractal dimension.

It can be assumed that the energy dissipated to produce a new free surface in the fragmentation process is provided by the product of specific energy absorbing capacity β_F and the total surface area A_f, for $2 < D < 3$ [9]:

$$W = \beta_F A_f = \beta_F A_f \frac{V}{V} \tag{2}$$

in which β_F should be have dimension of $[F][L]^{(D-1)}$. If we suppose that r_{max} is proportional to the size of the fragmented object, with k the constant of proportionality, V can be expressed as:

$$V = V^{D/3} V_f \frac{3-D}{-DB} k^{D-3} = \frac{r_{max}^3}{k^3}. \tag{3}$$

In this case it is possible to have:

$$W = \beta_F A_f \frac{V}{V} = \beta_F A_f \frac{V_f V^{D/3} \frac{3-D}{-DB} k^{D-3}}{\frac{r_{max}^3}{k^3}} = \left(\beta_f \frac{-BCD}{D-2} r_{min}^{2-D} r_{max}^D k^D \right) V^{\frac{D}{3}} = \mathcal{G}_F^* V^{\frac{D}{3}}. \tag{4}$$

The two extreme cases contemplated by eq.(4) are D=2, surface theory [10, 11], when the dissipation really occurs on a surface ($W \propto V^{\frac{2}{3}}$) and by D=3, volume theory [12], when the dissipation occurs in a volume ($W \propto V$). In this case $\mathcal{G}_F^*$ presents the following physical dimension:

$$[\mathcal{G}_F^*] = \left(\beta_f \frac{-BCD}{D-2} r_{min}^{2-D} r_{max}^D k^D \right) = [F][L]^{D-1}[L]^{2-D}[L]^{-D} = [F][L]^{1-D}. \tag{5}$$

For $D = 2 \rightarrow [\mathcal{G}_F^*] = [F][L]^{-1}$, which is the canonical dimension for fracture energy, while for $D = 3 \rightarrow [\mathcal{G}_F^*] = [F][L]^{-2}$, which is the physical dimension of stress. The experimental cases of fragmentation are usually intermediate ($D \cong 2.5$) [8], as well as the size distribution for concrete aggregates due to Fuller [13]. If we consider $V = l^3$, we can write the expression of the dissipated energy density, from eq.(4):

$$S = \frac{W}{V} = \mathcal{G}_F^* l^{D-3}. \tag{6}$$

The relationship of dissipated energy density related to different sizes can be posed in logarithmic form:

$$\log S = \log \mathcal{G}_F^* + (D-3) \log l. \tag{7}$$

Eq.(7) represents a straight line with slope $(D-3)$ in the log S versus log l plane (Fig. 2.a). If $D = 2$, the slope is -1, as well as $D = 3$ implies a vanishing slope.

As may be observed from Fig.2a, the slope of the dissipated energy density decrease proves to be equal to 0.97. The physical meaning reveals an energy dissipation on a fractal space of dimension 2.03, which appears to be very close to a 2-dimensional surface. It is therefore possible to obtain a constant (universal) dissipated energy density equal to 74 Nmm$^{-1.03}$ (Fig.2b). The graphic interpretation of the renormalization procedure is given in Fig.2. The assumption of a fractal physical dimension allows the determination of the dissipated energy density parameter $\mathcal{G}_F^*$, which results to be independent of the scale. As it is easy to observe, in the latter case the renormalized dissipated energy density tends to be a fracture energy, the dissipation occurring on a fractal set very close to a 2-dimensional surface. Such a result confirms the localization of the dissipation on a surface [14].

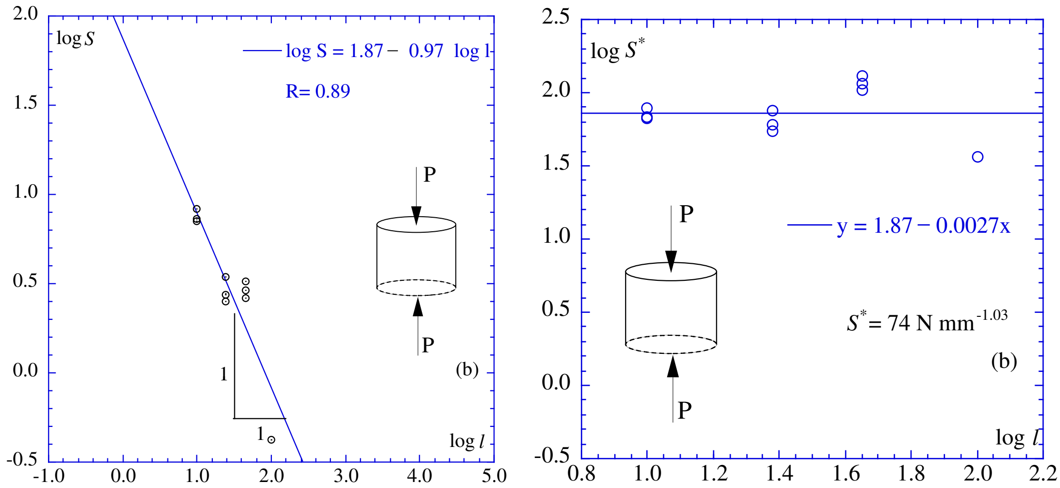

Figure 2. (a) Size effect on dissipated energy density (experimental tests); (b) Renormalized value of dissipated energy density for the experimental tests.

RENORMALIZED DIAGRAMS OF STRUCTURAL RESPONSE

The experimental curves σ vs ϵ, reported in Fig.3a, show a marked scale dependence, in particular for what concerns the post-peak part. These curves σ vs ϵ, or F vs. δ, are in fact characterized by two different regimes. The first regime corresponds to the pre-peak elastic behavior, when microcracks form randomly in the specimen. At this stage the external force linearly increases untill it reaches the peak value and the statistical fluctuations are very small. In the second regime, which could be called ”catastrophic”, the interactions between the microcracks begin to rule the process, untill macrofractures form and propagate through the whole specimen. In large specimens this phenomenon could occur with a sudden release of stored elastic energy.

In this section, a renormalization procedure is proposed to obtain a unique constitutive relationship for softening in compression. By assuming damage occurring in a fractal sub-domain inside the specimen, energy dissipation becomes scale-dependent. Hence it should be substituted by a fractal quantity, which is the true material constant. The assumption that the energy dissipation occurs in a sub-domain characterized by a fractal dimension, imposes the definition of fractal strain (or dilatation).

Let us consider the external work W, which presents the physical dimension of [F][L]. The nominal dissipated energy density, $S = W/V$, is usually the dissipated energy over the specimen volume, so that it presents the physical dimension of [F][L]$^{-2}$ and can be evaluated by integration:

$$S = \frac{W}{V} = \int_0^{\epsilon_{max}} \sigma(\epsilon)\mathrm{d}\epsilon, \tag{8}$$

which represents the area under the $\sigma - \epsilon$ curve. Supposing that the energy dissipation occurs not in the specimen volume ($V \propto l^3$) but in a fractal domain of dimension D ($V \propto l^D$), and considering $[\sigma] = [F][L]^{-2}$ as the nominal stress, in order to obtain a constant specific compression energy, the strain has to assume a physical dimension of $[L]^{-(D-3)} = [L]^{d_\omega}$ [15, 16]. In fact, in this hypothesis, if W is dissipated over a domain with physical dimension of [L]^D, we obtain:

$$[S] = \frac{[W]}{[V]} = \frac{[F][L]}{[L]^D} = [F][L]^{1-D}. \tag{9}$$

For D=2 (surface theory, dissipation occurring on a surface) $\rightarrow$ $S = [F][L]^{-1}$, while for D=3 (volume theory, dissipation occurring on a volume) $\rightarrow$ $S = [F][L]^{-2}$. Assuming to maintain the the nominal stress σ with physical dimension of $[F][L]^{-2}$, from eq.8 we have:

$$[S] = [\sigma][\epsilon^*] = [F][L]^{-2}[L]^x = [F][L]^{1-D}, \tag{10}$$

and than:

$$x = 3 - D = d_\omega. \tag{11}$$

In the monofractal hypothesis, the renormalized strain therefore assumes the physical dimension of $[L]^{3-D}$, defined as the displacements δ divided by l=$[L]^{D-2}$.

By considering the fractal strain, a scale-invariant constitutive relationship can be obtained. In other words, the experimental diagrams related to the different sizes can be rescaled by considering the strain renormalization, and a clear superposition of the curves is evidenced. In Fig.3.b the strains are renormalized for D=2.03. It is possible to observe how the curves tend to superpose one on each other and in particular how the variation in structural behaviour disappears.

Lastly, form Fig.3b, it can be observed how a renormalization (or a new definition) of the elastic modulus comes out. In fact, the elastic modulus is defined, from the classical Hooke Law's, as the ratio between the stress and the strain. In the present analysis we obtain:

$$[E^*] = \frac{[\sigma]}{[\epsilon^*]} = \frac{[F][L]^{-2}}{[L]^{-(D-3)}} = [F][L]^{D-5}, \tag{12}$$

and in the two limit cases for $D = 2$ (surface theory, dissipation occurring on a surface) $\rightarrow$ E=$[F][L]^{-3}$, and assumes the physical dimension of a density, while for $D = 3$ (volume theory, dissipation occurring on a volume) $\rightarrow$ E=$[F][L]^{-2}$ and we obtain the classical elastic modulus.

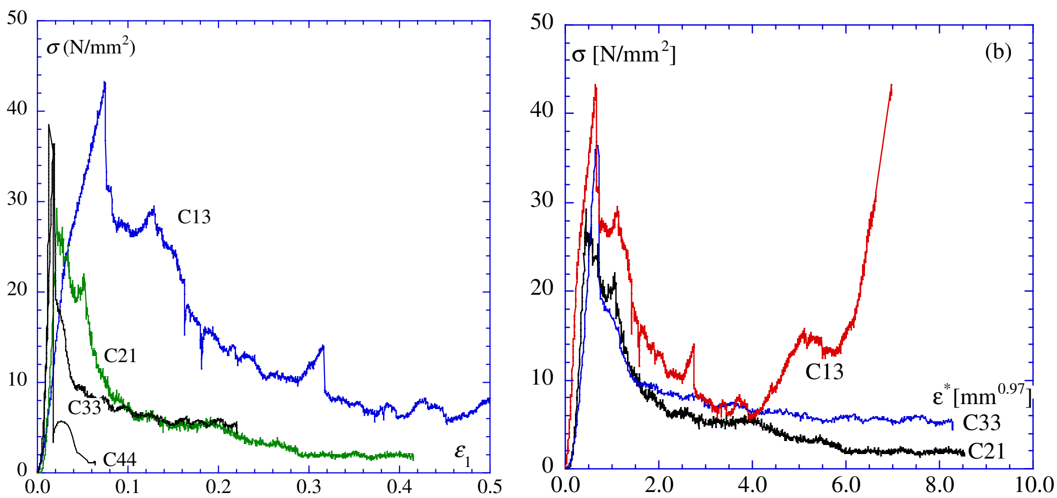

Figure 3. (a) Stress-strain curves for four different cylindrical specimen sizes; (b) Stress vs renormalized strain for four different specimen sizes D=2.03.

The renormalization strain has a physical dimension equal to 0.97, very close to 1, and then very close to a displacement. What is important to emphasize at this stage is that in compression we have dissipation of the energy over an area at small scales, while at large scales the energy dissipation occurs in a volume. This appears very interesting as it is the opposite trend with respect to tension, in which localization is evident for large specimens and not at small scales. Eventually, the renormalization procedure for large specimens (D=3) tends again to a stress-strain diagram, as $\epsilon^* = \epsilon$

CONCLUSIONS

The uniaxial compression tests performed under displacement control on drilled cylindrical specimens obtained by a unique concrete block over a very large scale range (1:19) have confirmed as the scale effect on compressive strength is not as evident as in traction. The experimental results have instead manifested a strong scale effect on dissipated energy density, showing a sharp decrease of that quantity by increasing specimen size.

The hypothesis of energy dissipation in a sub-domain with physical dimension between 2 and 3 can be effective to justify such a phenomenon. It can be observed how, when energy dissipation occurs in the volume (D=3) no scale effects are present, whereas when energy dissipation occurs over an area (D=2) the scale effects are characterized in the bilogarithmic diagram log S versus log l by a linear law with slope equal to -1. By fitting the experimental values, we obtain an intermediate case, and a renormalized value for dissipated energy density, invariant with scale, can be obtained. This scale invariant value is characterized by noninteger physical dimensions. This hypothesis works very well in the size range of the tested specimens.

A renormalization procedure for strain (or dilation) has been eventually proposed in order to obtain a scale-invariant stress vs renormalized strain diagram.

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REFERENCES

1. A. Carpinteri, G. Ferro, I. Monetto, Scale effects in uniaxially compressed concrete specimens, Magazine of Concrete Research 51 (1999) 217–225.
2. M. van Vliet, J. van Mier, Experimantal investigation of concrete fracture under uniaxial compression, Mechanics of Cohesive-Frictional Materials 1 (1996) 115–127.
3. G. Ferro, Scale effects on compressive tests for concrete specimens, Private Comunication .
4. G. Ferro, Effetti di scala sulla resistenza a trazione dei materiali, Ph.D. thesis, Politecnico di Torino (1994).
5. A. Carpinteri, B. Chiaia, G. Ferro, Size effects on nominal tensile strength of concrete structures: multifractality of materials ligaments and dimensional transition from order to disorder, Materials and Structures 28 (1995) 311–317.
6. A. Carpinteri, B. Chiaia, G. Ferro, A new explanation for size effects on the flexural strength of concrete, Magazine of Concrete Research 49 (1997) 45–53.
7. D. L. Turcotte, Fractals and chaos in geology and geophysics, Cambridge University Press, 1992.
8. D. L. Turcotte, Fractals and fragmentation, Journal of Geophysics Research 91 (1986) 1921–1926.
9. F. Borodich, Fractals and fractal scaling in fracture mechanics, International Journal of Fracture 95 (1999) 239–259.
10. P. R. Rittinger, Lehrbuch der aufbereitungskunde, Berlin, 1937.
11. F. Hönig, Grundgesetze der zerkleinerung, Tech. Rep. 378, VDI Forschungsheft (1936).
12. F. Kick, Das gesetz der proportionalen widerstande, Leipzig, 1885.
13. P. Stroeven, Fractals and fractography in concrete technology, in: Int. Symposium on Brittle matrix composities, Warsaw, Poland, 1991, pp. 1–10.
14. Y.-H. Lee, K. Willam, Mechanical properties of concrete in uniaxial compression, ACI Materials Journal 94 (1997) 457–471.
15. A. Carpinteri, P. Cornetti, A fractional calculus approach to the description of stress and strain localization in fractal media, Chaos, Solitons and Fractals, in print .
16. A. Carpinteri, B. Chiaia, P. Cornetti, A scale-invariant cohesive crack model for quasi-brittle materials, Engineering Fracture Mechanics 69 (2002) 207–217.