THERMO-MECHANICAL ANALYSIS OF ELASTIC CRACKS IN HETEROGENEOUS MATERIALS BY FEM AND VCFEM

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ABSTRACT

The present paper presents a numerical method for the evaluation of the stress intensity factor in two dimensional heterogeneous materials under mechanical and thermal loads. The proposed method uses both FEM (Finite Element Method) and VCFEM (Voronoi Cell Finite Element Method), and has been developed using the *Mathematica*^{TM} environment. The numerical models are obtained by combining the traditional finite elements, used to model the crack tip, and the Voronoi Cells, based on an hybrid formulation of the element, in order to represent the second phase. The use of this method in investigating the crack interaction in heterogeneous materials allows a simple mesh generation and a low computational cost, compared with traditional finite element method. In the present paper the proposed model has been verified by evaluating the stress intensity factors in an homogeneous material and by comparing them with those reported in literature; then the stress intensity factors for a heterogeneous material with periodic distribution of the second phase has been evaluated by introducing a variance in temperature.

1 INTRODUCTION

The use of heterogeneous materials to produce advanced structural components in mechanical, aerospace, automotive, shipbuilding and other branches of engineering, is constantly increasing. Furthermore the mechanical strength of brittle heterogeneous materials, like advanced ceramics, strongly depends on the presence of micro cracks dispersed in the materials, but their toughness can be increased by using reinforcement materials. The mechanical properties of these materials depend on the dimension, shape, distribution and properties of the second phase inclusion materials. The influence of a crack tip in heterogeneous materials has been investigated by means of continuum damage mechanics (Ortiz [1,2]), and semi analytical methods (Meguid [3], Gong[4]). The classical finite element method provides good results in evaluating the stress intensity factor, but unfortunately it requires a complex mesh in order to model micro-structural details of the heterogeneous materials. An alternative and less onerous numerical method consists in using the BEM (Boundary Element Method) and a special element for the analysis of the stress intensity factor (Luchi [5]), but it is too difficult to generate a mesh that takes into account the structural details of the heterogeneous materials. To overcome some of the limitations discussed above, the FEM can be integrated with an hybrid finite element method (VCFEM) in which the heterogeneity can be easily modelled by using n-sided polygonal elements with embedded inclusions, known as Voronoi Cell (Cesari [6-8], Ghosh [9], Zhang [10,11]). In this way a simple mixed mesh can be generated where the FEM is used to model the crack tip and the heterogeneities are modelled by using VCFEM. The aim of this paper consists in developing a numerical procedure which is able to analyze a two dimensional mesh containing both traditional and hybrid finite elements. By means of hybrid element formulation the heterogeneous materials with linearly elastic inclusions are analysed; the method is based on the principle of minimum complementary energy, while the stress distribution inside the element is interpolated by the full form of the Airy stress functions (Cesari [7], Ghosh [9]). In order to analyse two dimensional models under thermal loads a numerical procedure has been developed which is able to evaluate the equivalent thermal load vector in the hybrid elements. The reliability and accuracy of the proposed method have been initially verified by analysing hybrid models of two dimensional homogeneous materials and by comparing the stress intensity factors with those available in literature (Rooke [12], Brown [13]). Subsequently the effect of a variance in temperature on the stress intensity factor in an heterogeneous plate with a periodic distribution of the second phase has been evaluated. The material properties used in the simulation for the matrix and for the inclusions are those of alumina (Al_2O_3) and zirconia (ZrO_2), which are typical oxides used to obtain advanced ceramic composites.

2 NUMERICAL PROCEDURE

In order to integrate the FEM e VCFEM methods some numerical procedures have been developed in *Mathematica*^{TM} environment. The flow-chart in Fig. 1 show the sequence of the developed procedures.

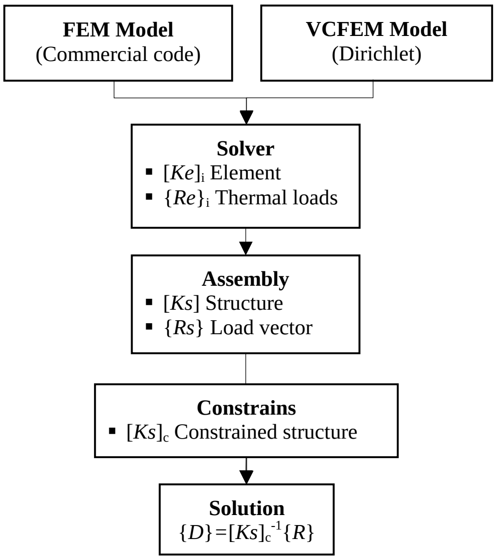

Figure 1: Flow chart of the numerical procedures.

The FEM model is created by using a commercial finite element pre-processor, while the VCFEM model is generated by using the pre-processor Dirichlet (Cesari [14]), based on the Dirichlet tessellation method. Subsequently a solver evaluates the stiffness matrices $[Ke]_i$ and load the vectors $\{Re\}_i$ for each element. Then the two models are merged in a single one by eliminating the common nodes and by building the structure stiffness matrix $[Ks]$ and the structure load vectors $\{Rs\}$. The mechanical loads are introduced in the structure load vector after the assembly of the two models while the thermal loads are introduced in the element load vectors. Finally, after constraining the structure and inverting the stiffness matrix, the displacement vector $\{D\}$ is evaluated.

3 VCFEM FORMULATION

Figure 2 reports an example of hybrid element, used in the VCFEM method, with an embedded inclusion. The element formulation, as reported in (Cesari [7], Ghosh [9]), is based on the stationary complementary energy principle $\pi=\pi_m+\pi_i$:

$$\pi_m = \frac{1}{2}\int_{Am}\{\sigma\}^T[C_m]\{\sigma\}dA - \int_{Am}\{\sigma\}^T\{\varepsilon_{m0}\}dA - \oint_S\{T\}^T\{u\}dS - \oint_{S_i}\{\sigma_m\}^T[n_i]^T\{u_i\}dS\,, \tag{1}$$

$$\pi_i = \frac{1}{2}\int_{Ai}\{\sigma\}^T[C_i]\{\sigma\}dA - \int_{Ai}\{\sigma\}^T\{\varepsilon_{i0}\}dA + \oint_{S_i}\{\sigma_i\}^T[n_i]^T\{u_i\}dS\,, \tag{2}$$

where the subscripts *i* and *m* indicate the inclusions and the matrix respectively; *A* and *S* represent respectively the area and the boundary of the matrix or of the inclusion; [*C*] is the elastic compliance matrix; {σ} is the stress field within the element and the inclusion; [*n*] contains the components of the outward normal unit vector to the element boundary; {*T*} represents the prescribed boundary traction; {*u*} are the displacements along the outer and inner boundary of the element; {ε_0} represents the thermal strain vector.

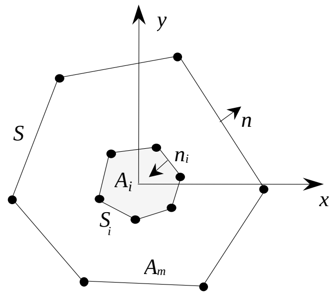

Figure 2: Hybrid element with an embedded inclusions.

The displacements {*u*} are interpolated by using the nodal displacements {*q*} and the boundary displacement interpolation functions [*L*] ({*u*}=[*L*]{*q*}), while the stress components within the element {σ} are assumed to be compatible with prescribed boundary tractions and satisfy the equilibrium conditions neglecting the body forces. The stress field {σ} is expressed as polynomial functions of coordinates *x-y*, by using complete forms of the stress Airy functions. This results in the product of an interpolation matrix [P_e] and an unknown vector of coefficients {β}, {σ}=[P_e]{β}. The stationary condition of the functional, with respect to the vector {β}, gives the expression of the stiffness matrix of the element:

$$[K_e] = \begin{bmatrix} [K_{11}] & [K_{12}] \\ [K_{12}]^T & [K_{22}] \end{bmatrix} \qquad \begin{aligned} &[K_{11}] = [G_m]^T[H_m]^{-1}[G_m] \\ &[K_{12}] = -[G_m]^T[H_m]^{-1}[G_i] \\ &[K_{22}] = [G_i]^T\left([H_m]^{-1} + [H_i]^{-1}\right)[G_m] \end{aligned} \tag{3}$$

where [*H*] and [*G*] are defined as follows:

$$[H] = \int_A [P_e]^T[C][P_e]dA\,, \qquad [G] = \int_S [R_e]^T[L]dS\,, \tag{4}$$

where [R_e]=[*n*][P_e]. A detailed description of the formulation is reported in (Cesari [7]).
In the present paper a particular focus will be done on a procedure developed to evaluate the equivalent load vector to be used in the cases of thermal loads.

$$[H_{mT}] = \int_{Am}[P_e]^T\{\varepsilon_{m0}\}dA\,, \qquad [H_{iT}] = \int_{Ai}[P_e]^T\{\varepsilon_{i0}\}dA\,, \tag{5}$$

where superscript *T* indicates the thermal loads, and the thermal strain vectors {ε_0} are expressed as follows:

$$\{\varepsilon_{m0}\} = \alpha_m \Delta T_m \begin{Bmatrix} 1 \\ 1 \\ 0 \end{Bmatrix}, \qquad \{\varepsilon_{i0}\} = \alpha_i \Delta T_i \begin{Bmatrix} 1 \\ 1 \\ 0 \end{Bmatrix}, \tag{6}$$

where α is the thermal expansion coefficient of the material and ΔT is the variance temperature. The resulting thermal load vector can be expressed as follows:

$$\{R_T\} = \begin{Bmatrix} \{R_{mT}\} \\ \{R_{iT}\} \end{Bmatrix}, \tag{7}$$

where $\{R_{mT}\}$ and $\{R_{iT}\}$ are given by the following equations:

$$\{R_{mT}\} = [H_{mT}][H_m]^{-1}[G_m], \quad \{R_{iT}\} = -[H_{mT}][H_m]^{-1}[G_i] + [H_{iT}][H_i]^{-1}[G_i]. \tag{8}$$

4 NUMERICAL EXAMPLES

4.1 Edge crack in an homogeneous plate

The accuracy of the proposed method has been initially verified by analysing a two dimensional plate of homogeneous material containing an edge crack under uniaxial tensile stress. The analysis have been performed for different values of the *a/w* ratio, where *a* and *w* represent the crack length and width of the plate respectively. In Fig. 3a) one of the model used in the simulation is reported. The upper part of the plate has been modelled by using 12 sixteen-noded and 4 twenty-noded Voronoi cells of homogeneous materials, while the bottom consists of 92 four-noded conventional isoparametric elements and a particular refinement was used to model the crack tip. The stress intensity factors have been evaluated by using the modified crack closure integral method as reported in (Rybicki [15]). The Fig. 3b) reports the K_I values, normalized with respect to $K_0 = \sigma\sqrt{\pi a}$, as function of the *a/w* ratio: the continuous line is the plot of an analytical expression reported in literature (Rooke [12], Brown [13]); the points represent the numerical results obtained by the proposed model. The graphics clearly show the good agreement between the results of the two methods, with difference never greater than 1%.

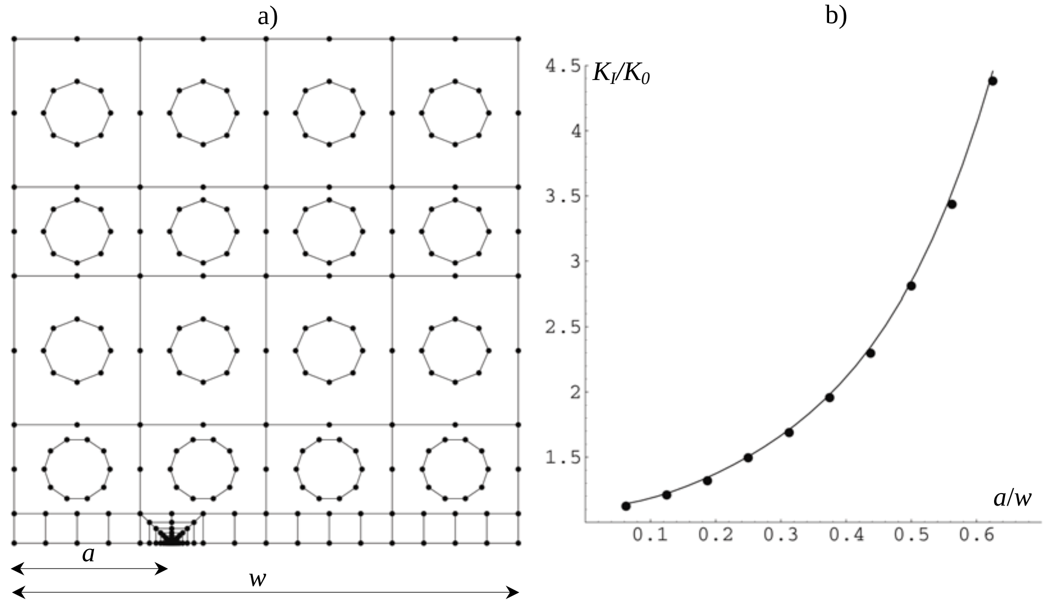

Figure 3: a) The numerical model; b) Comparison between the numerical results obtained by the proposed model (points) and those reported in literature (continuous line).

4.2 Edge crack in an heterogeneous plate

In the present example a plate of heterogeneous materials with an edge crack has been analysed, in particular the K_I values have been calculated under both mechanical and thermal loads. The material properties used in the simulation are those of zirconia, ZrO_2, (E=205 GPa, ν=0.32) and alumina, Al_2O_3, (E=356 GPa, ν=0.32) for the matrix and inclusions respectively, while the volume fraction of alumina is fixed to 20%. The thermal expansion coefficients are function of the absolute temperature T and are given by the following expressions:

$$\begin{aligned} &\alpha = 9.75\,10^{-6} + 4.0\,10^{-9}\,T - 1.44\,10^{-12}\,T^2 && \text{for } ZrO_2 \\ &\alpha = 6.6\,10^{-6} + 4.1\,10^{-9}\,T - 8.9\,10^{-13}\,T^2 && \text{for } Al_2O_3 \end{aligned} \tag{9}$$

The analysis have been performed by using the same models of the previous example, at different crack sizes ($0.0625 \le a/w \le 0.625$), and for two variance in temperature (100 K and 400 K). The results are reported in Fig. 4. In particular, Fig. 4a) shows the comparison between the homogeneous plate of the previous example and an heterogeneous plate (20% Al_2O_3 – 80% ZrO_2); the decrease of K_I is due to the higher Young's modulus of the second phase heterogeneity (Al_2O_3). This effect is more evident when the crack tip is far from the centroid of the second phase, and this trend is amplified when the volume fraction of the heterogeneities increases (see Cesari [8]). Figure 4b) reports the effects of the temperature on the K_I for the heterogeneous plate (20% Al_2O_3 – 80% ZrO_2); as it is shown in figure, K_I increases as the variance in temperature increases because the lower thermal expansion coefficient of the second phase induces a residual tensile stress in the matrix and in particular around the inclusions. This effect is more evident when the crack tip is located in front of the centroid of the second phase.

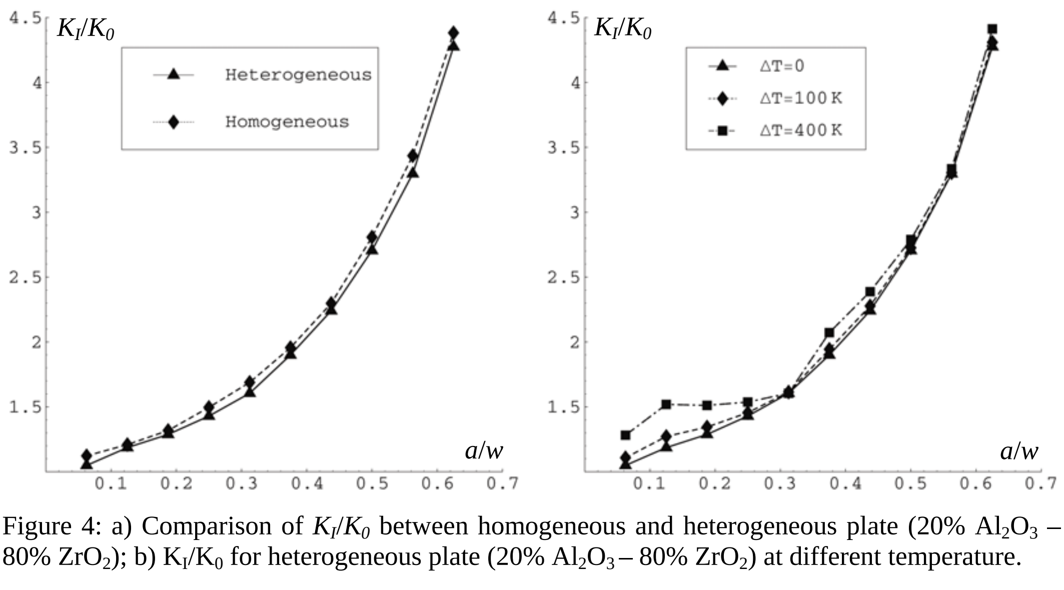

Figure 4: a) Comparison of K_I/K_0 between homogeneous and heterogeneous plate (20% Al_2O_3 – 80% ZrO_2); b) K_I/K_0 for heterogeneous plate (20% Al_2O_3 – 80% ZrO_2) at different temperature.

5 CONCLUSIONS

A mixed FEM-VCFEM method, which is capable of analysing elastic crack in two dimensional heterogeneous materials under thermo-mechanical loads, has been developed. The use of VCFEM allow the simple modelling of the heterogeneities, while the FEM is used to model the crack tip.

The accuracy and versatility of the proposed method have been illustrated by executing some numerical examples. The results obtained in this way are in good agreement with the corresponding theoretical results. As illustrated in the numerical examples an interesting application of the proposed method could be the analysis of the toughness of advanced ceramics materials under thermo-mechanical loads, and other models can be generated in order to analyse interface matrix-inclusion cracks.

6 REFERENCES

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