

A WEAKEST-LINK ANALYSIS FOR FATIGUE STRENGTH OF COMPONENTS CONTAINING DEFECTS

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ABSTRACT

A weakest-link model has been developed to assess the fatigue limit of three different strip steels. The analysis is based on a discretization of a component into FEM elements whose failure is correlated to the presence of internal inclusions inside the steel. The analysis of the inclusions content is carried out by means of the Statistics of Extremes: the three strips are then compared in terms of cleanliness and fatigue properties. For each steel fatigue limit distribution is obtained with a weakest-link model of the fatigue specimens: the obtained results are close to experimental outcomes.

1 INTRODUCTION

Defects, inclusions and inhomogeneities are detrimental to fatigue strength of steels. The behaviour of materials containing defects has been explained by Murakami [1], who clearly showed that the fatigue limit of specimens containing micro-holes is characterised by the presence of non-propagating cracks. It can then be said that the fatigue limit is not a limit stress for 'nucleation' but it rather is the threshold stress for non-propagation of the small crack which emanated from the original defects. Fatigue strength in presence of defects is characterised by the so-called Kitagawa diagram (i.e. the variation of fatigue strength with crack size), which is shown in Figure 1. Its main features are: *i*) fatigue strength tends to increase by decreasing defect (or crack) size; *ii*) there is a critical size – a_{cr} – below which defects (or cracks) are non-damaging and fatigue strength corresponds to the limit stress amplitude of smooth specimens. For defects larger than – a_{cr} – in the region of short cracks, the relationship between the fatigue limit and the maximum square root of the area of the internal defect is expressed by Murakami and Endo model (Murakami [1]):

$$S_{lim} = C \cdot \frac{(HV+120)}{(\sqrt{area_{max}})^{1/6}} \cdot (0.5 - R/2)^{0.226+HV \cdot 10^{-4}} \quad (1)$$

where C is constant (1.43 for surface and 1.56 for internal defects respectively) and \sqrt{area} is the square root of the defect area projected onto a plane perpendicular to applied stress.

After creating a FEM model of a component and using the Statistics of Extremes to assess the statistical distribution of inclusions inside the material, the fatigue limit of the component can be calculated developing a weakest-link model considering that the failure of a finite element, in presence of a detrimental defect inside it, causes the failure of all the component. The analysis has been applied to three high strength strip steels produced by Sandvik Materials Technology.

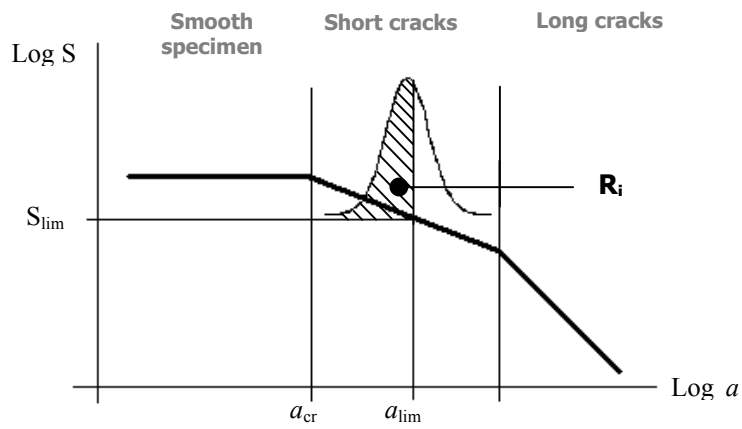


Figure 1: Kitagawa diagram and determination of a_{iim}

2 WEAKEST – LINK MODEL

A weakest-link model reproduces a system that fails just if one of its elements fails. If we consider a system consisting of n series elements (see Figure 2 a) we define:

- $P_{f,i}$ the probability of failure of an element,
 - P_f the probability of failure of the system,
 - R_i the reliability of an element,
 - R the reliability of the system,
- then:

$$\begin{cases} R = \pi R_i \\ P_f = 1 - \pi (1 - P_{f,i}) \end{cases} \quad (2)$$

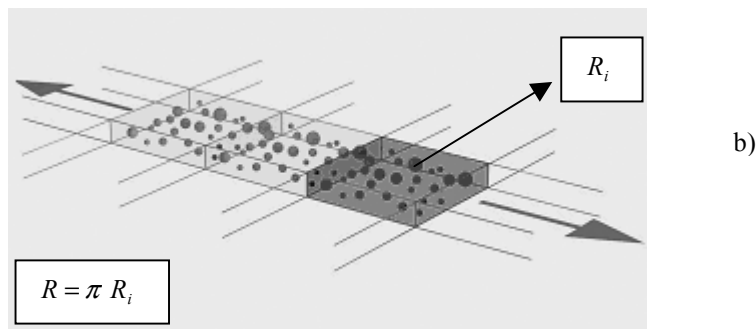
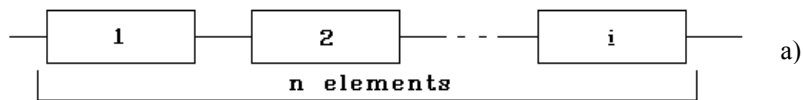


Figure 2: The weakest – link model a) and its application to the fatigue limit calculation b)

Considering specimens used in mechanical tests and dividing them in finite elements, the condition of failure is when the stress of a finite element overtakes its fatigue limit. Correspondingly the failure probability is the probability of the maximum expected inclusion in the element to exceed the critical inclusion corresponding to the limit stress through Murakami – Endo formula (Eq. (1)) in Figure 2. In particular, labelling with – a – the defect size (expressed in terms of $\sqrt{\text{area}}$):

$$P_{f,i} = P(S_i > S_{\text{lim}}) = P(a_{\text{max},i} > a_{\text{lim}}) = 1 - G_i(a_{\text{lim}}) \Rightarrow R_i = G_i \quad (3)$$


where $G_i(a)$ is the cumulative probability function of the maximum inclusions in the i -th finite element, calculated from the microscope analysis considering the dimension of the finite element (Beretta et al. [2]).

3 APPLICATION TO THREE MATERIALS

Three different strip materials have been investigated through the Statistics of Extremes using a weakest-link model to assess their fatigue limit. The analysed samples are the ones used for the fatigue characterisation.

The three materials called A, B and C differ for the mechanical properties (see Tab.1), the strip thickness and the inclusion content, that was investigated by adopting the ‘extreme value inclusion rating’.

Table 1: Properties of the strips and fatigue specimen

Material	Thickness [mm]	HV [kgf/mm ²]	Rm [MPa]	
strip A	0.305	539	1705	
strip B	0.305	556	1744	
strip C	0.381	581	1649	

The analysis of the inclusions content is carried out on polished section by detecting the maximum inclusion within control areas $S_o = 400 \text{ mm}^2$ (‘block maxima’ sampling [3]) and 40 maxima are collected for each strip. Data collected are plotted in a Gumbel plot (Figure 3) and interpolated with a Gumbel equation (Murakami [3]) for strip B:

$$G(a, \lambda, \delta) = \exp\left(-\exp\left(-\frac{a-\lambda}{\delta}\right)\right) \quad (4)$$

where λ, δ respectively are location and scale parameters. In the case of strips A and C, which show a kink in the Gumbel plot (Figure 3), inclusion data were analysed with Competing Risks model (Beretta et al. [4]):

$$F = G_1 \cdot G_2 \quad (5)$$

Considering the data collected in a control volume V_o (Murakami et al. [5]) $V_o = S_o \cdot h$, where h is the average dimension of extreme inclusions [3], the distributions of maximum inclusions inside a finite element with a volume V can be obtained using :

$$G_v = (G_{v_0})^{\frac{v}{v_0}}. \quad (6)$$

and the reliability of i -th element can be obtained by Eq. (3).

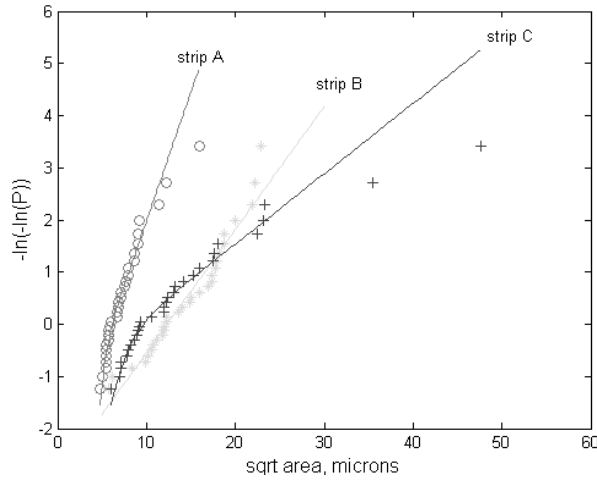


Figure 3: Inclusions distribution of the three strips

4 FEM ANALYSIS AND RESULTS

A finite elements analysis for the strips subjected to axial test has been carried out simulating loads and boundary conditions of the fatigue tests. The resulting map of maximum principal stress [MPa] is shown in Figure 4.

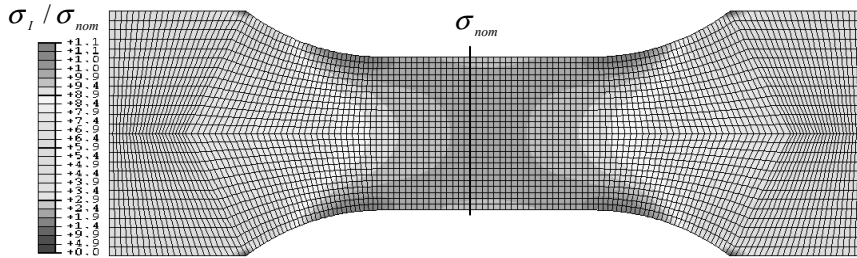


Figure 4: Finite elements model of the specimen and pattern of the maximum principal stress

By knowing the volumes of finite elements and their inclusion distribution, the reliability of each element and the specimen reliability can be calculated respectively with Eq. (3) and Eq. (2), as a function of the maximum applied load in the component.

The probability density function of the normalised fatigue stress limit of each strip is calculated and compared with the normal distribution obtained from stair-case fatigue tests in Figure 5a, b, c. A comparison among the normalised probability density functions of the strips shows the different mechanical fatigue properties of the strips in Figure 5d. The estimated density functions seem to estimate quite well the normal distribution obtained at from stair-case fatigue limit tests.

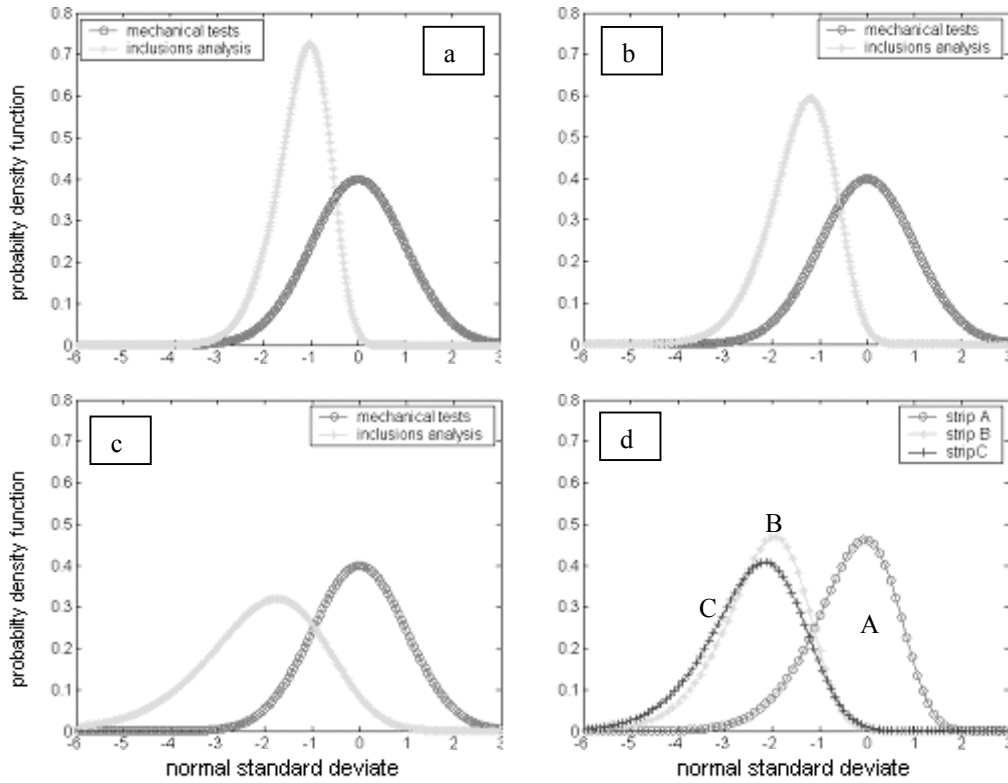


Figure 5: Fatigue strength estimation: a,b and c) probability density of the fatigue limit of strips A,B and C respectively; d) comparison among the three strips.

4.1 Defects density

The critical dimension of the inclusions over which the failure can happen is the one corresponding to a very high failure probability, and it can be evaluated using Eq. (1) and the maximum stress limit for each strip. The density of the critical dimension N_s is then achieved using the relationship (Shi et al. [6]):

$$\lambda = u + \delta \cdot \ln(N_s \cdot S_0) \quad (7)$$

where λ, δ are the parameters of the distribution of maximum inclusions of each strip over the critical threshold u , and S_0 is the collection area of control.

Table 2: Determination of the critical density

Strip A	Strip B	Strip C	
7	11	11	Critical threshold [μm]
0.606	1.361	0.722	Critical density / 400 mm^2

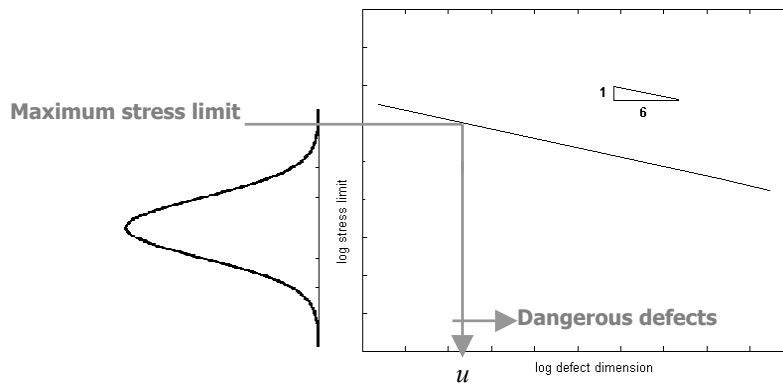


Figure 6: Determination of the dangerous defects from the Kitagawa diagram

Table 2 shows how, despite having the best mechanical properties (see Table 1), strip B is penalized by the highest density of inclusions bigger than the critical dimension, so that strip C is almost equivalent for the fatigue limit properties even showing a higher volume, that is a higher probability in finding inclusions.

5 CONCLUSIONS

The weakest – link model is able to reach results comparable to the ones obtained at mechanical tests but have a slightly smaller mean value and a narrower scatter, since residual stresses and superficial defects are not considered. Moreover it permits to make a comparison among the fatigue limit properties of the three strips considering the influence of the different cleanliness of the materials. Despite the absence of big inclusions, strip B is penalized by the presence of the highest density of inclusions $< 20 \mu\text{m}$ that are the most detrimental ones: the fatigue properties of the strips are more influenced by the density of a critical dimension of the inclusions rather than by the biggest size.

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