CRACK INITIATION IN CRACKED T-BEAMS

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ABSTRACT

In this paper, the S-theory is applied to determine crack initiation and direction for T-beams. It makes use of a parameter called strain-energy-density factor, *S*, which is a function of the stress intensity factors. A simple method for obtaining approximate stress intensity factors is also applied. It takes into account the elastic crack tip stress singularity while using the elementary beam theory. Basic loading conditions are studied.

SOMMARIO

In questo lavoro, viene impiegata la teoria basata sul "*fattore densità di energia di deformazione*", *S*, per determinare le condizioni iniziali di propagazione in una trave fessurata con sezione a T. Tale parametro dipende dai fattori di intensificazione degli sforzi (Stress intensity factors).
Viene inoltre proposto un metodo approssimato per la valutazione dei fattori di intensificazione degli sforzi, basato sulla condizione di equivalenza tra le caratteristiche della sollecitazione e la distribuzione delle tensioni in corrispondenza della sezione fessurata.

INTRODUCTION

In this paper the method advanced by Nobile [1] is used to determine crack initiation and direction for cracked T-beams .
As well known, Linear Elastic Fracture Mechanics is widely used to describe many aspects of crack behavior. Knowledge of the stress intensity factors plays an important role in fracture control. In structural applications, combined standard loading conditions often involve simultaneously K_I, K_{II} and K_{III} . Within the framework of brittle fracture, the well-known "*Strain energy density factor theory"*[2] allows to predict unstable crack growth in mixed mode. It makes use of a parameter called strain-energy-density factor, *S*, which is a function of the stress intensity factors.
For general loading, the strain energy density factor is

$$S = a_{11}K_I^2 + 2a_{12}K_I K_{II} + a_{22}K_{II}^2 + a_{33}K_{III}^2 \quad (1)$$

where

$$a_{11} = \frac{1}{16\pi\mu}\left[(3-4\nu-\cos\theta)(1+\cos\theta)\right], \quad a_{12} = \frac{1}{16\pi\mu}(2\sin\theta)\left[\cos\theta-(1-2\nu)\right]$$

$$a_{22} = \frac{1}{16\pi\mu}\left[4(1-\nu)(1-\cos\theta)+(1+\cos\theta)(3\cos\theta-1)\right], \quad a_{33} = \frac{1}{4\pi\mu}$$

with ν being the Poisson's ratio and μ the shear modulus of elasticity.
Stress intensity factors for many configurations are available. In most cases the results were obtained by means of analytical and numerical methods. In many cases the results were obtained by finite element methods and boundary element methods . Experimental methods have been applied to simple cases in order to determine the fracture toughness K_{IC} of engineering materials. Solutions for many structural configurations are not available in the handbooks.
Simple engineering methods which allow a fast but approximate determination of the stress intensity factors are highly valued to a design engineering.
Remarkably simple methods for close approximation of stress intensity factors in cracked or notched beams were proposed by Kienzler and Herrmann [3] and by Nobile [1]. The former has been based on elementary beam theory estimation of strain energy release rate as the crack is widened into a fracture band, the latter has been based on elementary beam theory equilibrium condition for internal forces evaluated in the cross section passing through the crack tip, taking in account the stress singularity at the tip of an elastic crack.
In this paper the latter method [1] is employed to compute stress intensity factors and to determine crack initiation and direction for cracked T-beams. Three basic loading conditions have been studied.

APPROXIMATE EVALUATION OF STRESS INTENSITY FACTORS

Consider a straight beam of constant cross section. The *z*-axis coincides with the geometrical axis, and the *x* and *y*-axes coincide with the principal axes of the cross section. The stress components due to stress resultants are well known. Suppose that the presence of an edge crack of initial length *a* doesn't alter the stress resultant on the cross section passing through the crack tip. The singular stress distribution at the crack tip takes the form

$$\sigma_{ij}^{s} = \frac{K_i}{\sqrt{2\pi r}} \qquad (2)$$

with the condition that σ_{ij}^{s} acts at a distance $r=b$ from the tip. The nominal stress is evaluated by the known stress distribution on the reduced solid cross section passing through the crack tip (ligament). The stress distribution doesn't take into account the presence of the crack. Then, the equivalent condition between singular stress and nominal stress resultant at the crack tip determines K_i approximately. Note that K_i-values are better approximated for $b< a$ such that the elastic singularity governs stresses at a distance from the tip lower compared to the geometric dimension of crack length.

BENDING

First consider the case when the beam is subjected to a bending moment M_x. The equivalent condition between singular stress and nominal stress resultant at the crack tip determines K_I approximately:

$$K_I = \frac{M_x}{h^{5/2}} F_I(\alpha, \beta, \delta) \qquad (3)$$

with

$$F_I(\alpha,\beta,\delta) = 2.893\sqrt{\frac{\left(2\beta+1-2\alpha-2\delta-2\alpha\beta+\alpha^2+2\alpha\delta-\beta\delta+\delta^2\right)}{\beta+1-\alpha-\delta}}(2\beta+1-$$

$$-2\alpha-2\delta-2\alpha\beta+\alpha^2+2\alpha\delta-\beta\delta+\delta^2)/(-4\alpha+6\alpha^2\delta^2+4\beta-4\beta\alpha\delta^2-6\beta\delta\alpha^2+$$

$$+12\beta\delta\alpha-4\delta+6\alpha^2+4\alpha^3\delta+4\delta^3\alpha+\delta^2\beta^2-2\beta\delta^3-4\beta\alpha^3-12\delta^2\alpha-12\delta\alpha^2+$$

$$+12\beta\alpha^2+4\beta\delta^2-4\alpha^3-4\delta^3+\alpha^4+\delta^4-12\alpha\beta+12\alpha\delta-6\beta\delta+1+6\delta^2)\delta$$

where $\alpha = a/h, \beta = B/h$ and $\delta = d/h$. A plot of normalized stress intensity factor as a function of *a/h* for different values of β and δ is shown in Fig.1.

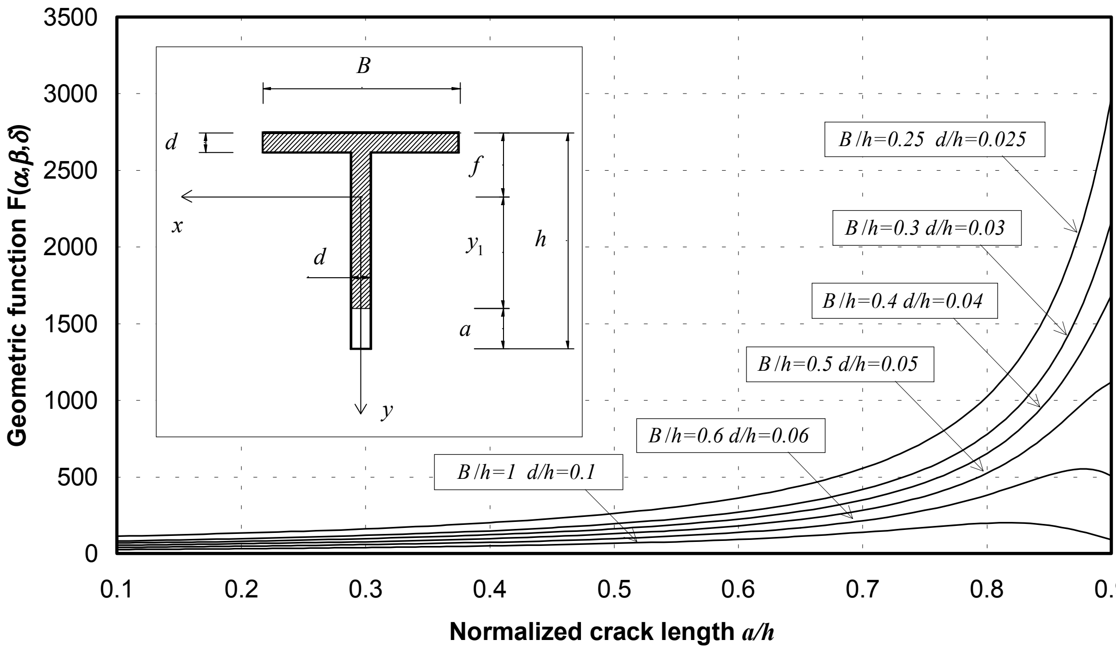

Figure 1: Stress-intensity factor vs. a/h for T-beam in bending.

Information on the direction of crack initiation can be obtained from the strain energy criterion [1]. It is assumed that the crack tends to run in the direction of S_{min}. Setting $K_{II}=0$ and $K_{III}=0$ in eq.(1) and putting eq.(3) in eq .(1), S can be obtained. The relative minimum of S corresponds to $\theta=0$. A plot of normalized strain-energy-factor as a function of θ is shown in Fig.2.

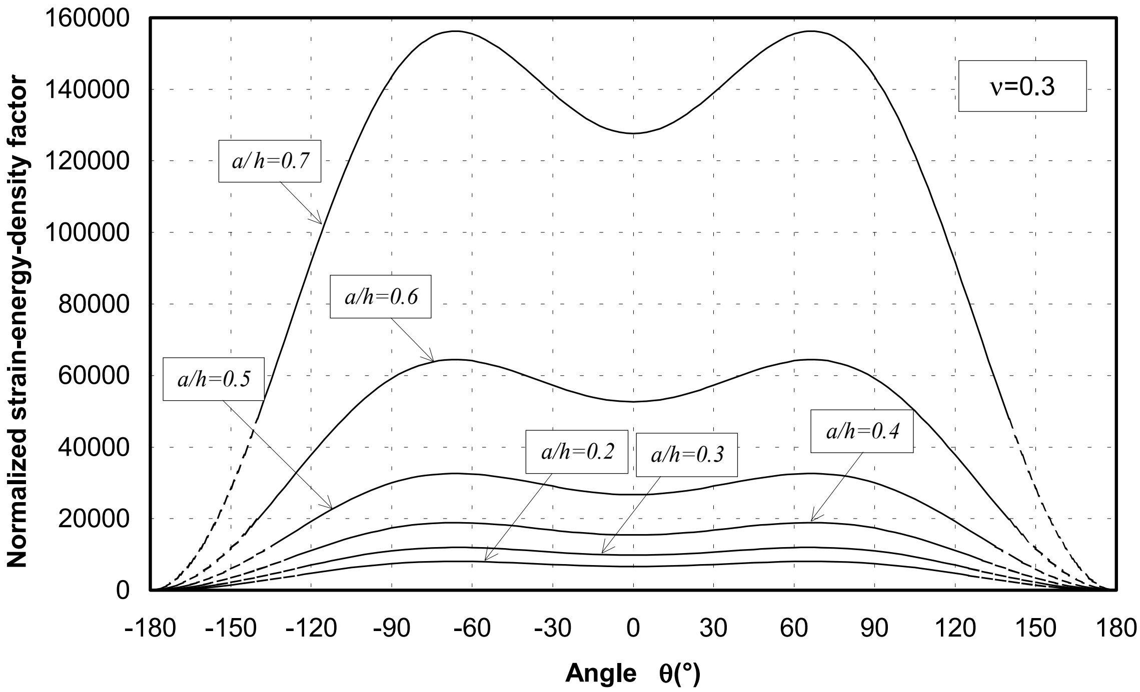

Figure2:Normalized strain energy-density factor vs. angle θ.

SHEAR LOADING

Consider the case when the beam is subjected to a shear loading V_y. The equivalent condition between singular stress and nominal stress resultant at the crack tip determines K_{II} approximately:

$$K_{II} = \frac{V_y}{h^{3/2}} F_2(\alpha,\beta,\delta) \qquad (4)$$

with

$$F_2(\alpha,\beta,\delta)=10.632\sqrt{\frac{\alpha\left(3\delta^2-6\delta-3\beta\delta+2\delta\alpha-2\alpha-2\beta\alpha+3+6\beta\right)}{\delta^2+\delta\alpha-2\delta-\beta\delta-\alpha+2\beta-\beta\alpha+1}}\ \alpha\,(+\delta^2+\delta\alpha+$$

$$-2\delta-\beta\delta-\alpha+2\beta-\beta\alpha+1)/\delta(\delta^4-4\delta^3-2\beta\delta^3+\beta^2\delta^2+4\beta\delta^2+6\delta^2-4\delta-$$

$$-6\beta\delta+1+4\beta)$$

A plot of normalized stress intensity factor as funcion of a/h for different values of β and δ is shown in Fig.3.

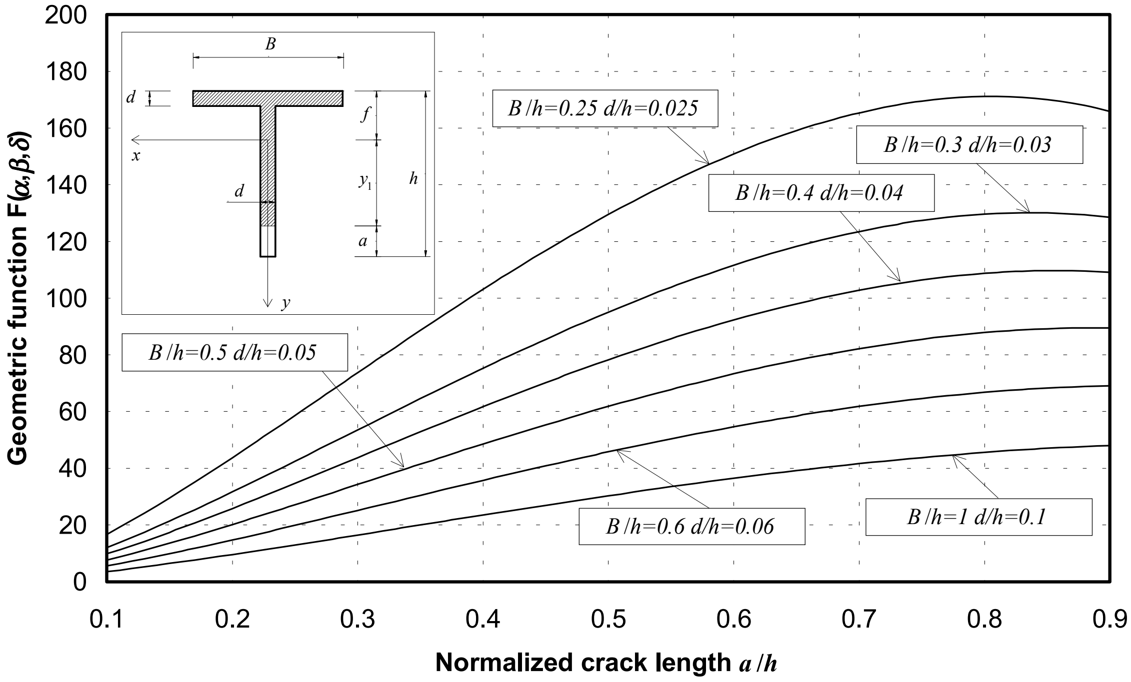

Figure 3: Stress-intensity factor vs. a/h for T-beam in shear.

Information on the direction of crack initiation can be obtained from the strain energy criterion [1]. It is assumed that the crack tends to run in the direction of S_{min}. Setting $K_I=0$ and $K_{III}=0$ in eq.(1) and putting eq.(4) into eq .(1), S can be obtained. The relative minimum of S corresponds to $\theta=\cos^{-1}[(1-2\nu)/3]$. A plot of normalized strain-energy-factor as a function of θ is shown in Fig.4.

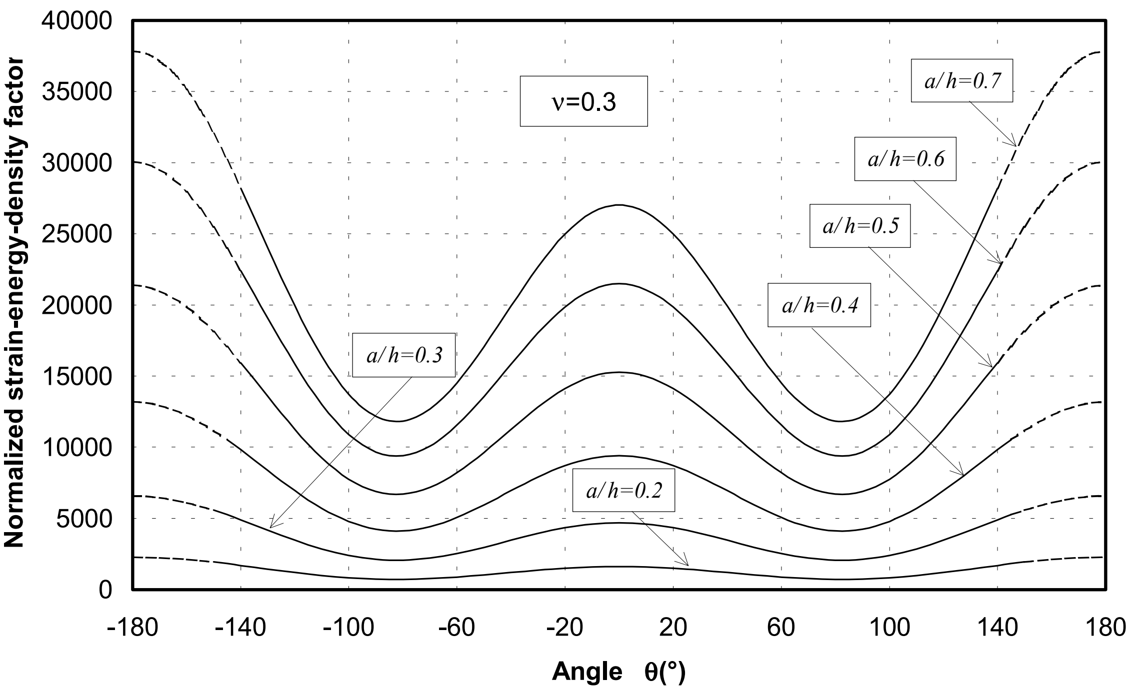

Figure4:Normalized strain energy-density factor vs. angle θ.

TORSION

Consider the case when the beam is subjected to a torque T. The equivalent condition between singular stress and nominal stress resultant at the crack tip determines K_{III} approximately:

$$K_{III} = \frac{T}{h^{5/2}} F_3(\alpha,\beta,\delta) \qquad (5)$$

with

$$F_3(\alpha,\beta,\delta) = 7.519\sqrt{\frac{\alpha}{(\beta+1-\alpha-\delta)(\beta+1-\delta)}}\,\frac{1}{\delta^2}$$

A plot of normalized stress intensity factor as funcion of a/h for different values of β and δ is shown in Fig.5.

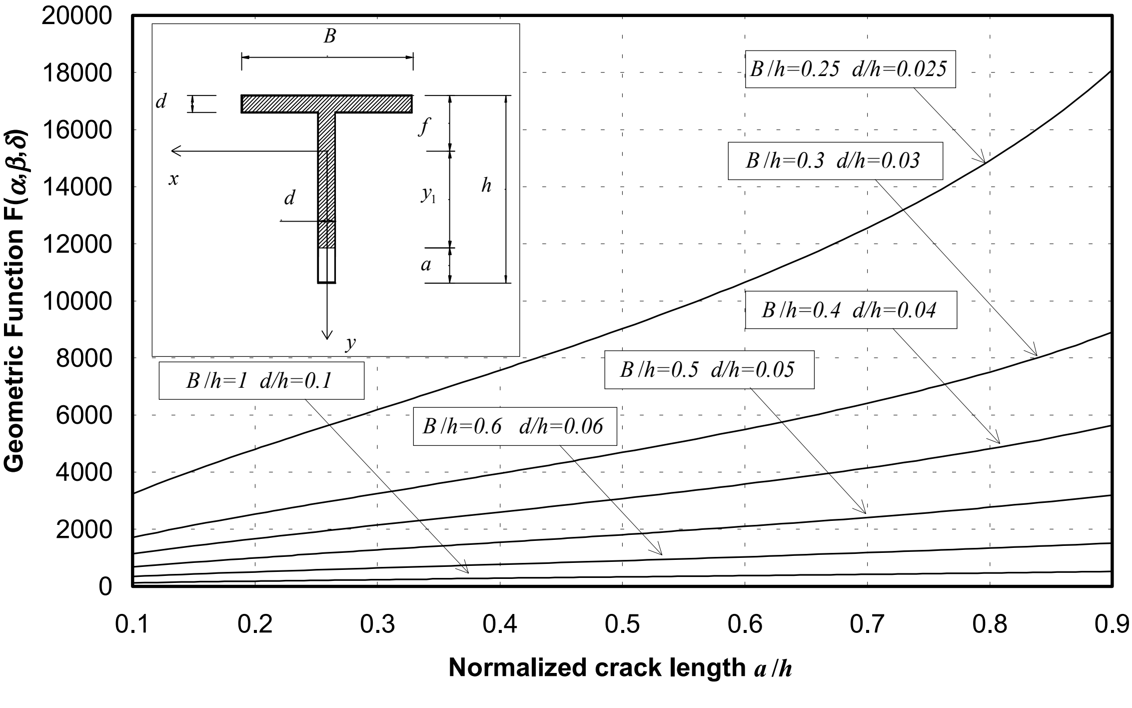

Figure 5: Stress-intensity factor vs. a/h for T-beam in torsion.

MIXED MODE

Consider the case where M_x and V_y are present. Setting $K_{III}=0$ in eq.(1) and putting eqs.(3) and (4) into eq.(1), S can be obtained as

$$S = \frac{M_x^2}{h^5}\left[F_1^2 a_{11} + F_2^2 \alpha^2 a_{22} + 2F_1F_2\alpha a_{22}\right] \tag{6}$$

with $\alpha = \frac{V_y h}{M_x}$.

Assume that the crack would initiate in the direction of S_{min} , i.e. $\frac{\partial S}{\partial \theta} = 0$. This corresponds to the direction where dilatation would dominate. Two plots of normalized strain-energy-density factors as a function of θ for constant ν and different values of α are shown in Figs. 6 and 7. Crack instability is then assumed to take place when S_{min} equal to a critical value S_c that depends only on the material.

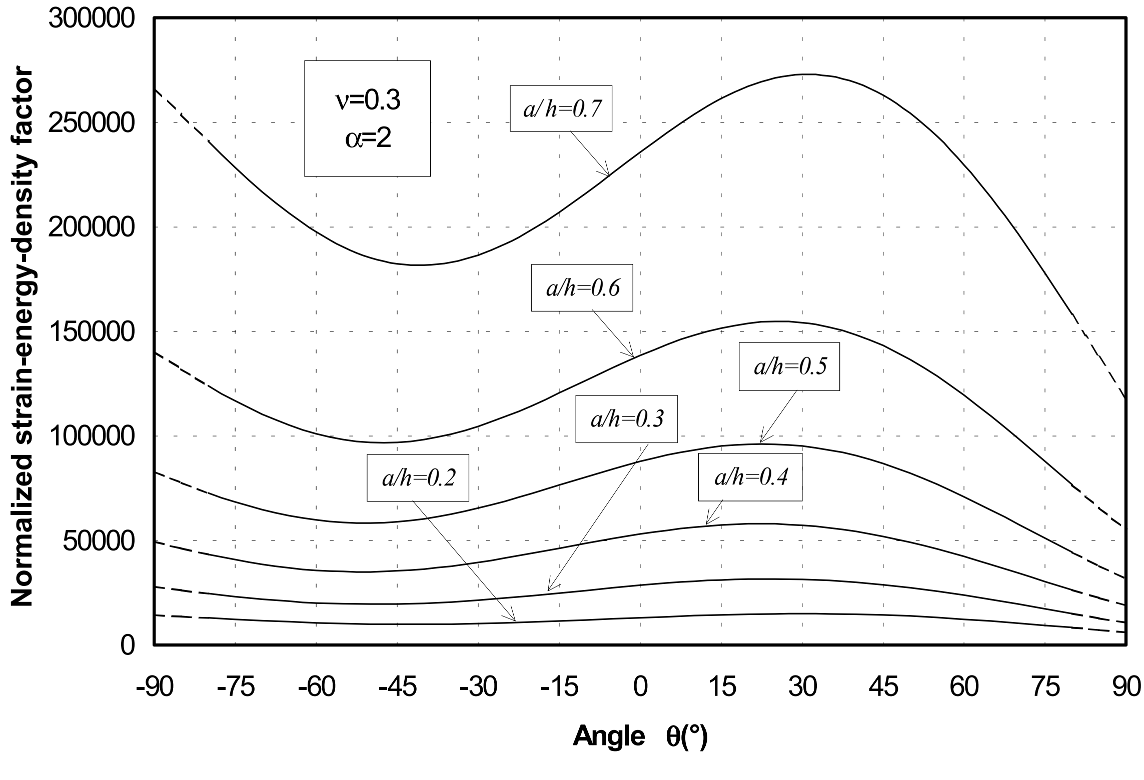

Figure6:Normalized strain energy-density factor vs. angle θ.

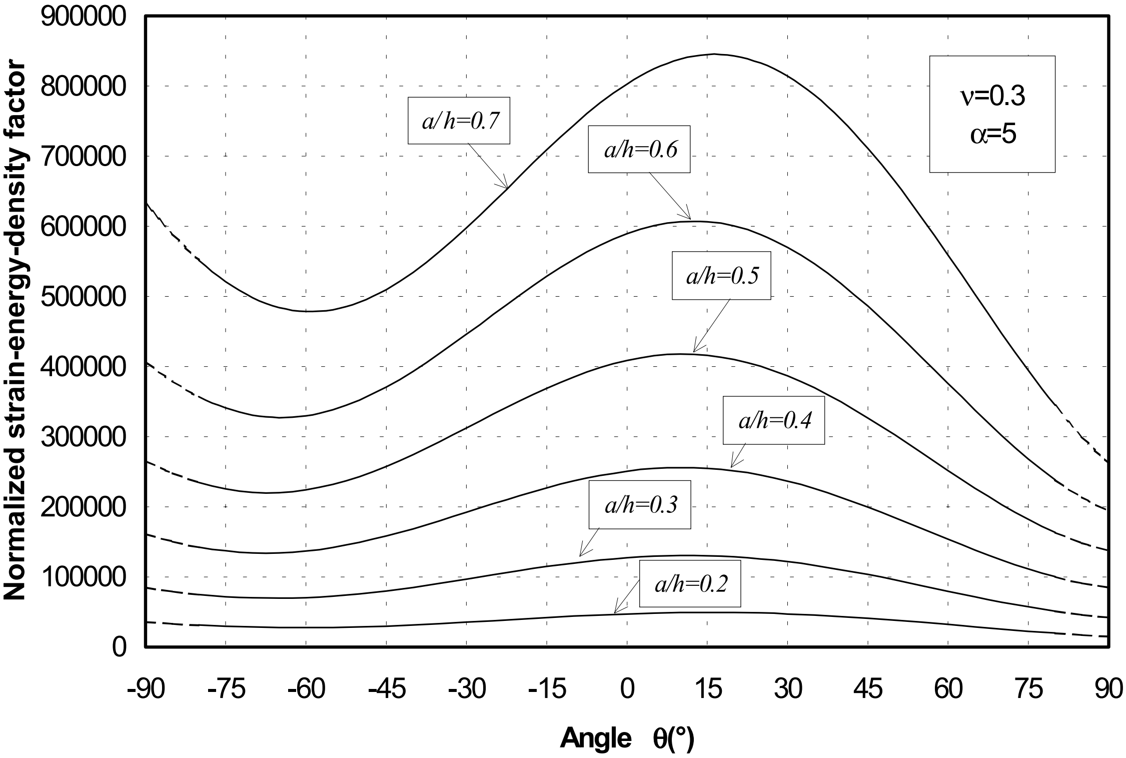

Figure7:Normalized strain energy-density factor vs. angle θ.

Acknowledgements

The authors gratefully acknowledge the financial support rendered by the Italian Ministry of University.

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