A STRESS-BASED PLASTIC DAMAGE MODEL FOR CONCRETE-LIKE MATERIALS

S. MARFIA[1], E. SACCO[1], N. VALOROSO[2]

[1] *Dipartimento di Meccanica, Strutture, Ambiente e Territorio, Università di Cassino, Cassino*
[2] *Istituto per le Tecnologie della Costruzione, Consiglio Nazionale delle Ricerche, Roma*

SOMMARIO

Si presenta un modello elastoplastico con danno per lo studio delle risposta meccanica dei materiali lapidei. Il degrado del materiale viene simulato attraverso l'introduzione di due variabili scalari di danno la cui evoluzione è governata dalla deformazione elastica. In accordo con l'evidenza sperimentale, si assume inoltre che le deformazioni irreversibili (plastiche) si manifestino per stati tensionali di prevalente compressione; la loro evoluzione viene descritta introducendo la funzione di snervamento e le leggi di flusso in termini di variabili di tensione effettiva. Tale scelta consente una notevole semplificazione nella implementazione numerica in quanto gli incrementi delle variabili interne connesse con l'evoluzione del fenomeno plastico possono essere valutati indipendentemente da quelli dalle variabili di danno mediante una tecnica di splitting ed un algoritmo di return mapping. Allo scopo di illustrare le caratteristiche del modello si presentano due applicazioni numeriche.

ABSTRACT

A plastic damage model for concrete-like materials is presented. Material degradation is modelled introducing two scalar damage variables whose evolution is driven by the elastic strain. The spread of irreversible (plastic) deformations which, according to experimental evidence, are significant for mainly compressive stress states, is described by introducing a suitable yield function and evolution laws for the plastic strain and the kinematic internal variables which are postulated in terms of effective stress-like variables. This allows one to greatly simplify the numerical implementation of the proposed model since the evolution of plasticity can be computed independently from that of damage based upon an elastic-plastic-damage operator splitting technique and a return mapping scheme. Numerical simulations are provided for evaluating the basic features of the proposed model.

1. PLASTIC-DAMAGE MODEL

In recent years a growing attention has been devoted to the modelling of concrete and geomaterials and a number of descriptions accounting for the sharp tensile-compressive asymmetric response typically exhibited by this class of materials have been proposed within the context of damage mechanics and plasticity-based formulations, see e.g. [1, 2, 3, 4].

Even though a rank-four tensor should be used to describe full states of damage, scalar damage models are useful for practical applications because of their simplicity; moreover, a description of damage based on the introduction of scalar variables can be significantly improved if irreversible (plastic) deformations are introduced.

In the framework of linearized kinematics a plastic-damage model can be derived based on a stored energy function introduced in decoupled form as:

$$\psi(\mathbf{e},\zeta,D_t,D_c)=\psi_e(\mathbf{e},D_t,D_c)+\psi_h(\zeta,\mathbf{p}) \tag{1}$$

where ε is the infinitesimal strain measure, characterized by the decomposition:

$$\varepsilon=\mathbf{e}+\mathbf{p} \tag{2}$$

being $\mathbf{e}$ and $\mathbf{p}$ the elastic and plastic shares, ζ a scalar strain-like internal variable while D_t, D_c denote two damage variables which account for stiffness degradation for stress states of prevailing tension and compression, respectively [2].

In the present formulation the functions $\psi_e(\mathbf{e},D_t,D_c)$ and $\psi_h(\zeta,\mathbf{p})$, which represent the elastic-damage energy and the plastic hardening potential, are introduced as follows :

$$\begin{aligned}
&\psi_e(\mathbf{e},D_t,D_c)=\frac{1}{2}(1-D_s)[2G\,\mathrm{tr}(\mathrm{dev}(\mathbf{e}))^2+K<\mathrm{tr}(\mathbf{e})>_+^2+1/(1-D_t)K<\mathrm{tr}(\mathbf{e})>_-^2]\\
&\psi_h(\mathbf{q},\zeta)=\tfrac{1}{2}h_{iso}\,\zeta^2+(k^\infty-k^o)[\zeta+\tfrac{1}{\gamma}\exp{(-\gamma\zeta)}]+\tfrac{1}{2}h_{kin}\mathrm{tr}(\mathbf{p}^2)
\end{aligned} \tag{3}$$

where the symbols $<\cdot>_+$ and $<\cdot>_-$ indicate the positive and negative part of $<\cdot>$, defined as $<x>_\pm=1/2(x\pm|x|)$, K and G are the elastic bulk and shear moduli, h_{iso} and h_{kin} denote the plastic hardening moduli, k^∞ and k^o are the plastic yield stresses and it has been set $(1-D_s)=(1-D_t)(1-D_c)$.

Use of the standard thermodynamic argument allows one to identify the state laws for the stress $\boldsymbol{\sigma}$ and the thermodynamic forces $\mathbf{q}$ and θ conjugate to the internal variables $\mathbf{p}$ and ζ as:

$$\begin{cases}
\boldsymbol{\sigma}=(1-D_s)2G(\mathrm{dev}\mathbf{e})+(1-D_s)K<\mathrm{tr}(\mathbf{e})>_+\mathbf{1}+(1-D_c)K<\mathrm{tr}(\mathbf{e})>_-\mathbf{1}\\
\mathbf{q}=h_{kin}\mathbf{p}\\
\theta=h_{iso}\,\zeta+(k^\infty-k^o)[1-\exp{(-\gamma\zeta)}]
\end{cases} \tag{4}$$

where $\mathbf{1}$ is the rank-two identity tensor.

Since local stresses act only on the undamaged material, a well-grounded conjecture is that plastic strains should develop on the intact matrix material and not to the voids in between, i.e. they should be driven by the effective stresses [5].

Following this assumption, the evolution of plastic deformations can be described by introducing a yield function of the form:

$$\phi(\hat{\boldsymbol{\sigma}},\mathbf{q},\theta)=\hat{\phi}(\hat{\boldsymbol{\sigma}}-\mathbf{q})-Y(\theta) \tag{5}$$

where $Y(\theta)$ denotes the instantaneous yield limit and $\hat{\boldsymbol{\sigma}}$ is the effective stress tensor, for which the following expression is taken:

$$\hat{\boldsymbol{\sigma}} = 2G(\text{deve}) + K(<\text{tr}(\text{e})>_+ + <\text{tr}(\text{e})>_-)\mathbf{1} \tag{6}$$

The flow equations for the plastic variables are assumed as:

$$\begin{cases} \dot{\text{p}} &= \dot{\lambda}_p \mathbf{d}_{\hat{\boldsymbol{\sigma}}} \phi \\ \dot{\zeta} &= -\dot{\lambda}_p \mathbf{d}_\theta \phi \end{cases} \tag{7}$$

which are non-associated in character since they express normality only in the effective stress space. In the previous equations $\dot{\lambda}_p$ represents the continuum plastic consistency parameter satisfying the loading/unloading conditions in Kuhn-Tucker form:

$$\phi(\hat{\boldsymbol{\sigma}}, \theta) \leq 0; \qquad \dot{\lambda}_p \geq 0; \qquad \dot{\lambda}_p \phi(\hat{\boldsymbol{\sigma}}, \theta) = 0. \tag{8}$$

The characterization of damage adopted in the present work originates from the one proposed by Comi and Perego in [2]. In particular, the damage surface in stress space is defined through two limit fuctions accounting for compressive ad tensile damage as:

$$f_t = J_2(\sigma) - a_t I_1(\sigma) + b_t h_t(D_t) I_1(\sigma)(1-\alpha D_c) - k_t h_t^2(D_t)(1-\alpha D_c)^2 \tag{9}$$

$$f_c = J_2(\sigma) + \left[a_c I_1(\sigma) + b_c h_c(D_c) I_1(\sigma) - k_c h_c^2(D_c)\right](1-\beta D_t)^2 \tag{10}$$

with

$$h_i(D_i) = \begin{cases} 1 - \left[1 - \frac{\sigma_{ei}}{\sigma_{0i}}\right]\left(1 - \frac{D_i}{D_{0i}}\right)^2 & \text{for} \quad D_i < D_{0i} \\ \left[1 - \left(\frac{D_i - D_{0i}}{1 - D_{0i}}\right)^{c_i}\right]^{0.75} & \text{for} \quad D_i \geq D_{0i} \end{cases} \tag{11}$$

where a_t, b_t, k_t, a_c, b_c, k_c, c_i, α and β are material parameters , I_1 is the first stress invariant, J_2 is the second deviatoric invariant, σ_{ei} and σ_{0i} are the elastic limit and the peak stress in an uniaxial test, while the subscript i stands for tension $i = t$ or compression $i = c$.

Growth of damage is associated with the attainment of the limit surface; accordingly, the kinematics of damage is completely characterized by the relationships:

$$\begin{cases} \dot{D}_t \geq 0; & f_t \leq 0; & \dot{D}_t f_t = 0 \\ \dot{D}_c \geq 0; & f_c \leq 0; & \dot{D}_c f_c = 0 \end{cases} \tag{12}$$

The outlined equations lead to an initial value problem for the effective stress-like variables which, can be recast in the form:

$$\begin{cases} \dot{\hat{\boldsymbol{\sigma}}}(t) = 2G(\text{dev}\dot{\varepsilon}(t)) + K(<\text{tr}(\dot{\varepsilon}(t))>_+ + <\text{tr}(\dot{\varepsilon}(t))>_-)\mathbf{1} - \dot{\lambda}_p \mathbf{d}_{\hat{\boldsymbol{\sigma}}} \phi(\hat{\boldsymbol{\sigma}}(t), \theta(t)) \\ \dot{\theta}(t) = \dot{\lambda}_p \left[h_{iso} + (k^\infty - k^o)\gamma \exp\left(-\gamma\zeta(t)\right)\right] \end{cases} \tag{13}$$

From the computational standpoint, for all $t \geq t_0$ it is assumed that the strain history $t \to \dot{\varepsilon}$ is a given function of time; accordingly, the stress update algorithm will be *strain driven*.

As first shown in [6], the updating procedure for the above problem can be defined based upon an *elastic - plastic - damage* split in which the damage corrector part of the algorithm is implemented separately from the plastic corrector part. This is made possible by the fact that, in the present context, the plastic response can be computed independently from the damage evolution in terms of the effective stress variables, so that the standard return mapping algorithms for plasticity turn out to be directly applicable.

2. NUMERICAL APPLICATIONS

In this section we report some representative numerical simulations for evaluating the model response under cyclic loading. For the analyses carried out in the sequel we consider the following expression for the yield function:

$$\phi(\hat{\boldsymbol{\sigma}}, \mathbf{q}, \theta) = (\hat{\boldsymbol{\sigma}} - \mathbf{q})\mathbf{n} \cdot \mathbf{n} - (k^o + \theta) \leq 0 \tag{14}$$

where $\mathbf{n}$ is a unit vector, and the following choice for the model parameters [2]:

$$\begin{array}{llll} E = 31000\text{MPa} & \nu = 0.1 & & \\ \sigma_{et} = 2.08\text{MPa} & \sigma_{0t} = 2.66\text{MPa} & \sigma_{ec} = 26.6\text{MPa} & \sigma_{0c} = 38.0\text{MPa} \\ D_{0t} = 0.1 & D_{0c} = 0.3 & \alpha = 0.8 & \beta = 1.0 \\ c_t = 2.0 & c_c = 2.0 & & \\ a_t = 0.3333\text{MPa} & b_t = 3.5\text{MPa} & k_t = 9.1\text{N}^2/\text{mm}^4 & \\ a_c = 0.00025\text{MPa} & b_c = 3.0\text{MPa} & k_c = 370.0\text{N}^2/\text{mm}^4 & \\ h_{iso} = 10000\text{MPa} & h_{kin} = 10000\text{MPa} & k^0 = 25\text{MPa} & k^\infty = 25\text{MPa} \end{array} \tag{15}$$

The material response under uniaxial tensile-compressive loading is depicted in figure 1, which corresponds to the following loading program:

time [s]	0	1	2	3	4	5	6
strain ε_{11}	0	0.0002	0	-0.006	-0.00027	-0.01	-0.0055

(16)

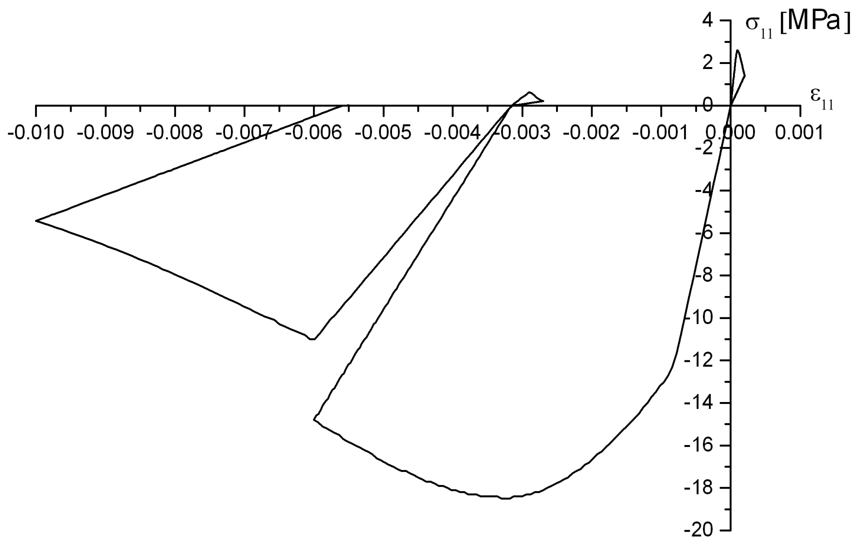

Figure 1: Material response under tensile-compressive loading.

Figure 1 shows the ability of the model to describe the unilateral phenomenon, i.e the partial crack closure subsequent to unloading in tension and reloading in compression.

The material response under pure shear loading is plotted in figure 2. In particular, the following loading program is considered:

time [s]	0	1	2	3	4
strain ε_{12}	0	-0.0008	0	0.001	0

(17)

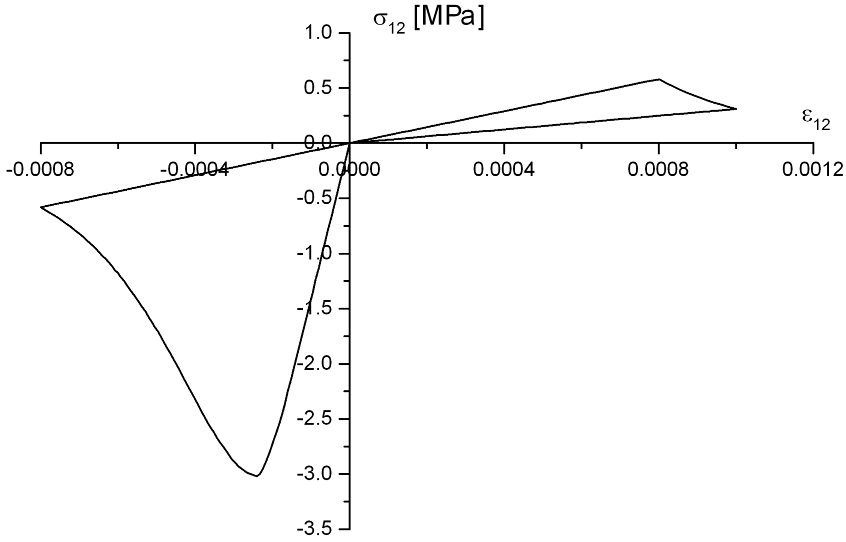

Figure 2: Material response under pure shear loading.

It is worth noting that, as a consequence of the form of the adopted yield function, no residual strain is exhibited during unloading, so that in this case the material behaviour turns out to be quasi-brittle.

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