Crack analogue in fretting fatigue. New results

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Abstract

Giannakopoulos et al (1998, 2000) have recently proposed analogies between the stress ...eld induced in fretting contact situations, with those of a crack. Here, improvements of the fretting "crack analogue" are given considering full-sliding and partial slip conditions, and analyse the correct e¤ect of bulk stress.

1 Introduction

Fretting Fatigue (FF) has been found almost 100 years ago (Eden et al., 1911, Tomlinson, 1927), but has mostly been seen, until recently, as a "separate" area of fatigue, where the mechanical damage over the surface was considered to have a dominant role in decreasing the fatigue performance of the material. Therefore, parameters as microslip amplitude and surface energy dissipated by friction were considered (Nishioka and Hirakawa, 1969, Nowell and Hills, 1990), but their determination remained empirical and unrelated to more classical fatigue literature. More recently, the role of the contact stress ...eld in provoquing fatigue from a stress raiser feature, has been recognized more in details, and indeed a crack analogue model for the case where the contact is complete (singular pressure and frictional shear tractions) and a notch analogue for cases where we expect a smooth transition to zero pressure at the contact area edges, and correspondingly a ...nite stress concentration, have been proposed (Giannakopoulos et al., 1998, 2000). It was recognized that the stress ...eld induced by the contact is very similar to the square-root singular stress ...eld around an external crack — the singular stress ...eld can be quanti...ed by a stress intensity factor, and the bulk stress in the contacting materials becomes a T ¡ stress in the fracture mechanics terminology. Cracks developed at the contact site are kinked cracks, and the condition of initiation is rather a condition for non-propagation over stress intensity factors amplitudes $\Delta K < \Delta K_{th}$.

However, in the original crack analogue perfect stick is assumed, for simplicity, and no detailed analysis was produced on the possible conditions for microslip to arise, depending on applied loads (and in particular on the effect of bulk loads into one of the contacting bodies). In the present paper, this assumption is removed. The effects on the predicted stress intensity factors, on the complete stress field, and on the consequences for fretting fatigue life methodology, are all analyzed.

2 Crack Analogue (CA)

Many fretting conditions may be idealized as a first approximation as square-ended feet pressing over a fatigue specimen. Consider therefore the geometry in Fig.1.

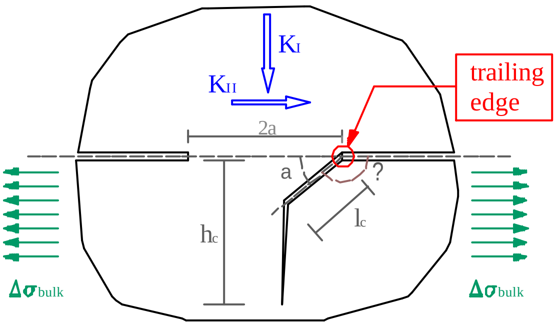


Figure 1: The CA model a flat punch under normal load

For a constant mode I load and a varying mode II load (constant normal load P and varying Q), we have (for a 2D geometry)

$$K_I = -\frac{P}{\sqrt{\pi a}} = -\frac{2}{\pi}\bar{p}\sqrt{\pi a}; \quad \Delta K_{II} = \pm\frac{Q}{\sqrt{\pi a}} = \pm\frac{2}{\pi}\bar{q}\sqrt{\pi a} \tag{1}$$

where $\bar{p} = P/2a$ and $\bar{q} = Q/2a$.

The contact problem in Fig. 1 is governed by the equation (Hills et al., 1993)

$$\frac{g'(x)}{A} = -\frac{1}{\pi}\int_{-a}^{a}\frac{q(t)}{x-t}dt - \frac{\sigma_b}{2\gamma} \tag{2}$$

where Coulomb's friction law implies conditions on the relative tangential displacements function $g'(x)$, A is the "composite compliance" of the bodies

$$A = \frac{2(1-\nu_1^2)}{E_1} + \frac{2(1-\nu_2^2)}{E_2} = 2\left(\frac{1}{E_1^*} + \frac{1}{E_2^*}\right); \tag{3}$$

We recollect here that we have assumed $\beta = 0$, and $\gamma = \left(\frac{E_2^*}{E_1^*} + 1\right)$; i.e. $\gamma = 2$ for identical materials, but $\gamma = 1$ for the only case where the treatment is strictly rigorous, of rigid puch and incompressible material. Once shear traction are determined, the resulting surface stress in the half-plane can be obtained as

$$\sigma_{xx}(x;0) = p(x) + \frac{2}{\pi}\int_{-a}^{a}\frac{q(t)}{x-t}dt + \sigma_b \tag{4}$$

which we will be using in the various cases – notice that here γ has no effect, whereas it has effect on the determination of the stick region, as in eqt. (2).

2.1 Tangential load only

The shear traction are given by the solution of the integral equation above (2), where $g'(x)$ depends on relative displacements in tangential direction, in the following manner: when a tangential load Q is applied sequentially to the normal load only, obviously $g'(x) = 0$ in the stick region. The equation is the same found for the normal load only, so the solution is as correctly derived in the original CA model. This solution in fact satisfies Coulomb's law for friction, and in particular $|q(x)| \leq f\,|p(x)|$ in the entire contact area, as long as $|Q| \leq f\,|P|$. At this limit, there is suddenly full sliding in the entire contact area.

Near the contact edge, the asymptotic form for σ_{xx} is, for example for the right edge (and symmetrically for the left edge)

$$\begin{aligned}
\sigma_{xx}(x \to a^{+}) &= \frac{2Q}{\pi\sqrt{x^2 - a^2}} = 2\frac{K_{II}}{\sqrt{2\pi r}} \\
\sigma_{xx}(x \to a^{-}) &= -\frac{P}{\pi\sqrt{x^2 - a^2}} = \frac{K_I}{\sqrt{2\pi r}}
\end{aligned} \tag{5}$$

as predicted by the CA model, where K_I and K_{II} are defined in (1):

2.2 Bulk stress only

In the case of normal force P and bulk load σ_b the solving equation in the hypothesis of full stick gives

$$q(x) = \frac{\sigma_b/2\gamma}{\pi\sqrt{a^2 - x^2}}\int_{-a}^{a}\frac{\sqrt{a^2 - t^2}}{x-t}dt = \frac{x/a}{\sqrt{1-(x/a)^2}}\frac{\sigma_b}{2\gamma} \tag{6}$$

which holds if $|q(x)| \leq f\,p(x)$ or

$$\frac{|x|/a}{\sqrt{1-(x/a)^2}}\frac{\sigma_b}{2\gamma} \leq \frac{2}{\pi}\frac{f\,\bar{p}}{\sqrt{1-(x/a)^2}} \tag{7}$$

Hence, there is complete stick only for small bulk load when

$$\sigma_b \le \frac{4}{\pi} \gamma f \bar{p} \tag{8}$$

For larger bulk loads, if $\sigma_b \ge \frac{4}{\pi} \gamma f \bar{p}$, two slip zone take place next to the contact edge in symmetrical position, and the solution of the integral equation is obtained using a procedure similar to Spence's solution (Spence, 1971). We find

$$q(x) = \begin{cases} q^*(x); & |x| \le b \\ f p(x) \operatorname{sign}(x); & b \le |x| \le a \end{cases} \tag{9}$$

where b is the semidimension of the stick area, given by the consistency condition

$$K'(b/a) = \frac{\pi^2 \sigma_b}{8 \gamma f \bar{p}} \tag{10}$$

where $K'(b/a) = K(1 - (b/a)^2)$, and $K(*)$ is the complete elliptic integral of the second kind. In the figure 3 the stick area semiwidth is plotted as a function of dimensionless bulk load $\sigma_b / \frac{4}{\pi} \gamma f \bar{p}$ and it can be noticed that for $\sigma_b / \frac{4}{\pi} \gamma f \bar{p} \le 1$ the entire contact area is in full stick.

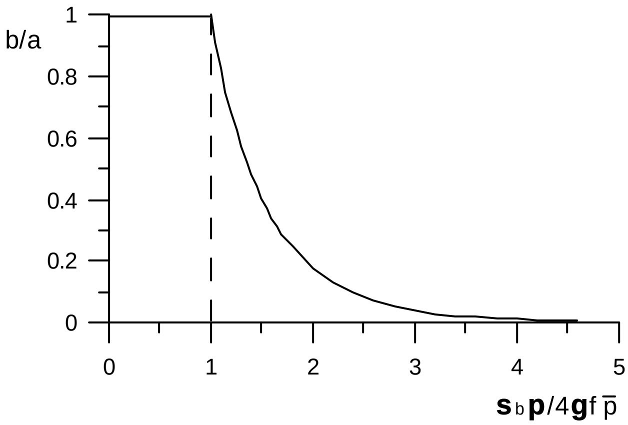
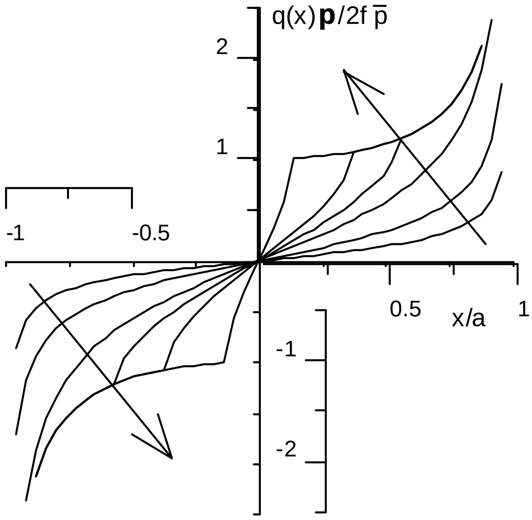


Figure 3: stick area semiwidth as a fuction of bulk load

In figure 4 the tangential tractions are plotted for different values of bulk load $\sigma_b / \frac{4}{\pi} \gamma f \bar{p}$: The K_{II} factors in such case also define the asymptotic stresses in the regions near the edges, and are found as

$$K_{II} = \pm \frac{\sqrt{\pi a} \sigma_b}{2 \gamma}; \qquad \sigma_b \le \frac{4}{\pi} \gamma f \bar{p}$$

$$K_{II} = \pm f K_I; \qquad \sigma_b \ge \frac{4}{\pi} \gamma f \bar{p} \tag{11}$$



$\frac{\pi \sigma_b}{4\gamma f \bar{p}}$ =0.25,0.5,1.0,1.25,1.5,2.0

Figure 4: tangential tractions for several values of bulk stress

2.3 Tangential & bulk load

In the case of tangential and bulk load applied simultaneously, as it is typical of fretting fatigue, the solution of integral equation(2) is

$$q(x) = \frac{x/a}{\sqrt{1-(x/a)^2}}\frac{\sigma_b}{2\gamma} + \frac{Q}{\pi a\sqrt{1-(x/a)^2}} \tag{12}$$

in the assumption complete stick situation, when $|q(x)| \le f p(x)$ in the entire contact region.

Accordingly, we can write that (12) is the correct solution if the following condition is satisfied

$$\frac{\sigma_b}{2\gamma} + \frac{Q}{\pi a} \le f\frac{P}{\pi a} \quad \Rightarrow \quad \sigma_b \le \frac{4}{\pi}\gamma f \bar{p}\left(1 - \frac{Q}{fP}\right) \tag{13}$$

If the condition is not satisfied we can expect the slipping region to be next to the left edge, in opposite direction with respect to the tangential load Q. We can therefore write the tangential traction as the sum of a component of complete sliding $f p(x)$ and a corrective contribute $q^*(x)$, different from zero only in the stick area. If we indicate b as the coordinate of left edge of the stick area, we find

$$q(x) = \begin{cases} f|p(x)| - \frac{\sigma_b}{2\gamma}\sqrt{\frac{b-x}{a+x}}; & -a \le x \le b \\ f|p(x)|; & b \le |x| \le a \end{cases}$$

where, from the equilibrium condition,

$$b = \frac{4\gamma(fP - Q)}{\pi\sigma_b} - a \qquad (14)$$

If the bulk load is large enough

$$\frac{fP}{\pi a\sqrt{1-(x/a)^2}} - \frac{\sigma_b}{2\gamma}\sqrt{\frac{b-x}{a+x}} \approx -\frac{fP}{\pi a\sqrt{1-(x/a)^2}} \qquad x \to -a \qquad (15)$$

and so

$$\sigma_b \geq \frac{4}{\pi}\gamma f\bar{p}\frac{1}{1-Q/fP} \qquad (16)$$

there is also slip next to the right corner.

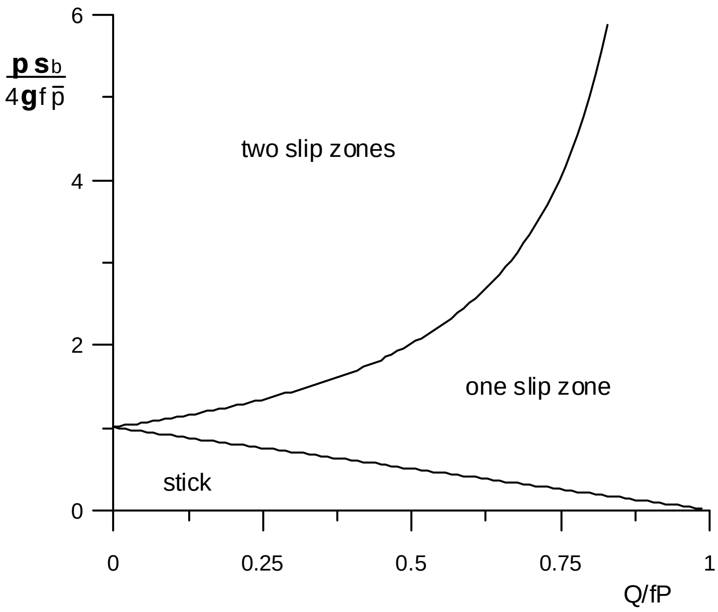


Figure 7: stick zone boundaries according to the load conditions

Hence, for bulk load larger than the above limit the only approach at the moment is to develop a numerical method to find the tractions, but having a knowledge of the asymptotic stresses (from the sliding condition at both ends) there is no need for this. Finally, we have three possibly situations according to the bulk load, as we can see in the figure 7:

Accordingly, the K_{II} factors in general case of bulk and tangential load applied for $\sigma_b \leq \frac{4}{\pi}\gamma f\bar{p}(1 - Q/fP)$ becomes

$$K_{II} = \frac{\sqrt{\pi a}\,\sigma_b}{2\gamma} + \frac{Q}{\sqrt{\pi a}} \qquad x = a$$

$$K_{II} = -\frac{\sqrt{\pi a}\,\sigma_b}{2\gamma} + \frac{Q}{\sqrt{\pi a}} \qquad x = -a \qquad (17)$$

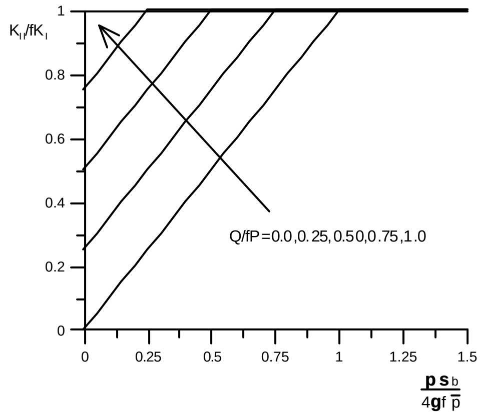
whereas for $\frac{4}{\pi}\gamma f\sqrt{p}(1-Q/fP) \le \sigma_b \le \frac{4}{\pi}\gamma f\sqrt{p}/(1-Q/fP)$

$$K_{II} = fK_I; \qquad x = a$$
$$K_{II} = fK_I - \frac{\sqrt{2\pi\sigma_b}}{2\gamma}\sqrt{\frac{4\gamma(fP-Q)}{\pi\sigma_b}}; \quad x = -a \tag{18}$$

and finally for $\sigma_b \ge \frac{4}{\pi}\gamma f\sqrt{p}/(1-Q/fP)$

$$K_{II} = fK_I; \qquad x = a$$
$$K_{II} = -fK_I; \quad x = -a \tag{19}$$

Figure 8 and figure 9 give the variation of the mode II stress intensity factor as a function of bulk and tangential loads, at the trailing and leading edge, respectively: the latter is given only for completeness (as the trailing edge value is always greater), and notice that it varies sign, for large enough bulk loads.



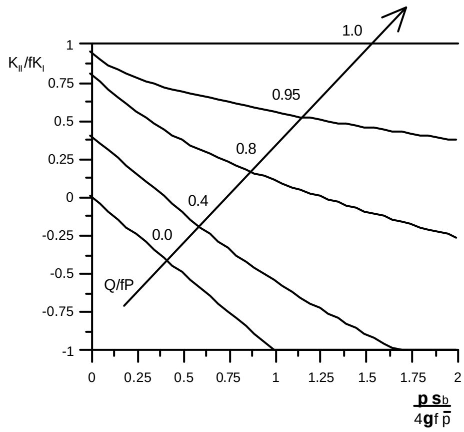


Figure 8: SIF ratio in the trailing edge — Figure 9: SIF ratio in the leading edge

The new factors are compared with the previous CA model K_{II}, where only the contribute due to the tangential load was taken into account

$$K_{II} = (K_{II})_Q = \frac{Q}{\sqrt{\pi a}}$$

The difference is reported in figure 10. Clearly, the error is larger for large bulk loads and low Q/fP, i.e. away from the friction limit, when the e¤ect of bulk load dominates over the tangential load.

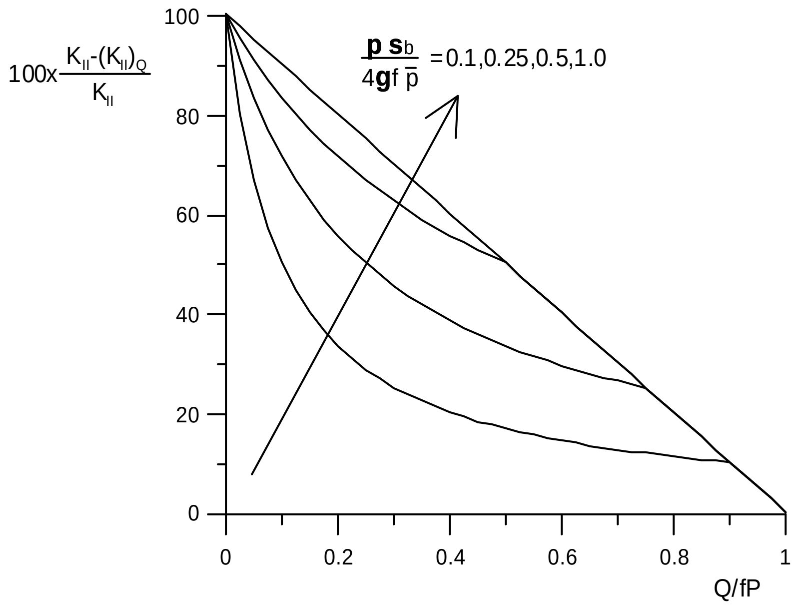


Figure 10: Correction (in %) in the mode II SIF with respect to the original CA model

2.4 Initiation angle

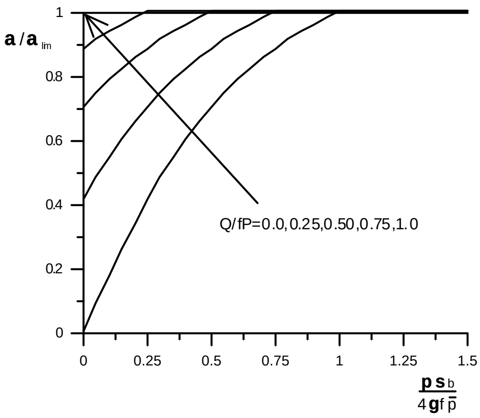
According to the CA model, to determine the inclination direction of the ...rst propagation phase, we consider that K_I is ...xed while K_{II} depends on the load case and is an oscillating term. The CA model suggest to impose the maximum SIF value to ...nd the angle with the x axis, Á in ...gure 1b, (® is the complementary Á to ¼). In the partial slip condition, clearly the ratio $K_{II}=K_I$ is constant and is equal to the friction coe¢cient f, whereas it is variable in the situation of complete stick. In the latter case we have to impose k_2 to be zero. In either condition, we compute the predicted initiation angle as

$$\frac{\sin\frac{Á}{2}+\sin\frac{3Á}{2}}{\cos\frac{Á}{2}+3\cos\frac{3Á}{2}} = ¡\frac{K_{II}}{K_I} \tag{20}$$

In the case of complete stick, and $¾_b \cdot \frac{4}{¼}°f\bar{p}(1 ¡ Q=fP)$

$$\frac{\sin\frac{Á}{2}+\sin\frac{3Á}{2}}{\cos\frac{Á}{2}+3\cos\frac{3Á}{2}} = \frac{¼(¾_b)_{max}}{4°\bar{p}} + \frac{Q_{max}}{P} \tag{21}$$

whereas for larger bulk loads, we have to impose that the second term is equal to the limit value f . In the ...gure 11 the initiation direction is shown as a ratio with ®$_{lim}$; the limit value depending on friction coe¢-cient f which is in turn plotted in the …gure 12.



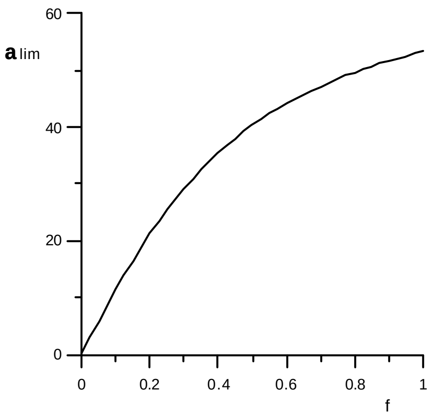


Figure 11: initiation direction of crack in the trailing edge

Figure 12: limit initiation direction

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